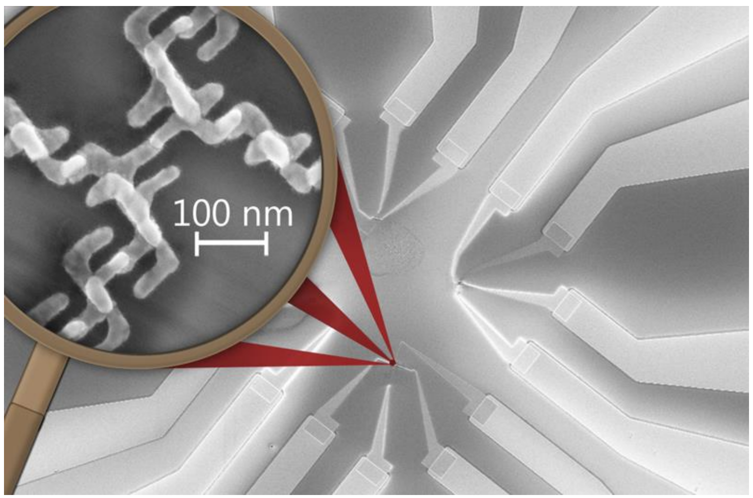
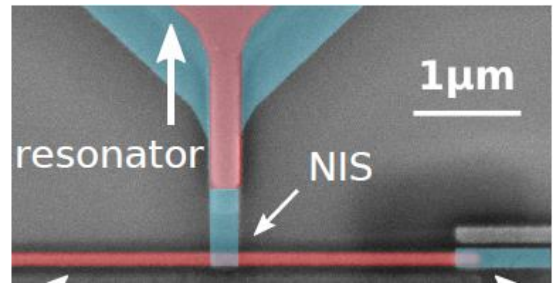
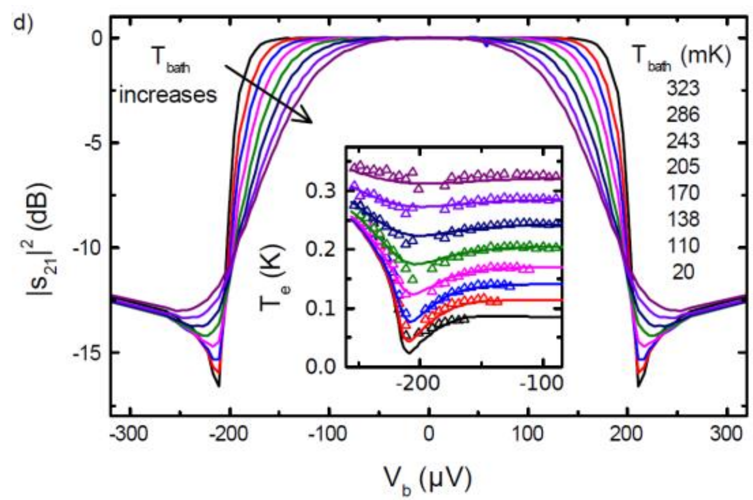
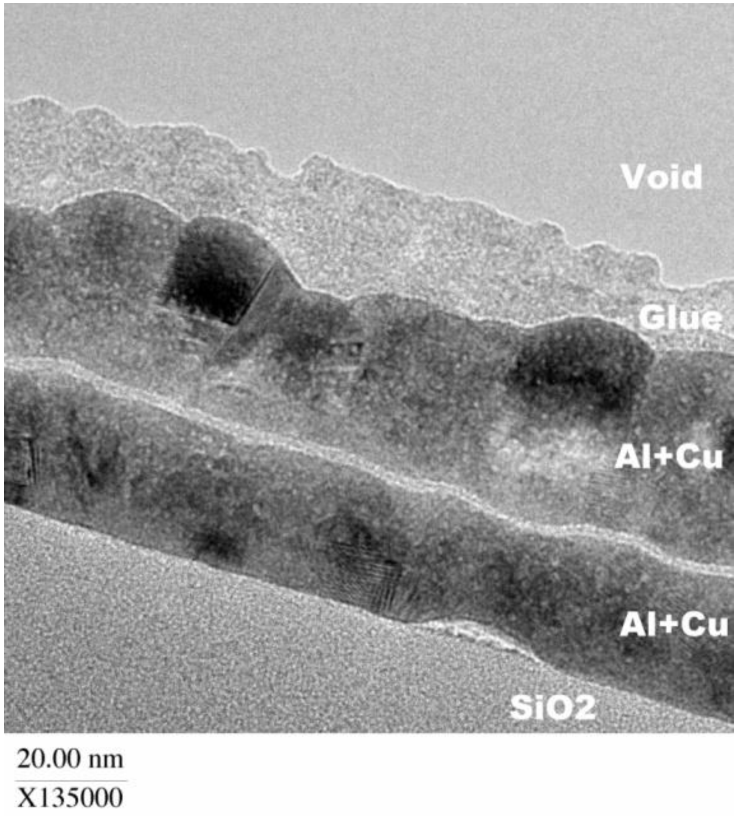
LECTURE 2: Thermometry

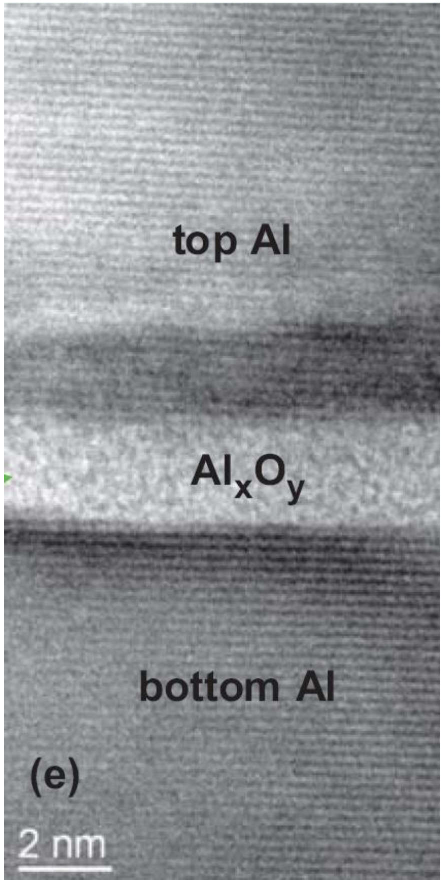






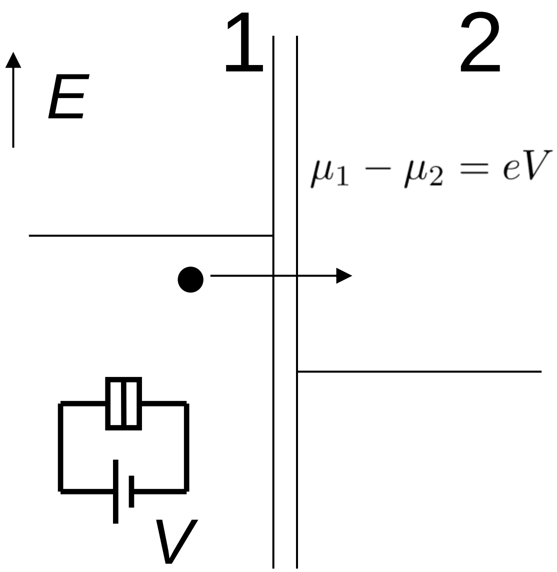
Tunnel barrier





Examples of aluminium-oxide tunnel barriers

Basics of tunnel junctions



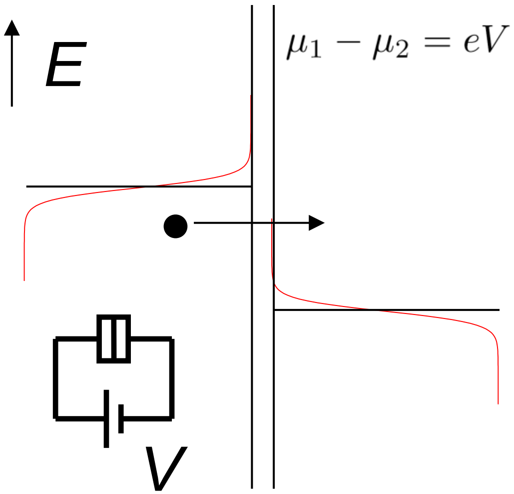
Tunneling from occupied states to empty states

$$I_{1\to 2}(V) = \int \mathcal{T}^2 e N_1(E-eV) f_1(E-eV) N_2(E)[1-f_2(E)]dE$$

$$I_{2\to 1}(V) = \int \mathcal{T}^2 e N_2(E) f_2(E) N_1(E-eV)[1-f_1(E-eV)]dE$$

$$\boxed{I(V) = \mathcal{T}^2 \int e N_1(E-eV) N_2(E)[f_1(E-eV) - f_2(E)]dE}$$

Metal – Insulator – Metal (NIN) tunnel junction



$$I(V) = \mathcal{T}^2 \int eN_1(E-eV)N_2(E)[f_1(E-eV) - f_2(E)]dE$$

Now, density of states (DOS) is almost constant over the small energy interval:

$$I(V) = e\mathcal{T}^2 N_1(0)N_2(0) \int [f_1(E-eV) - f_2(E)]dE$$

Quite generally:

$$\int [f_1(E-eV) - f_2(E)]dE = eV$$

$$\boxed{I = V/R_{\mathrm{T}}}$$

Ohmic, no temperature dependence

$$R_{\mathrm{T}} = [e^2\mathcal{T}^2 N_1(0)N_2(0)]^{-1}$$

NOT A THERMOMETER!

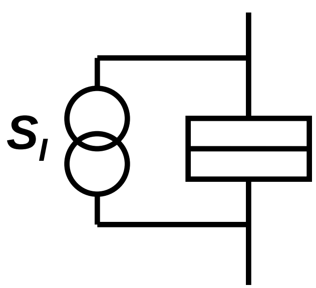
Thermal Johnson-Nyquist noise?

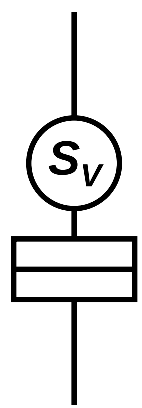
Average current through a basic tunnel junction is not sensing *T*. How about fluctuations around the mean current?

Equilibrium noise: $I_{\mathrm{ave}} = 0$

Tunnel junction behaves like a resistor

$$S_I = 4k_BT/R_T \quad S_V = 4k_BTR_T$$

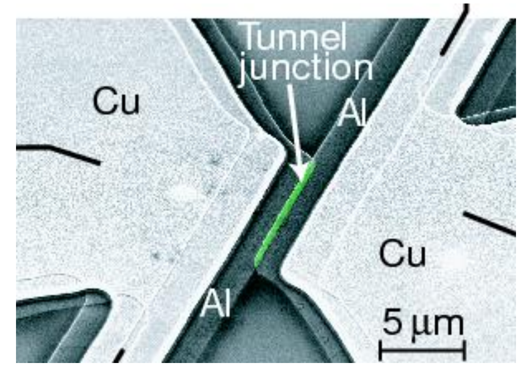




$$\langle \delta V^2 \rangle = \int_{f_{\min}}^{f_{\max}} df\, S_V(f) = 4k_BTR_T\Delta f$$

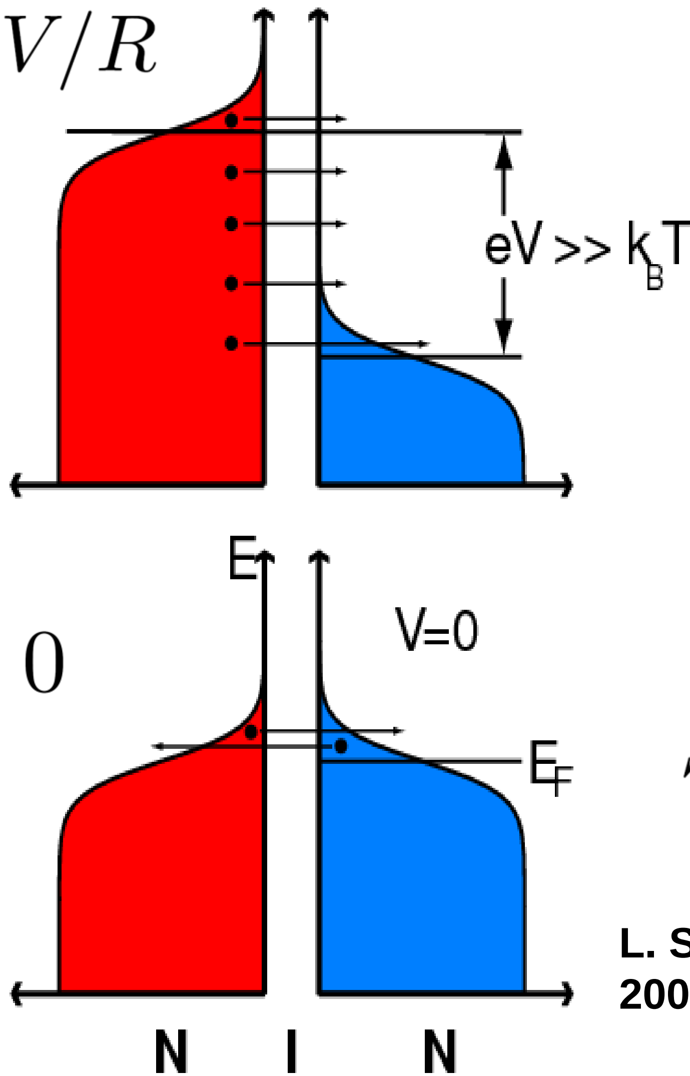
Noise thermometry is also possible and employed with on-chip resistors.

Non-equilibrium: Shot noise thermometry (SNT)



$$I = V/R$$

$$I = 0$$



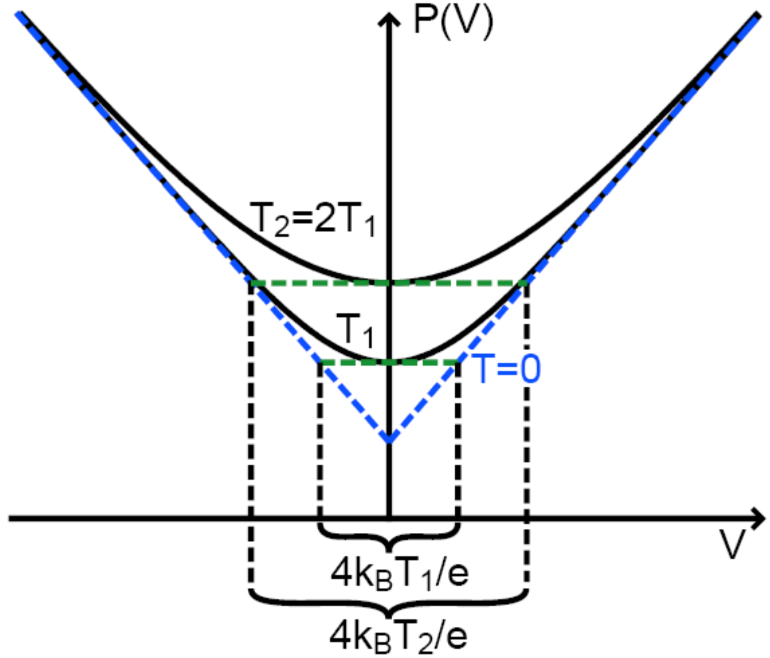
$$S_I \simeq 2eI$$

$$S_I \simeq \frac{4k_B T}{R}$$

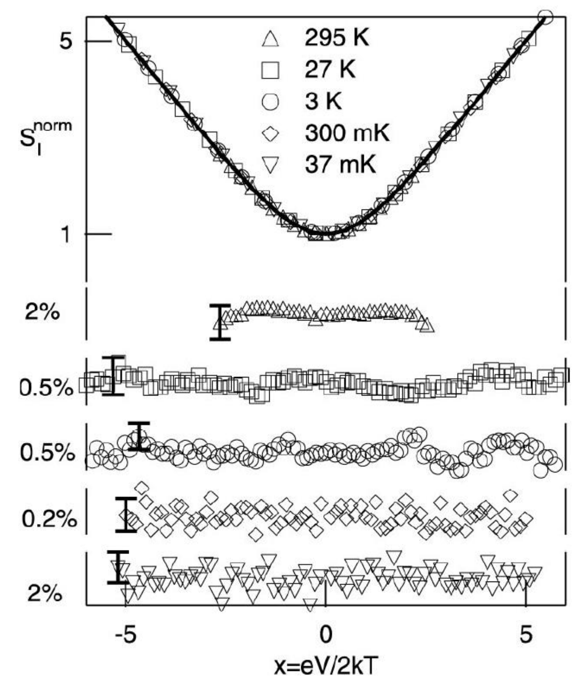
L. Spietz et al., Science 300, 1929 (2003); PhD thesis 2006; APL 89, 183123 (2006).

SNT

$$S_I(V) \simeq \frac{2eV}{R}\coth\left(\frac{eV}{2k_BT}\right)$$

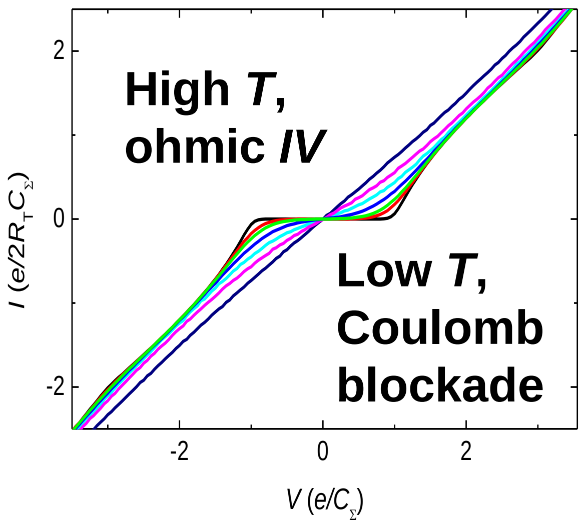


Idea: measure the crossover voltage between thermal noise and shot noise; not necessary to know the absolute calibration for noise measurement.

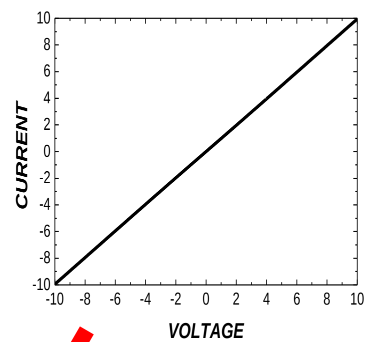


$$P(V) \propto S_I(V)\Delta f$$

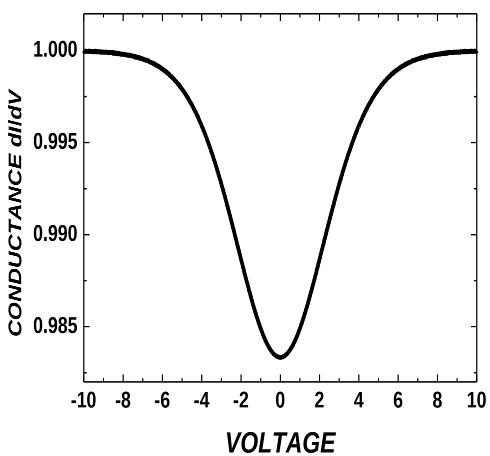
Coulomb Blockade Thermometry – principal idea

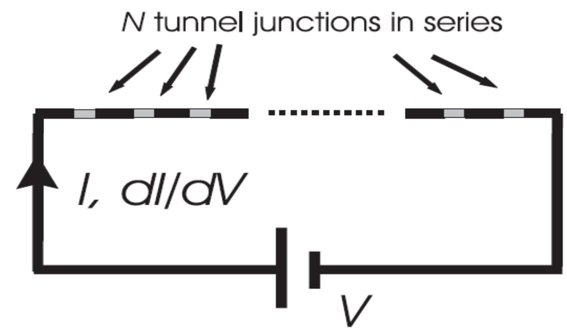


IV curve

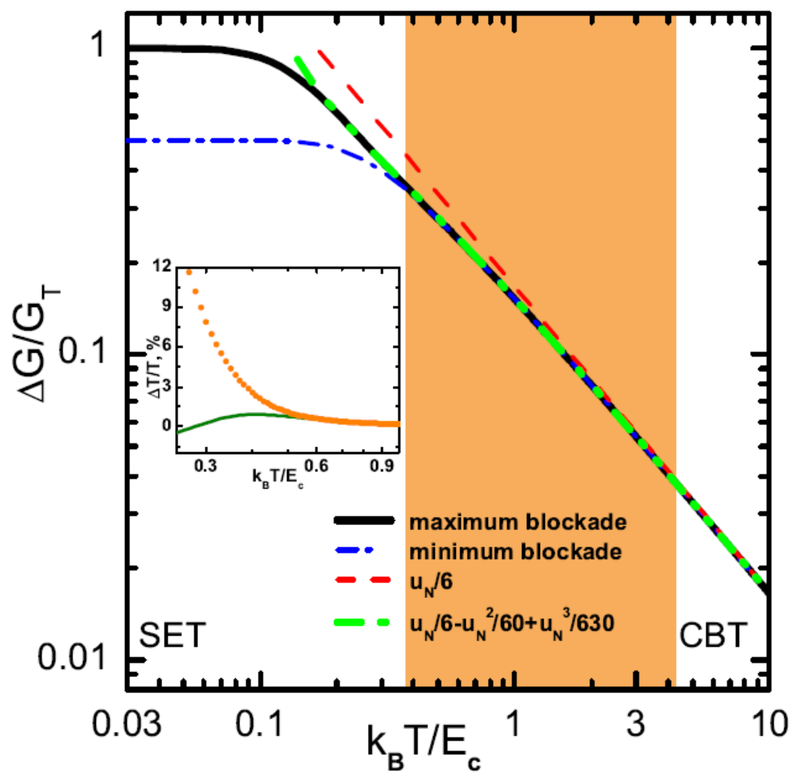


conductance

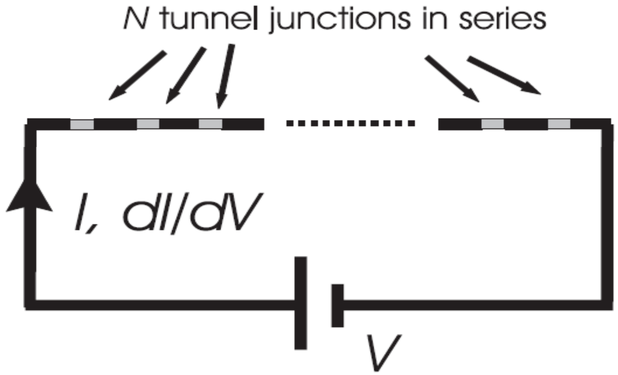




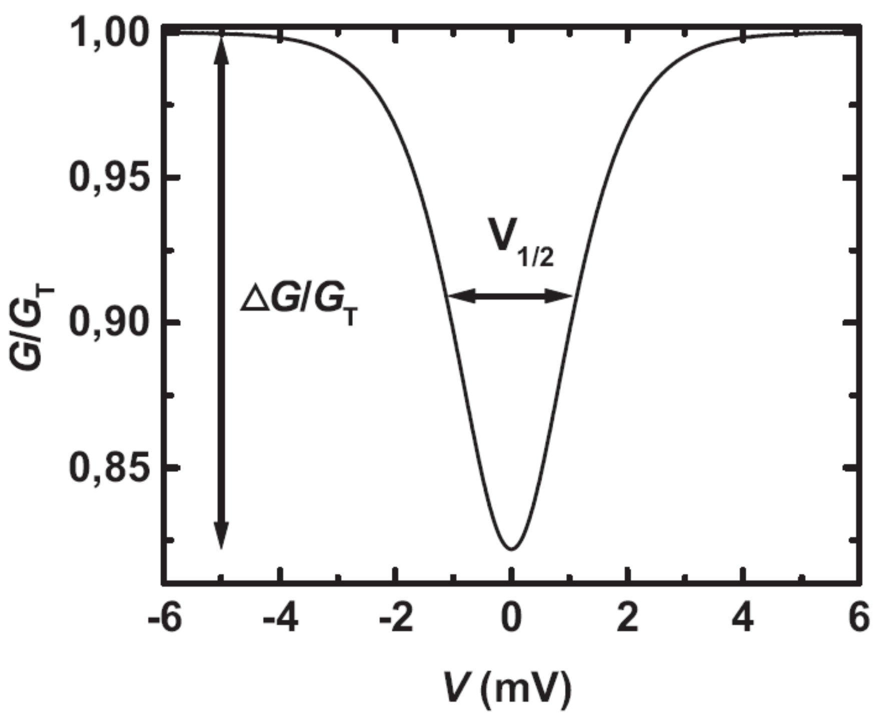
Different temperature regimes



Coulomb blockade thermometer (CBT)



An array of N tunnel junctions in weak Coulomb blockade, $E_C \ll k_B T$



$$V_{1/2} \simeq 5.439 N k_{\mathrm{B}} T / e$$

primary thermometer

$$\Delta G / G_{\mathrm{T}} = \frac{1}{6} \frac{E_{\mathrm{C}}}{k_{\mathrm{B}} T}$$

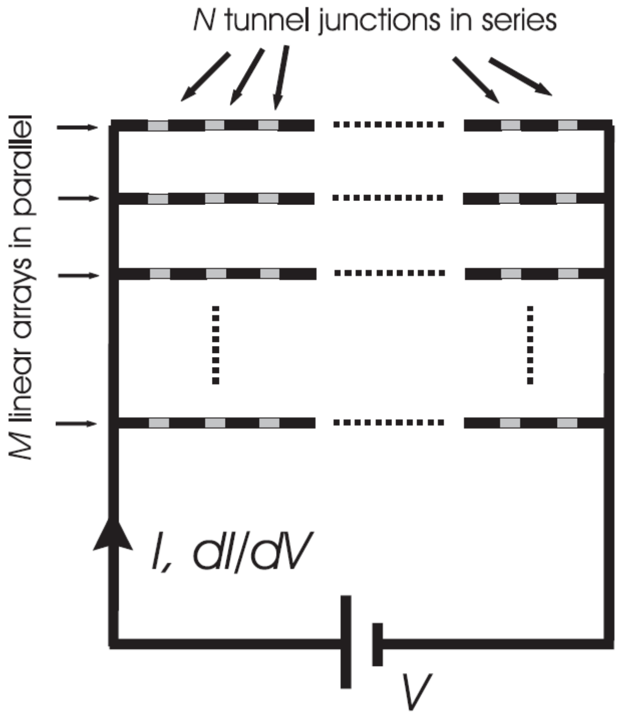
~~secondary thermometer~~

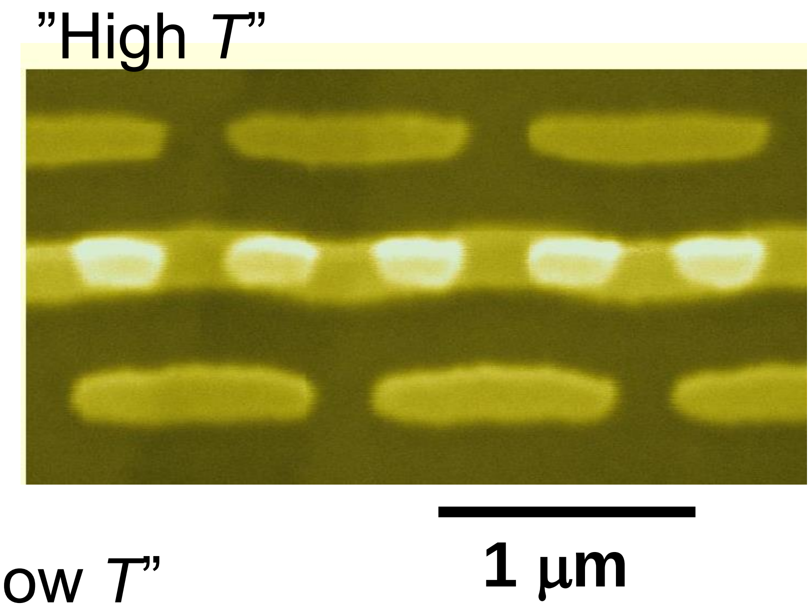
relative primary thermometer

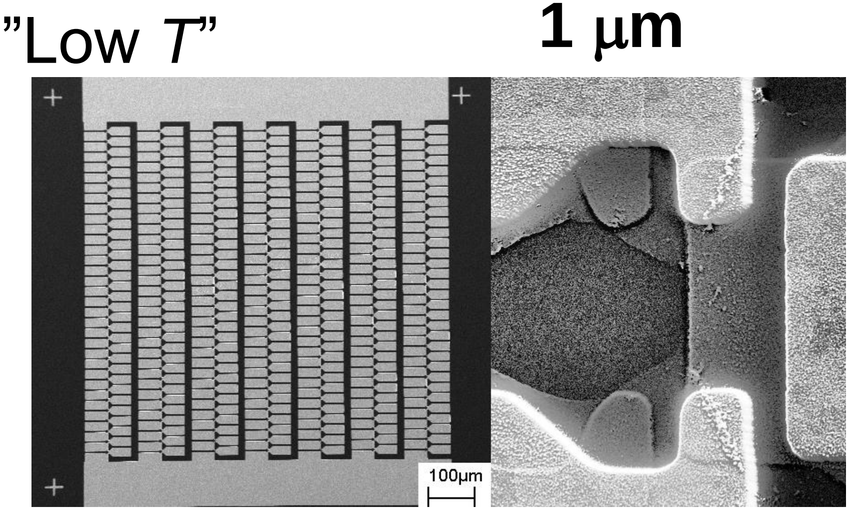
$$E_{\mathrm{C}} \equiv [(N-1)/N]\, e^2 / C$$

JP, K. P. Hirvi, J. P. Kauppinen, and M. A. Paalanen, PRL 73, 2903 (1994).

CBT sensors







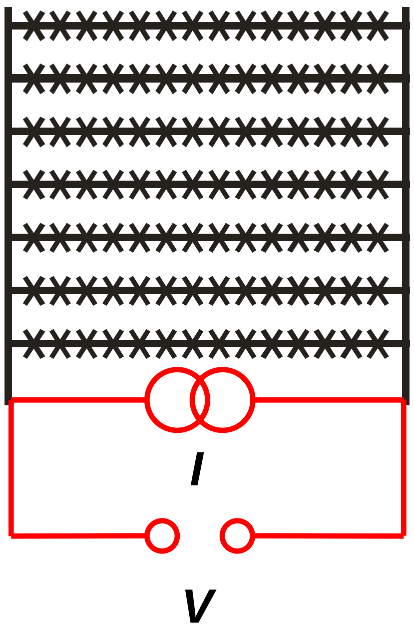
Long arrays suppress efficiently the errors due to co-tunnelling and finite impedance of the electromagnetic environment.

Parallel arrays lower the sensor impedance to a convenient value.

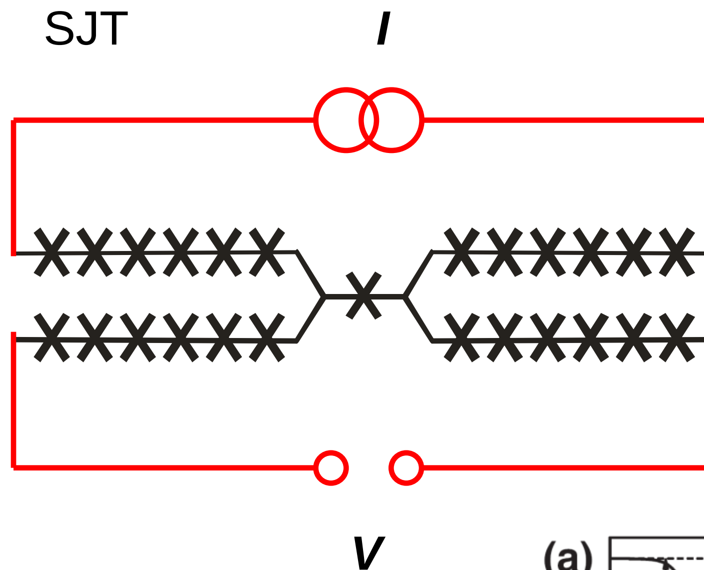
Another configuration is a 2D array, employed by T. Bergsten et al., APL 78, 1264 (2001).

Single junction thermometer (SJT)

CBT

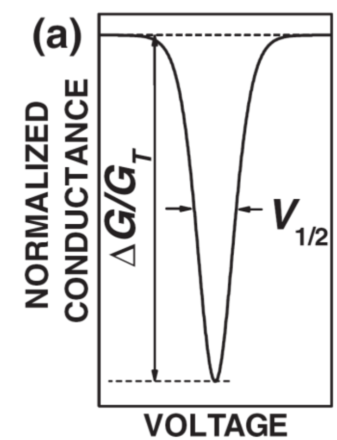


SJT

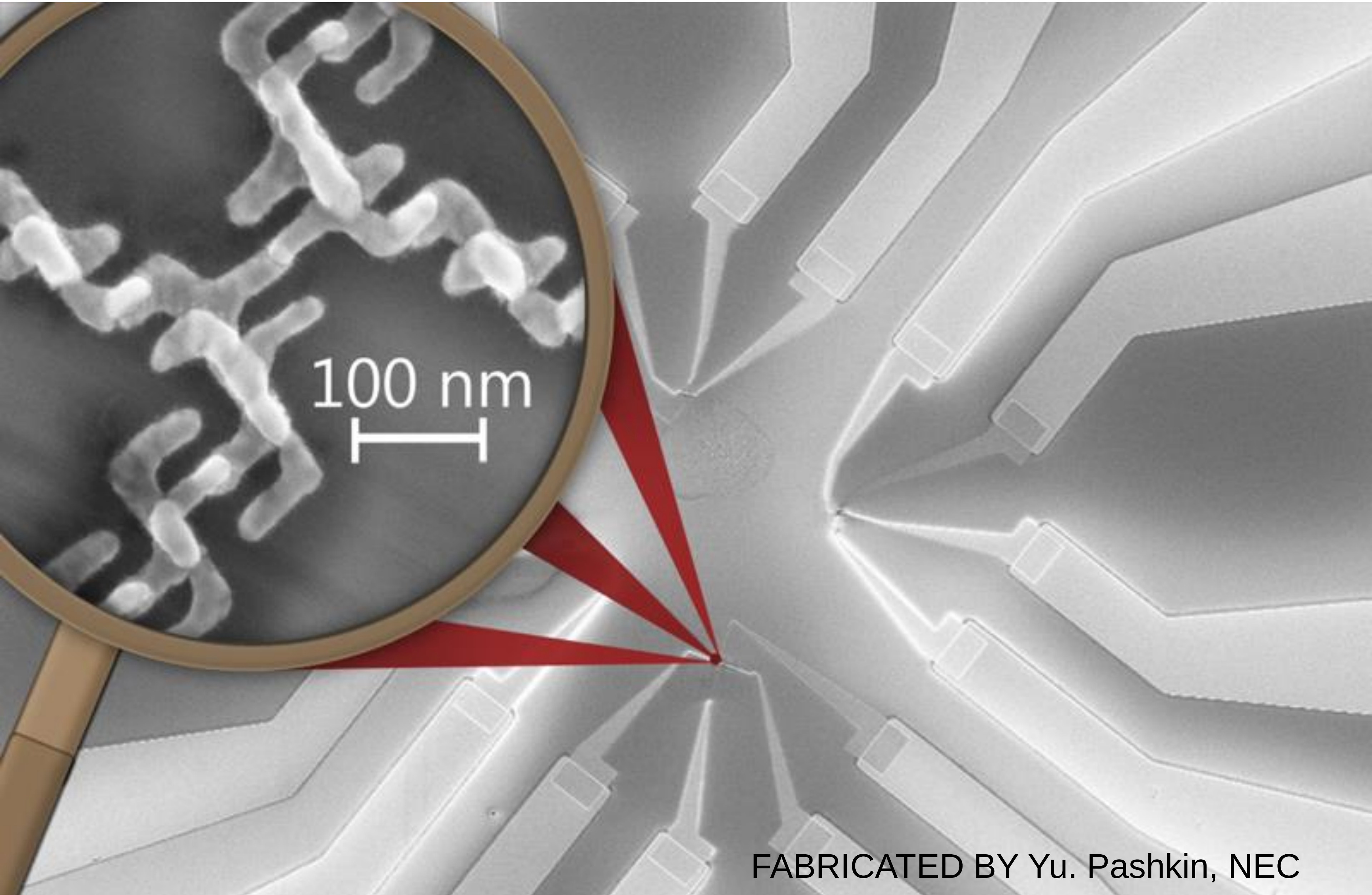


In both cases:

$$V_{1/2} = 5.439 \frac{N k_B T}{e}$$

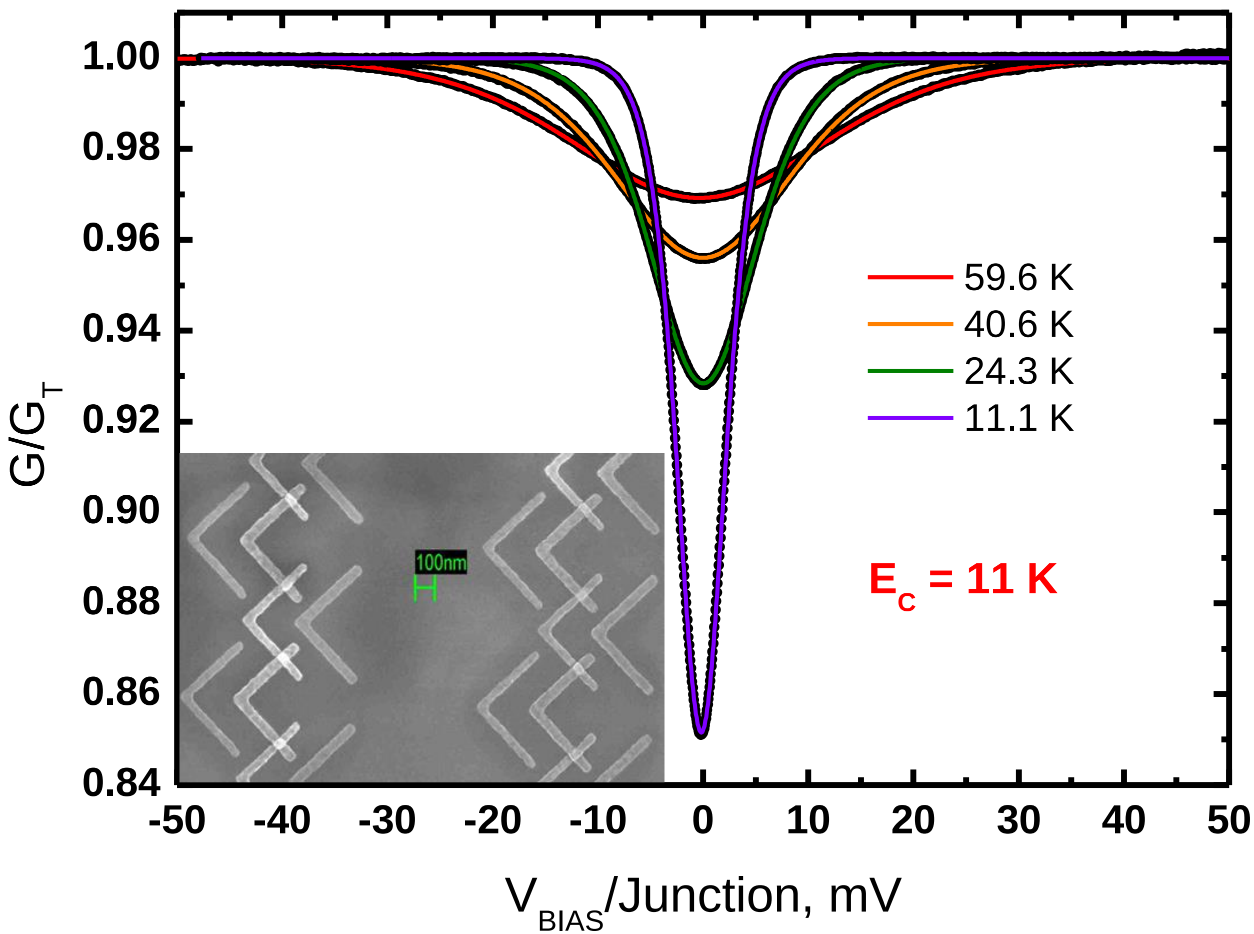


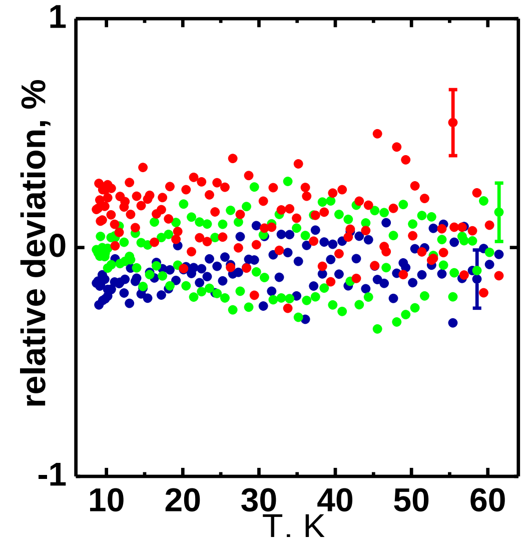
Single Junction Thermometer for *T* > 77 K



CBT optimized for T > 10 K

Fabrication with suspended germanium mask:
Good resolution, high E_C (>10 K)
Homogenity of R_T (~ 5%) for absolute T accuracy of ~ 0.5%





3 sensors agree within 0.2 % (relative deviation)

M. Meschke et al., Phil. Trans. R. Soc. A 374, 20150052 (2016).

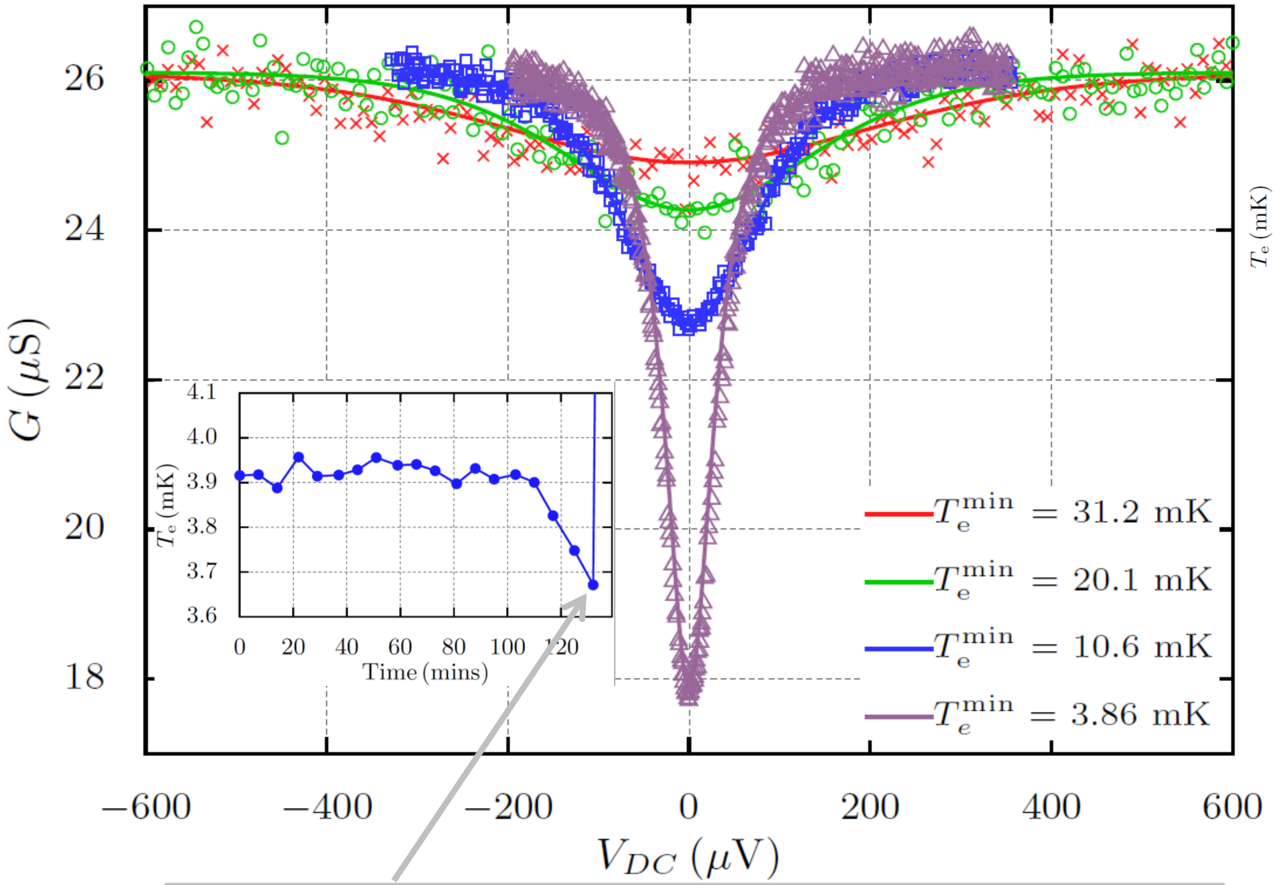
Aalto University

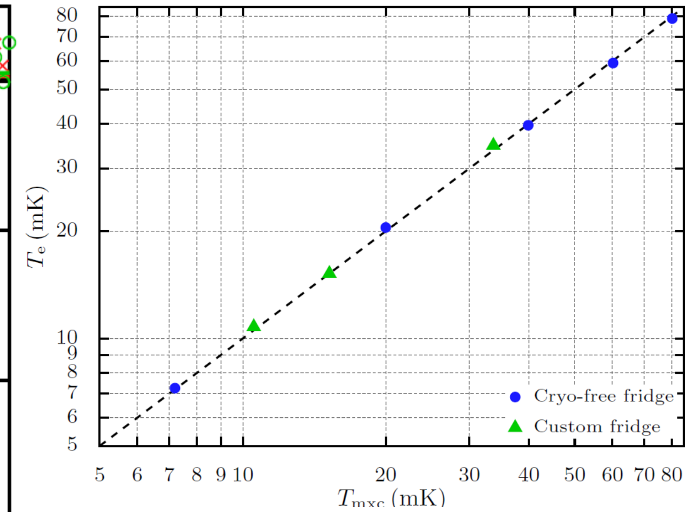
MIKES METROLOGIA

EMRP

EURAMET

CBT optimised for T < 10 mK





J. R. Prance et al., Nature Communications 7, 10455 (2016).

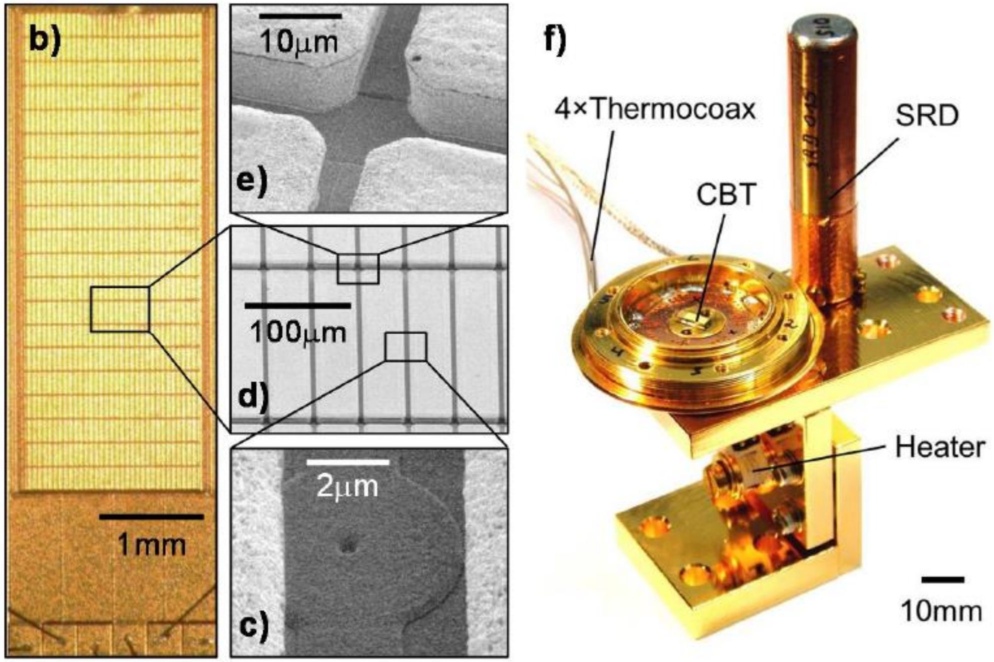
~ 3.7 mK, lowest ever measured by CBT

Lancaster Quantum Technology Centre

aivon

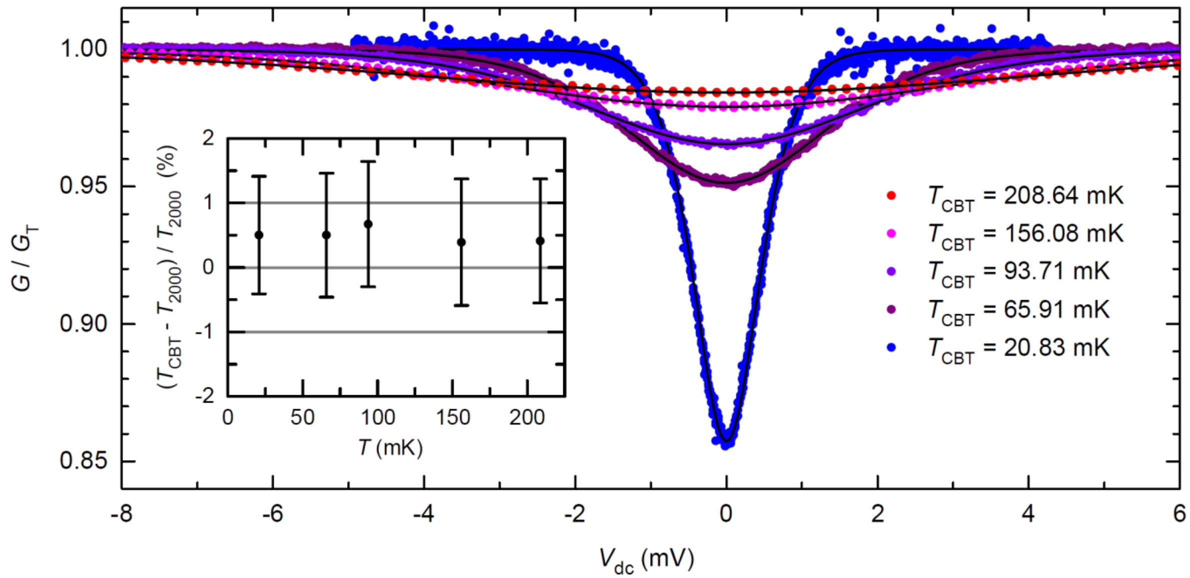
VTT

Traceable CBT at MIKES



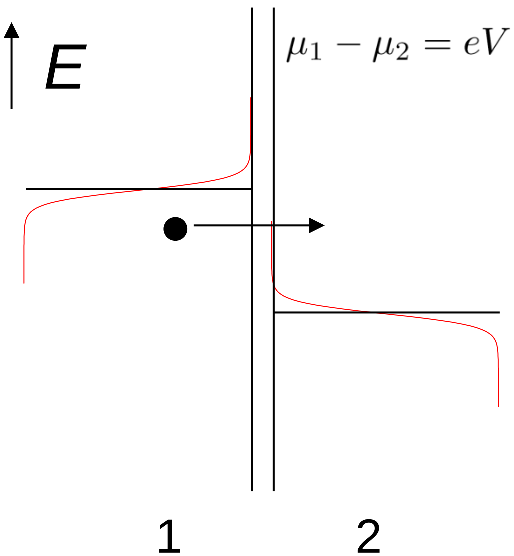
Comparison with the PLTS 2000 temperature scale

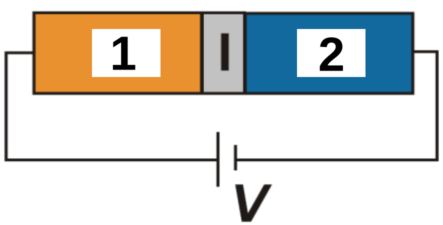
O. Hahtela et al., arXiv:1609.06943



NIS-THERMOMETRY

Basics of transport through a barrier





$$I_{1\to 2}(V) = \int \mathcal{T}^2 e N_1(E-eV) f_1(E-eV) N_2(E)[1-f_2(E)]dE$$

$$I_{2\to 1}(V) = \int \mathcal{T}^2 e N_2(E) f_2(E) N_1(E-eV)[1-f_1(E-eV)]dE$$

$$\boxed{I(V) = \mathcal{T}^2 \int e N_1(E-eV) N_2(E)[f_1(E-eV) - f_2(E)]dE}$$

For **normal conductors (NIN junction)**, density of states (DOS) is almost constant over the small energy interval:

$$I(V) = e\mathcal{T}^2 N_1(0) N_2(0) \int [f_1(E-eV) - f_2(E)]dE$$

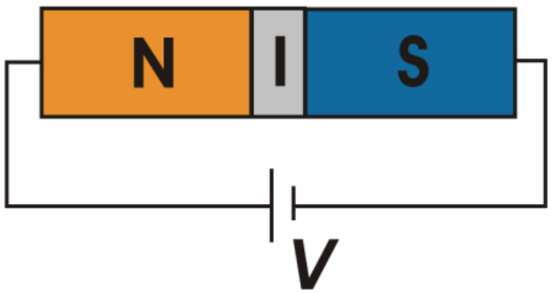
Quite generally: $\int [f_1(E-eV) - f_2(E)]dE = eV$

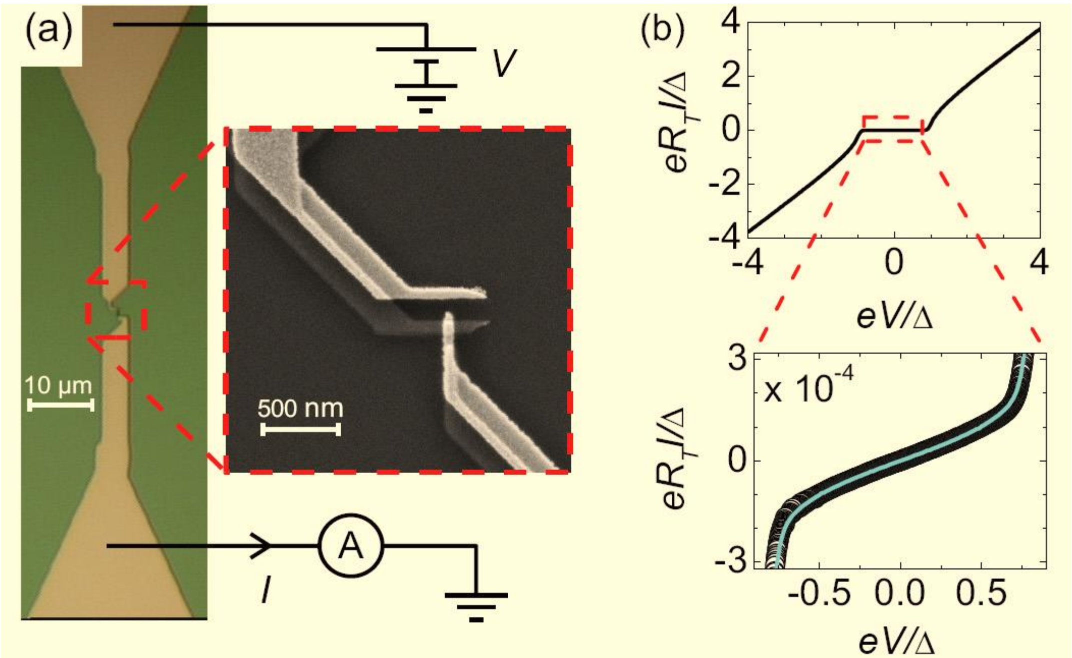
$$I = V/R_{\mathrm{T}}$$

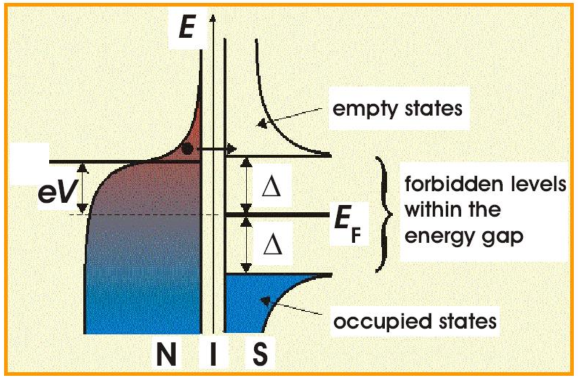
Ohmic, no temperature dependence

$$R_{\mathrm{T}} = [e^2 \mathcal{T}^2 N_1(0) N_2(0)]^{-1}$$

Normal – Insulator – Superconductor (NIS) junction



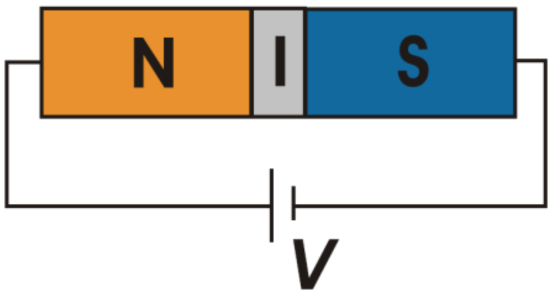


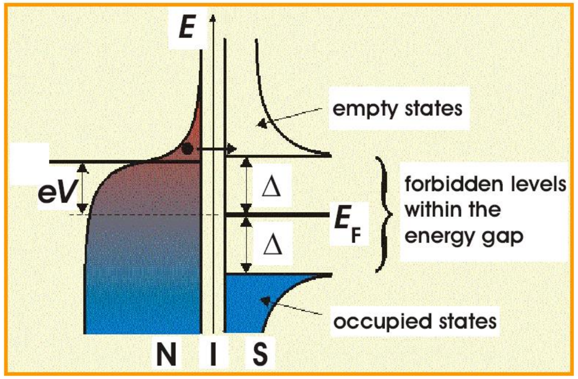


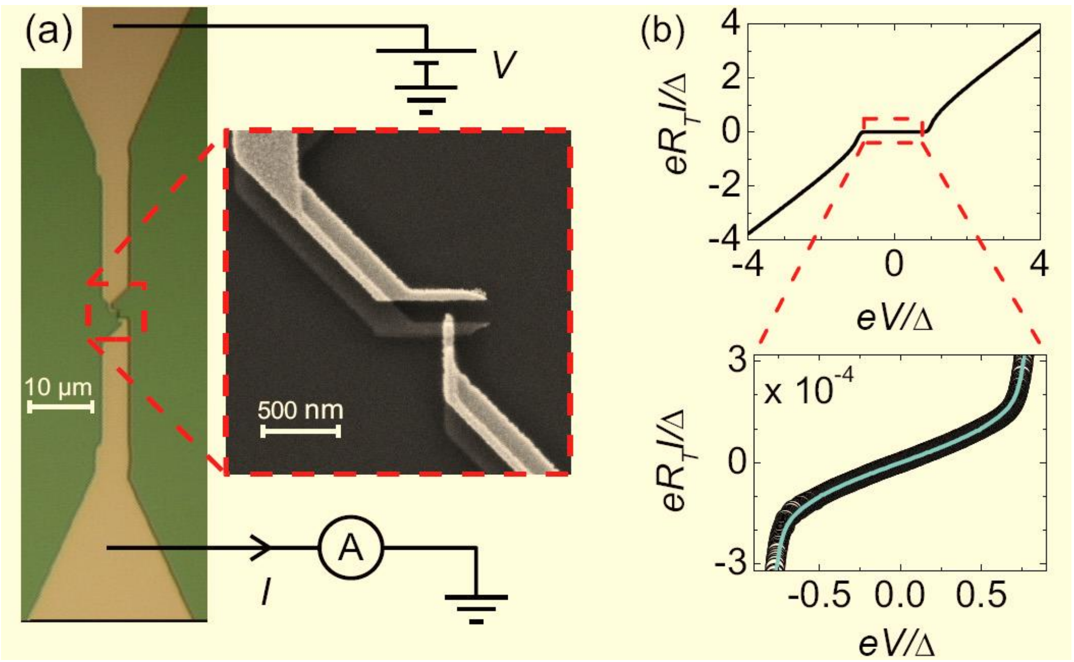
Current through an ideal NIS-tunnel junction with low-transparency

$$I = \frac{1}{eR_T} \int n_S(E)[f_N(E - eV) - f_S(E)]dE$$

Normal – Insulator – Superconductor (NIS) junction







Current through an ideal NIS-tunnel junction with low-transparency

$$I = \frac{1}{eR_T}\int n_S(E)[f_N(E-eV) - f_S(E)]dE$$

$$= \frac{1}{2eR_T}\int n_S(E)[f_N(E-eV) - f_N(E+eV)]dE$$

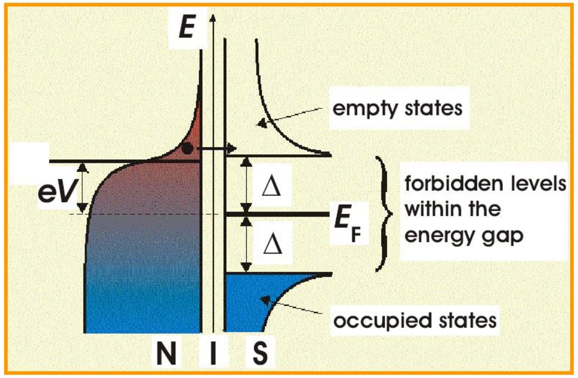
Thermometry by a hybrid tunnel junction (NIS)

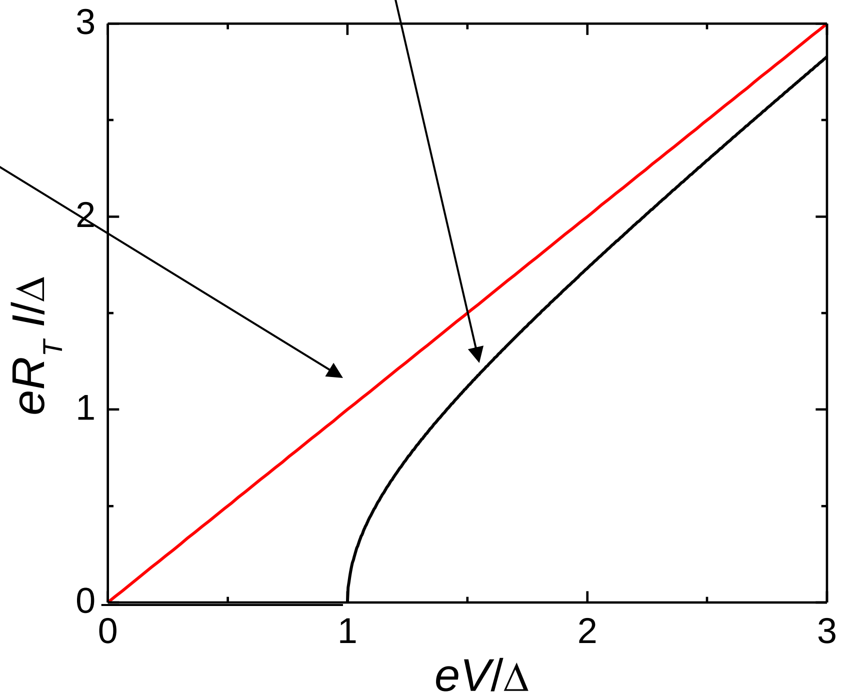
$$I = \frac{1}{eR_T}\int N_S(E)[f_N(E-eV) - f_S(E)]dE = \frac{1}{2eR_T}\int N_S(E)[f_N(E-eV) - f_N(E+eV)]dE$$

$$N_S(E) = \frac{|E|}{\sqrt{E^2-\Delta^2}}$$

$$I = \frac{1}{eR_T}\int_0^{eV}\frac{E}{\sqrt{E^2-\Delta^2}}dE = \frac{1}{R_T}\sqrt{V^2-(\Delta/e)^2},\ T=0$$

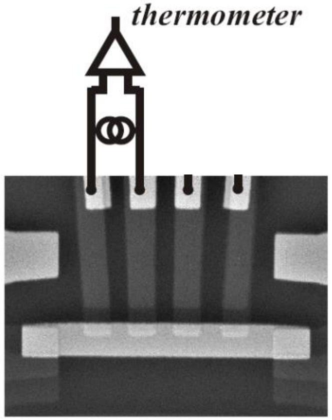
$$I = \frac{V}{R_T},\ T \geq T_C$$



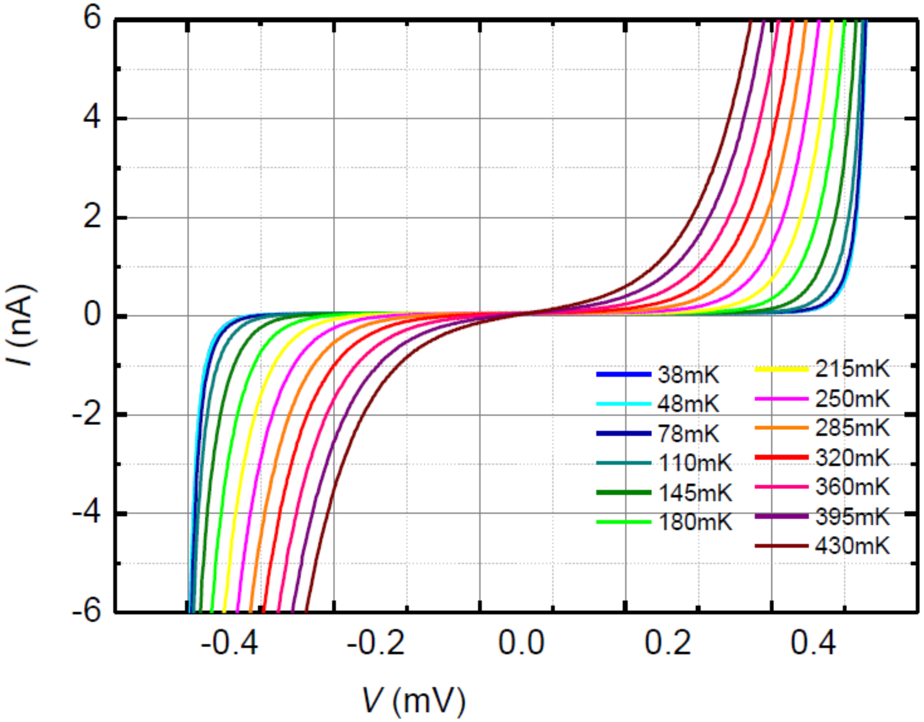


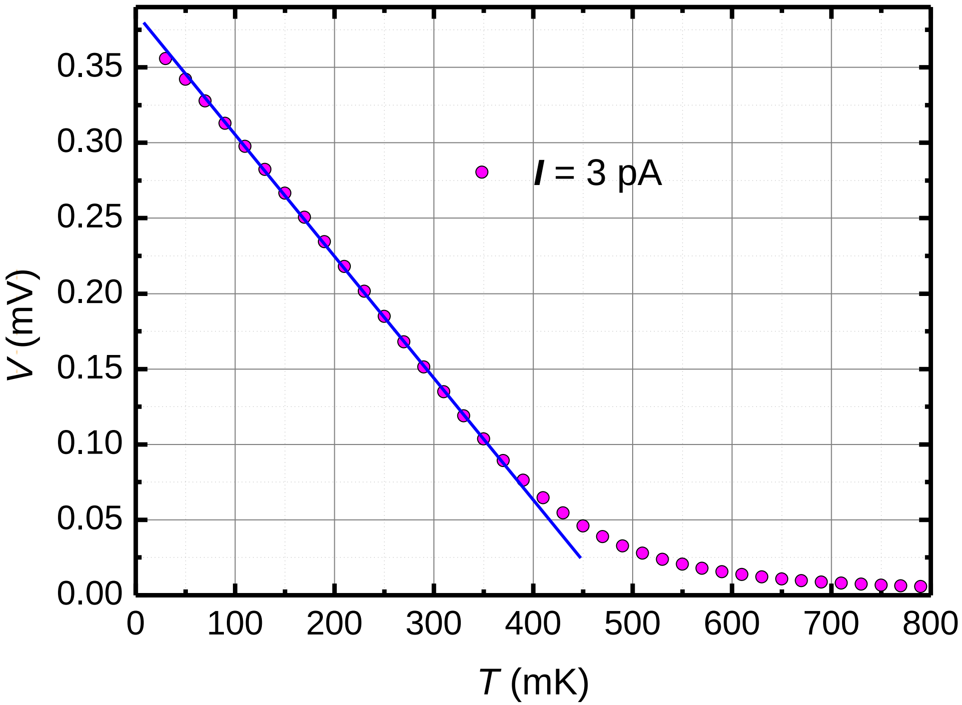
NIS-thermometry

$$I = \frac{1}{2eR_T}\int n_S(E)[f_N(E-eV) - f_N(E+eV)]dE$$

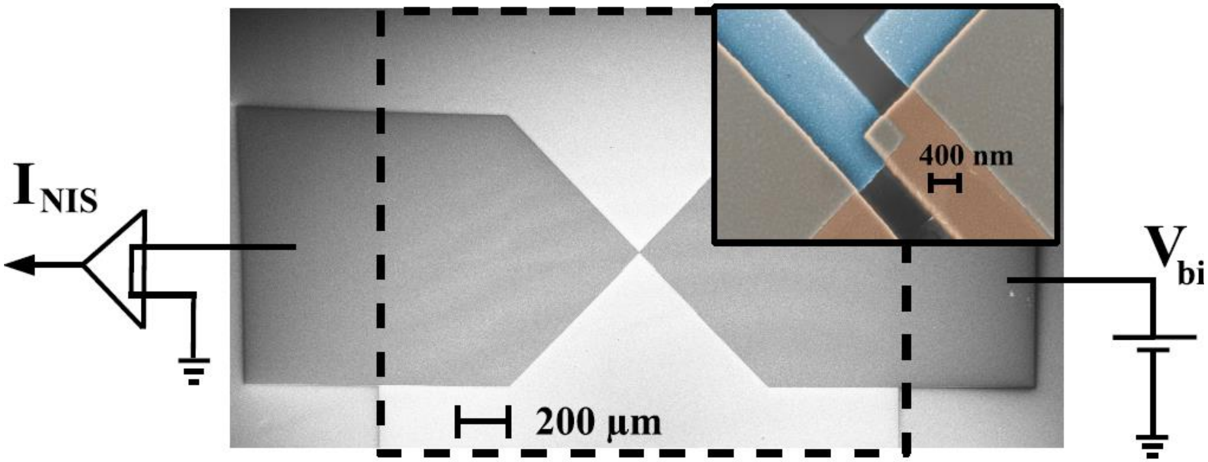


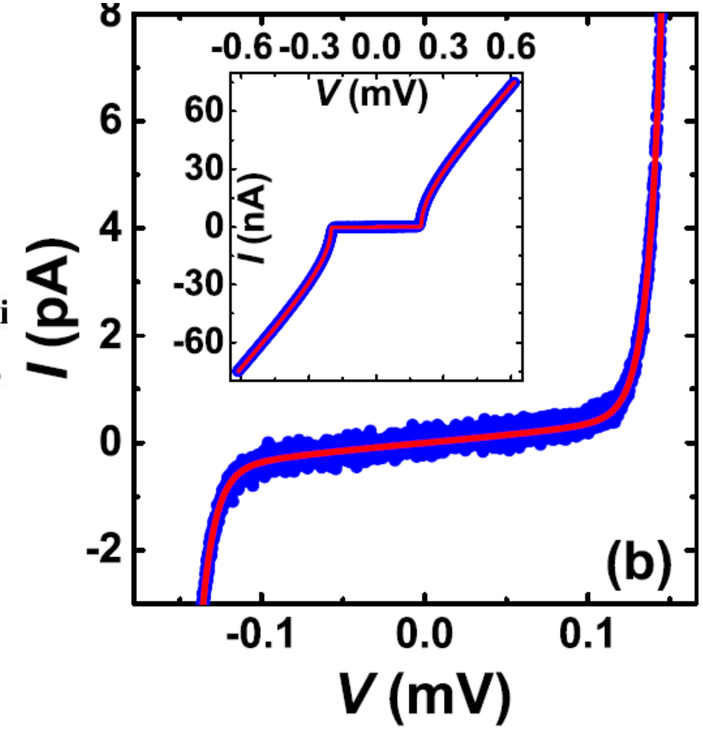
Probes electron temperature of N island (and not of S!)

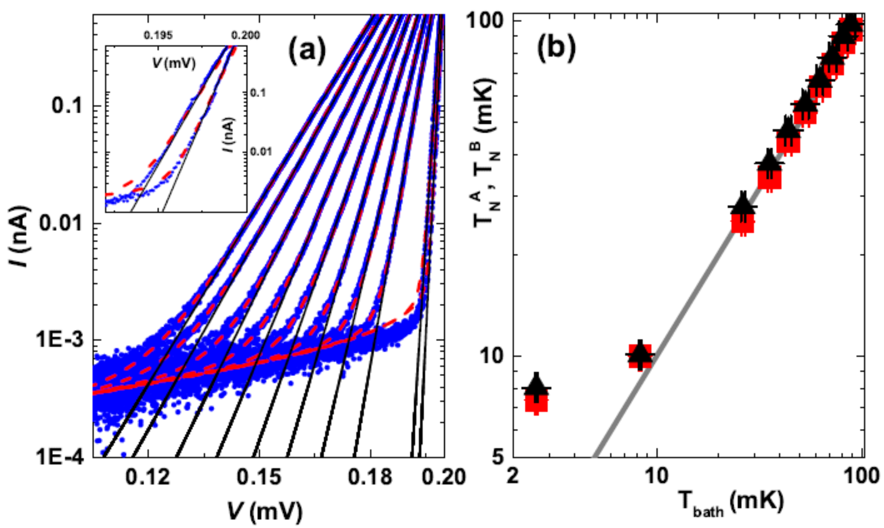




NIS-thermometry at mK temperatures





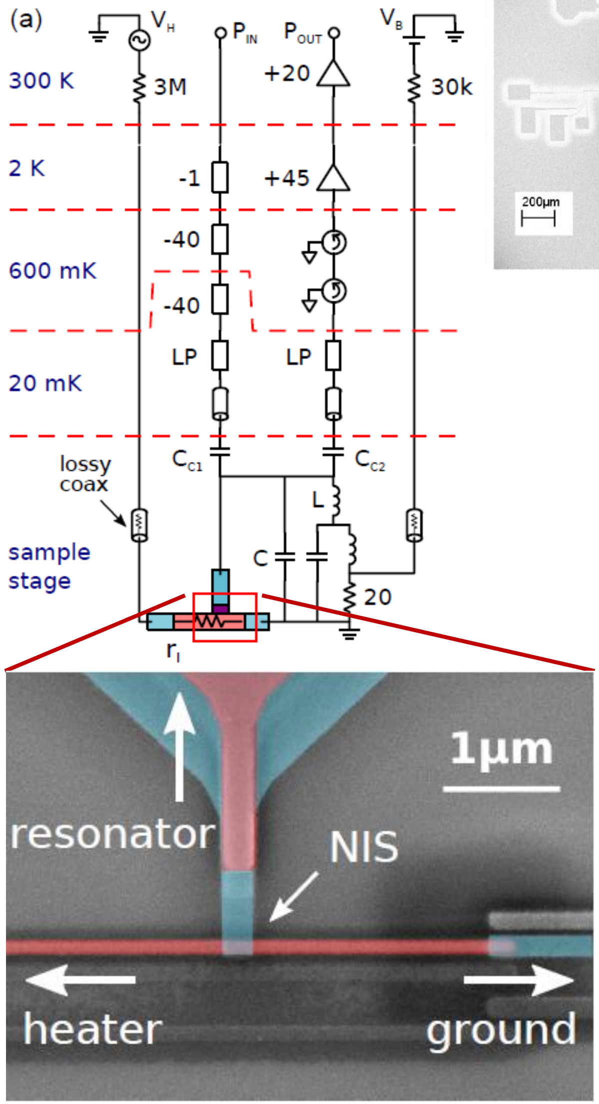


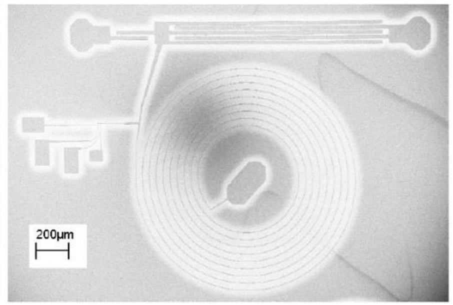
A. V. Feshchenko et al. (Aalto - Basel Univ. collaboration), arXiv:1504.0384

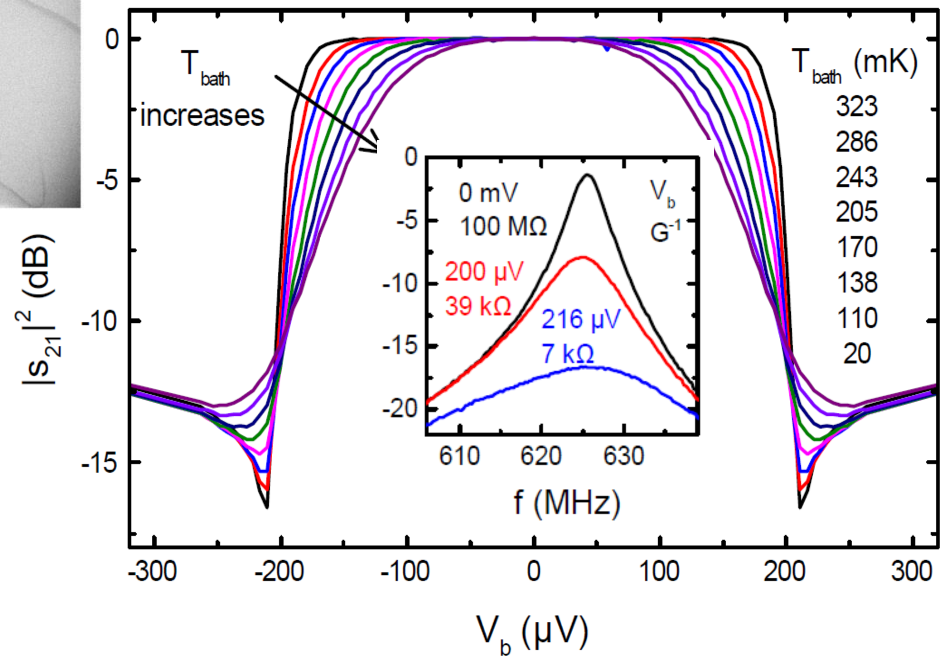
Potential to operate down to 1 mK

Fast NIS thermometry on electrons

Read-out at 600 MHz of a NIS junction, 10 MHz bandwidth

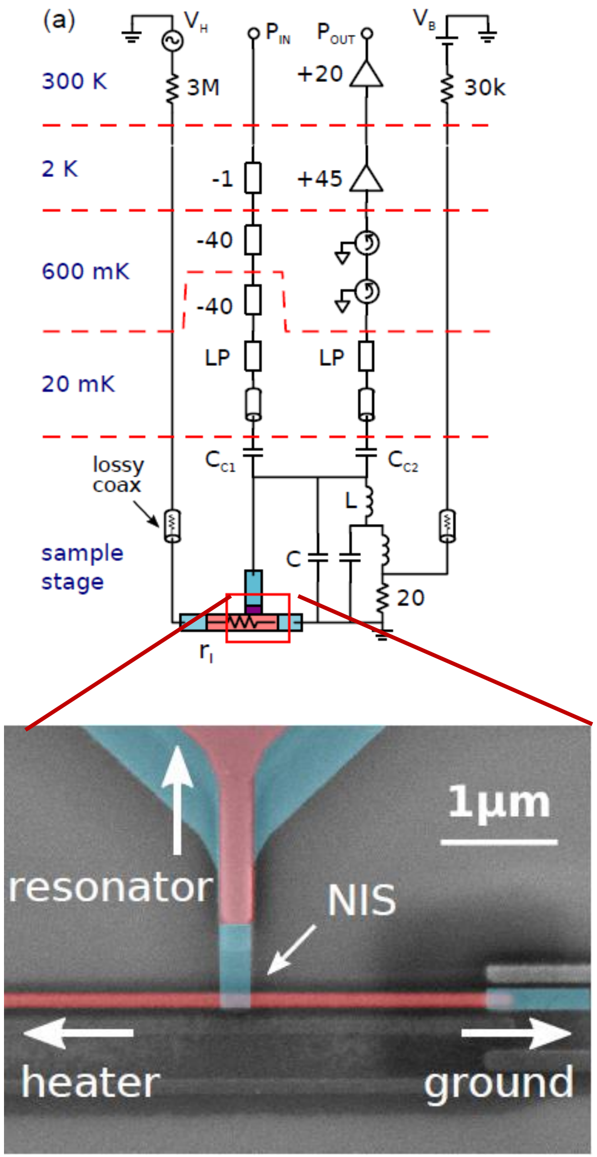




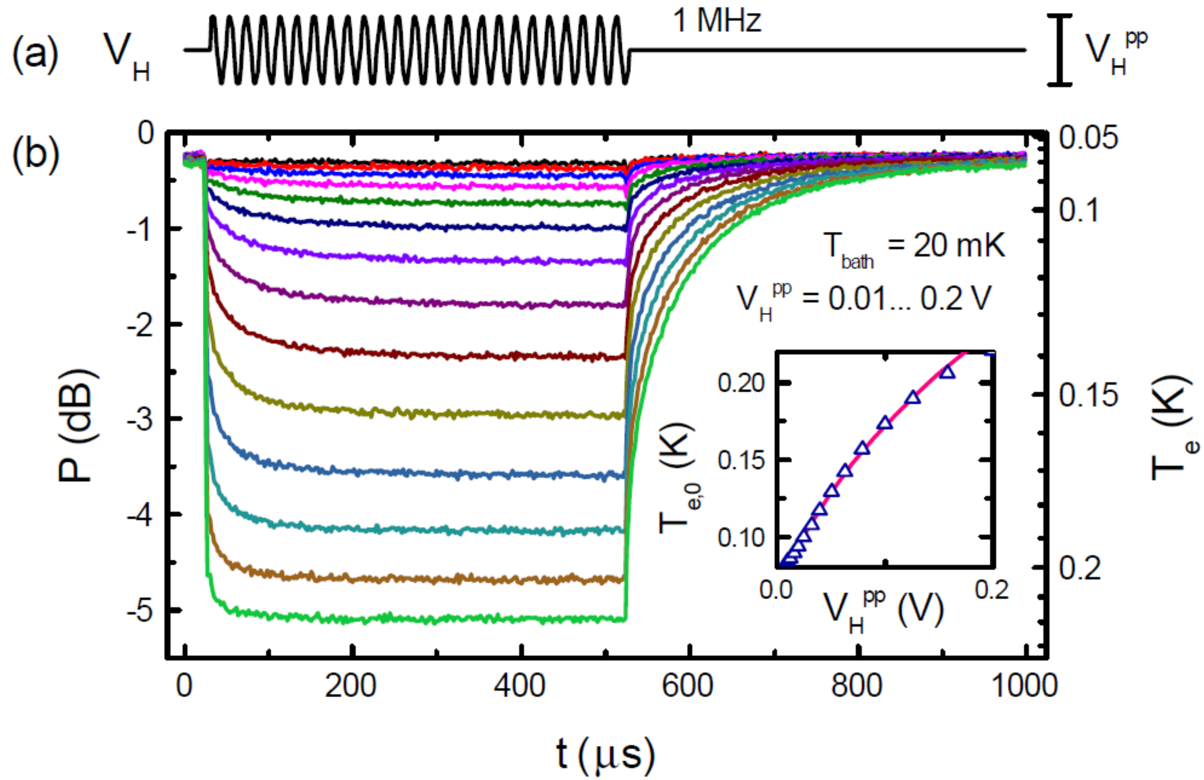


S. Gasparinetti et al., Phys. Rev. Applied 3, 014007 (2015); K. L. Viisanen et al., New J. Phys. 17, 055014 (2015).
Proof of the concept: Schmidt et al., 2003

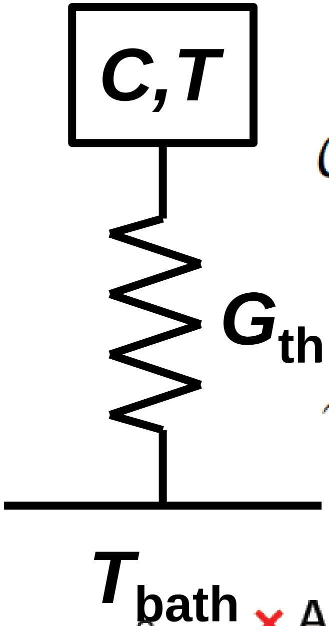
Fast NIS thermometry



Transmission read-out at 600 MHz of a NIS junction

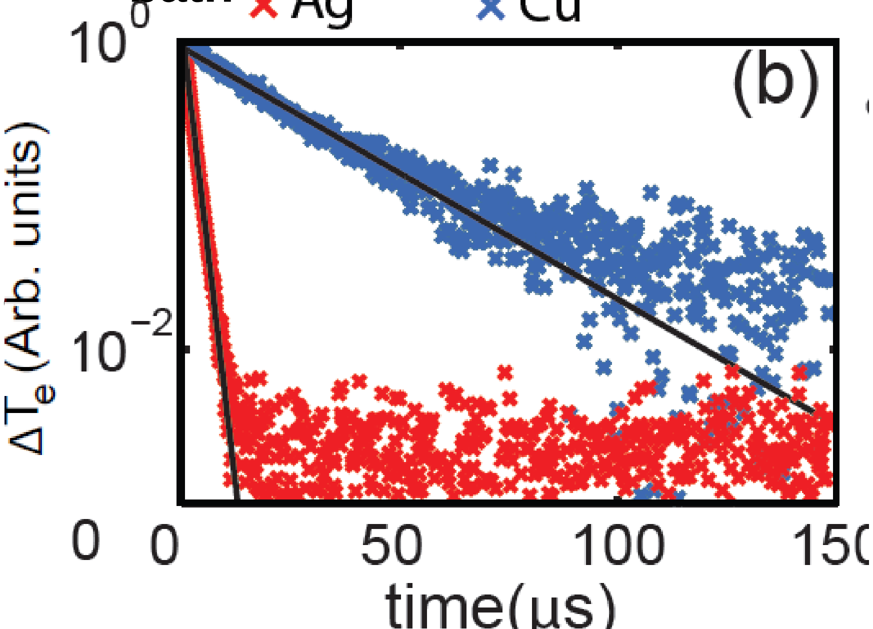


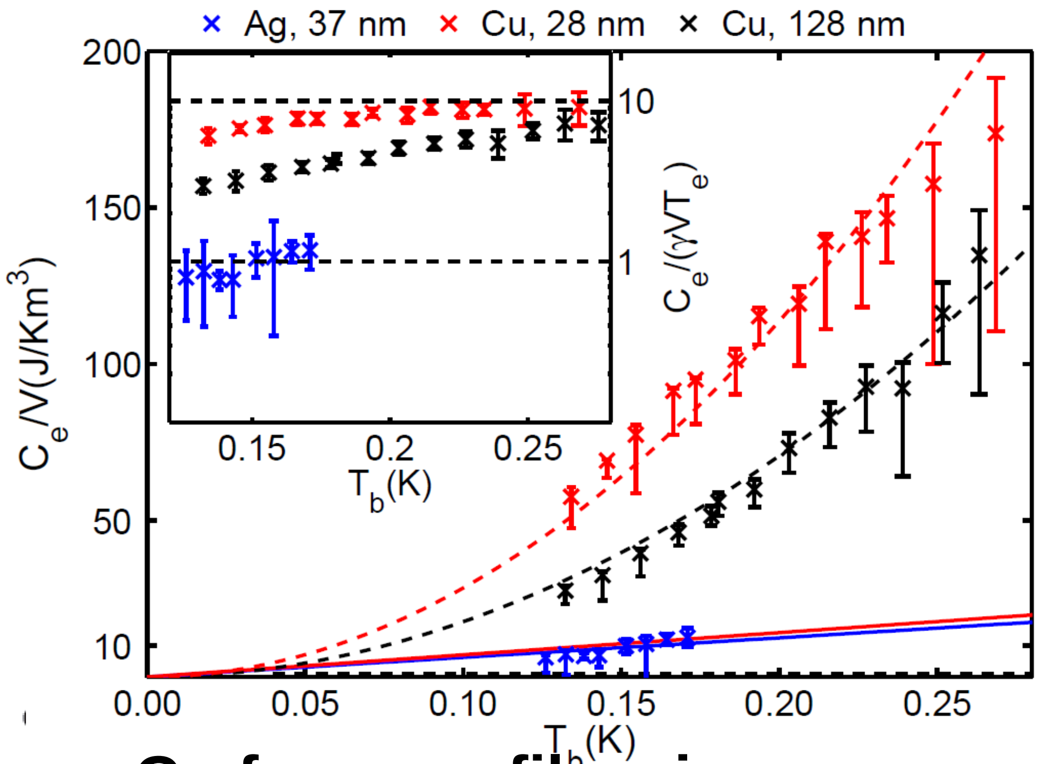
Heat capacity C of metallic films



$$\mathcal{C}\frac{d\delta T}{dt} = -G_{\rm th}\delta T$$

$$\tau = \mathcal{C}/G_{\rm th}$$





C of copper films is anomalously high (x10)

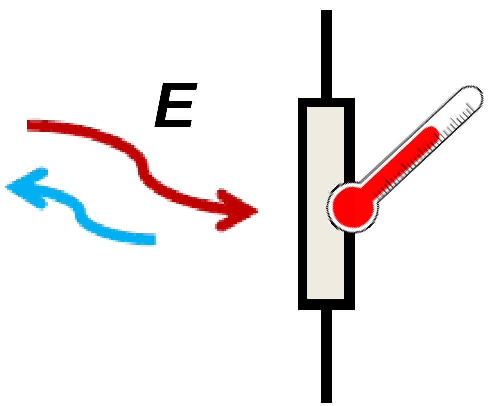
Silver follows free-electron Fermi-gas model

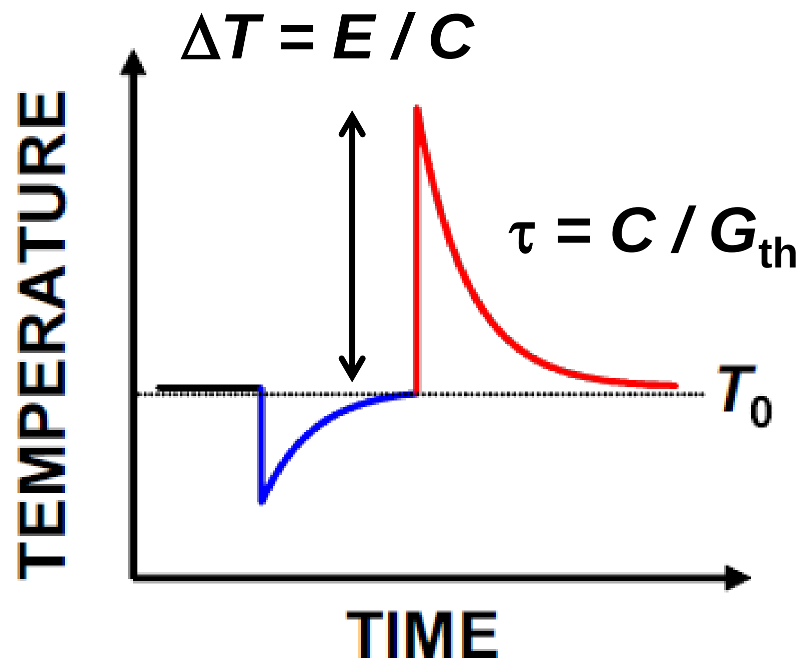
$C = (\pi^2/3)\, N(0) k_B^2 V\, T$

K. L. Viisanen and JP, arXiv:1606.02985

Calorimetry

Aims at measuring single quanta of energy *E* by an absorber with finite heat capacity *C*.



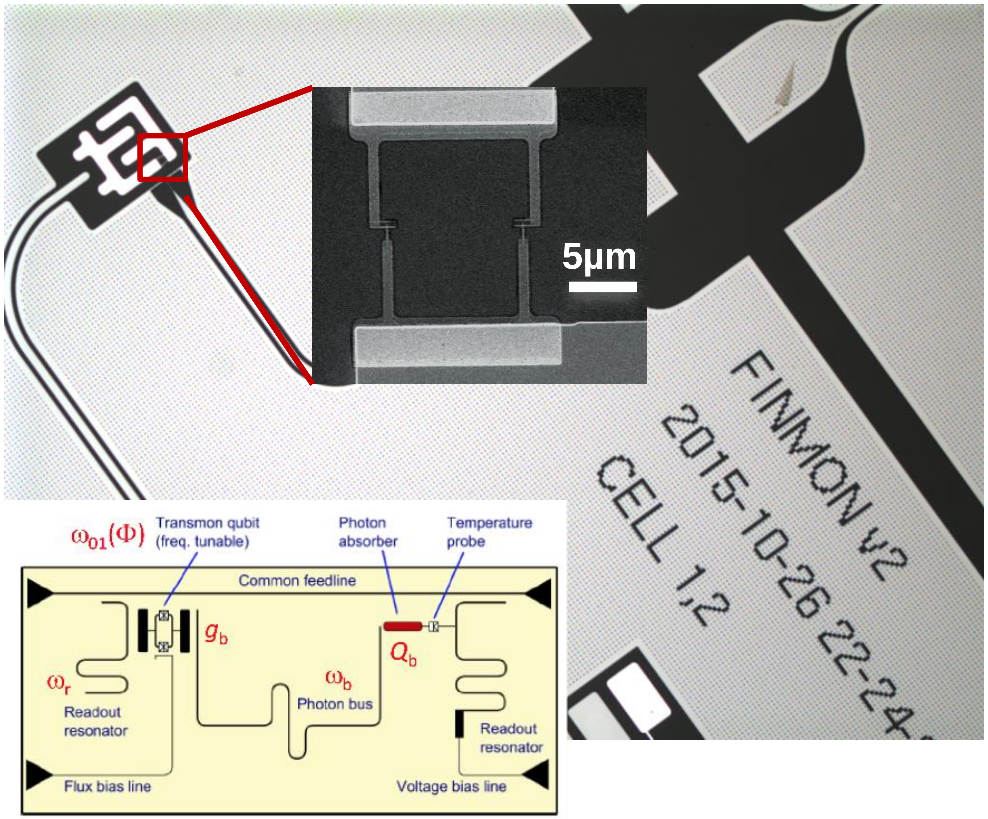


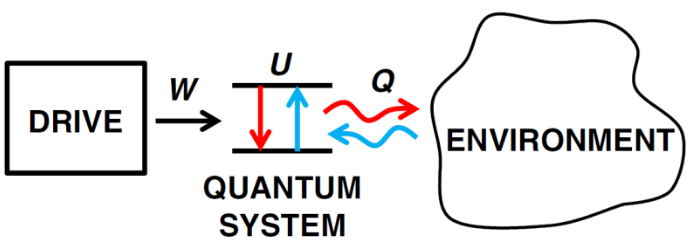
Energy resolution:

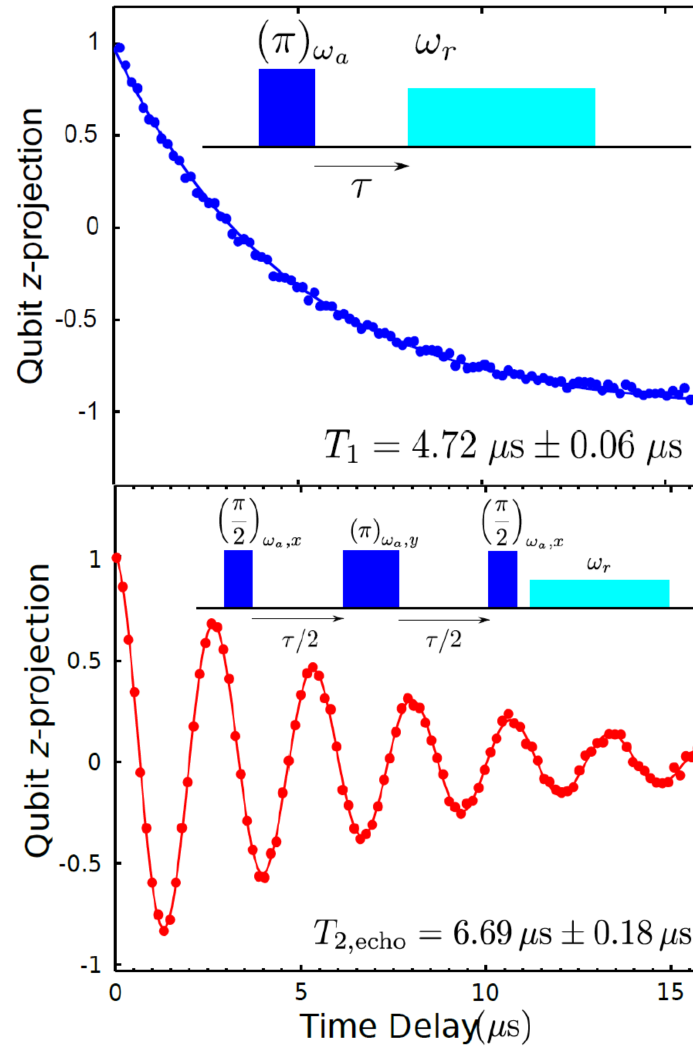
$$\delta E = \mathbb{NET}\ (C\ G_{th})^{1/2}$$

Calorimetry on superconducting qubits

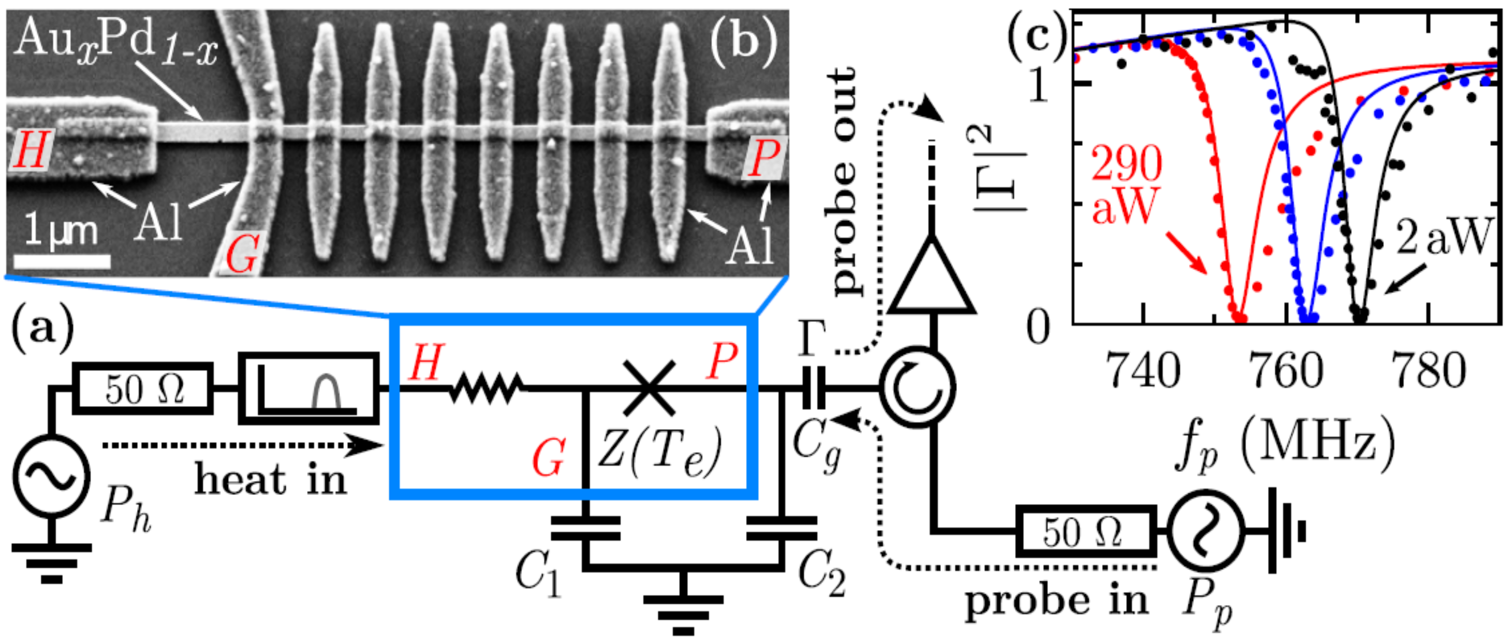
R. George et al., [arXiv:1609.07057]







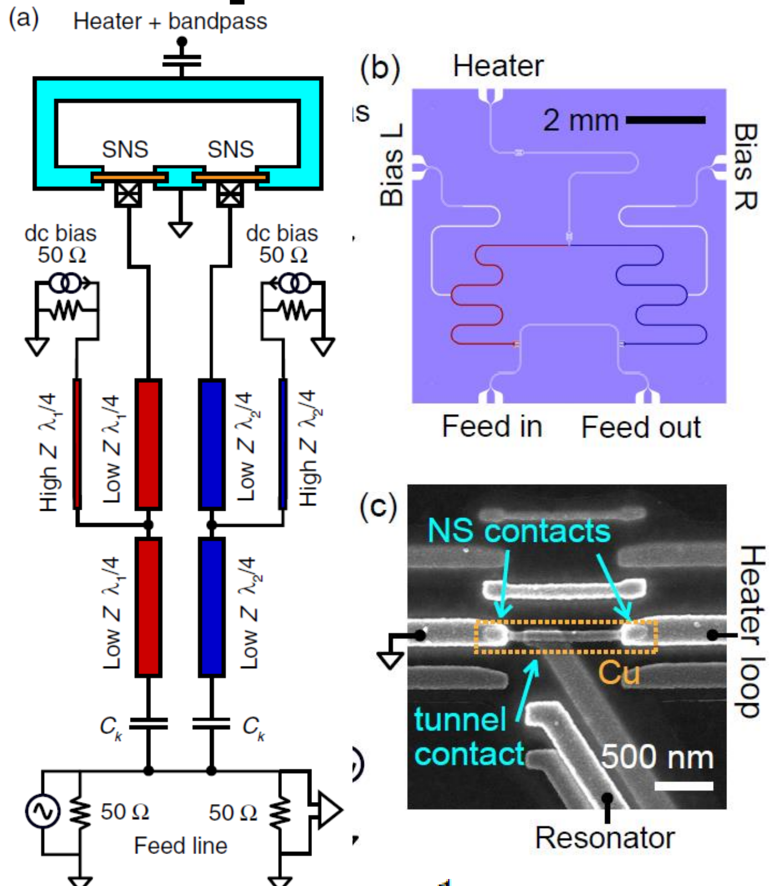
Proximity superconductor detector

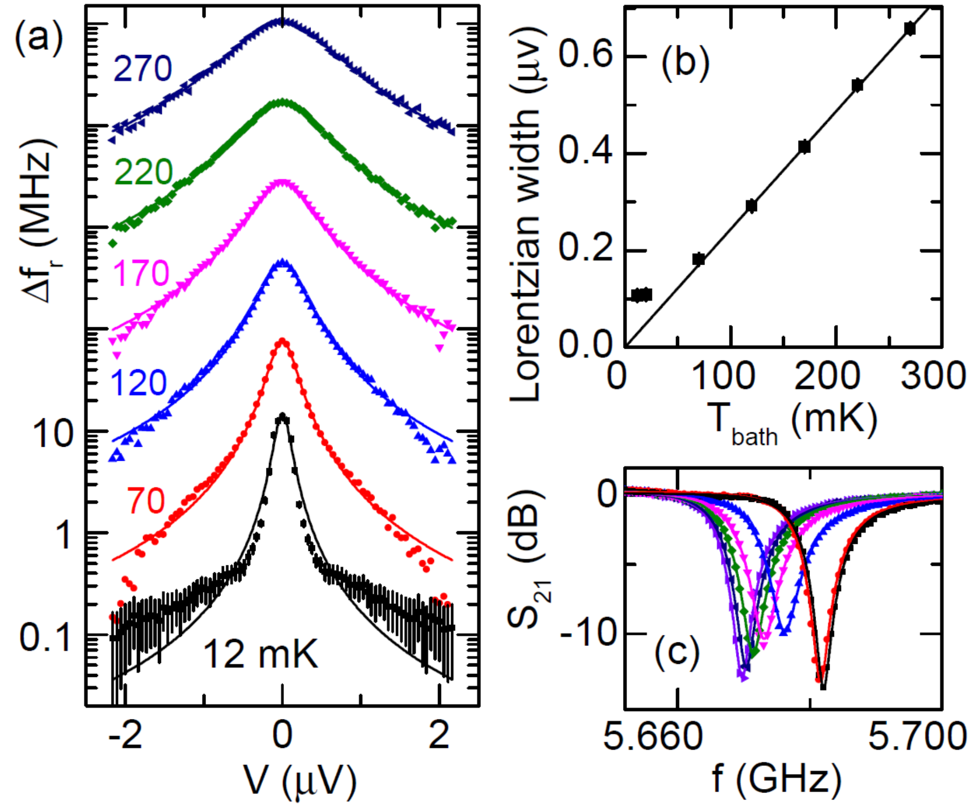


Capable of resolving 10^{-21} J energy absorption, which corresponds to about 200 microwave photons at 8 GHz.

J. Govenius et al., PRL 2016

Josephson thermometer (at 5 GHz)



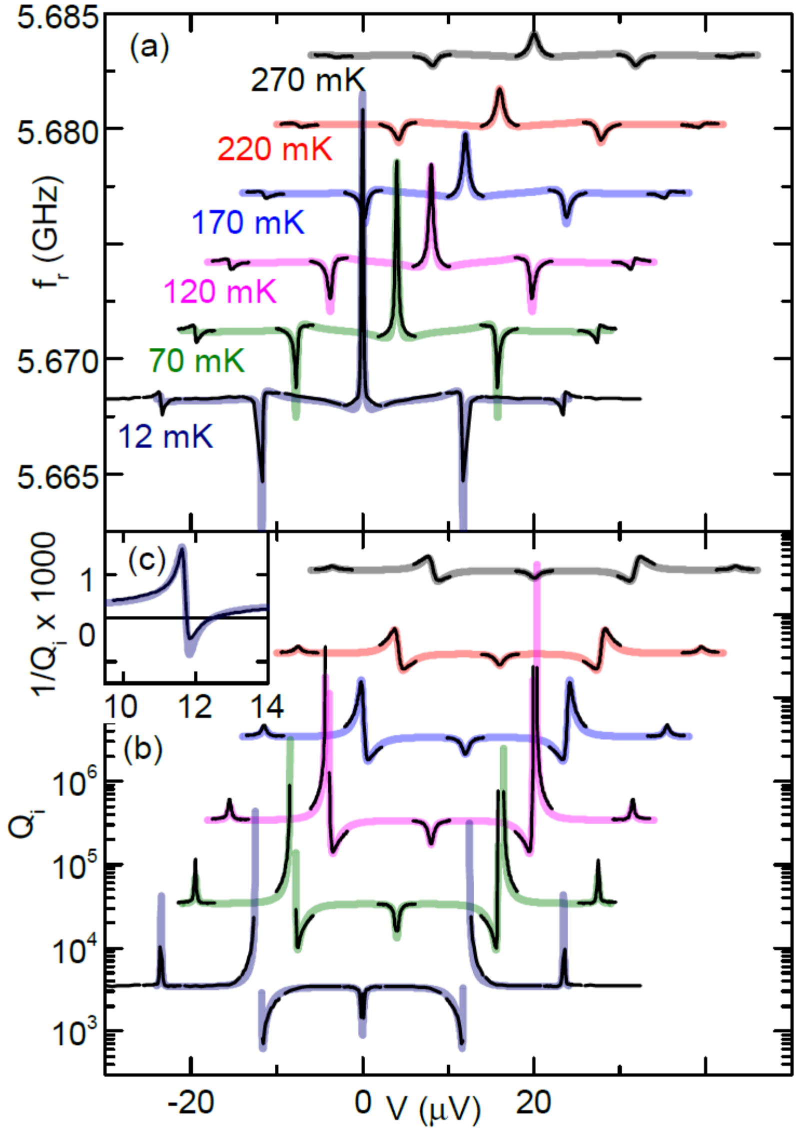


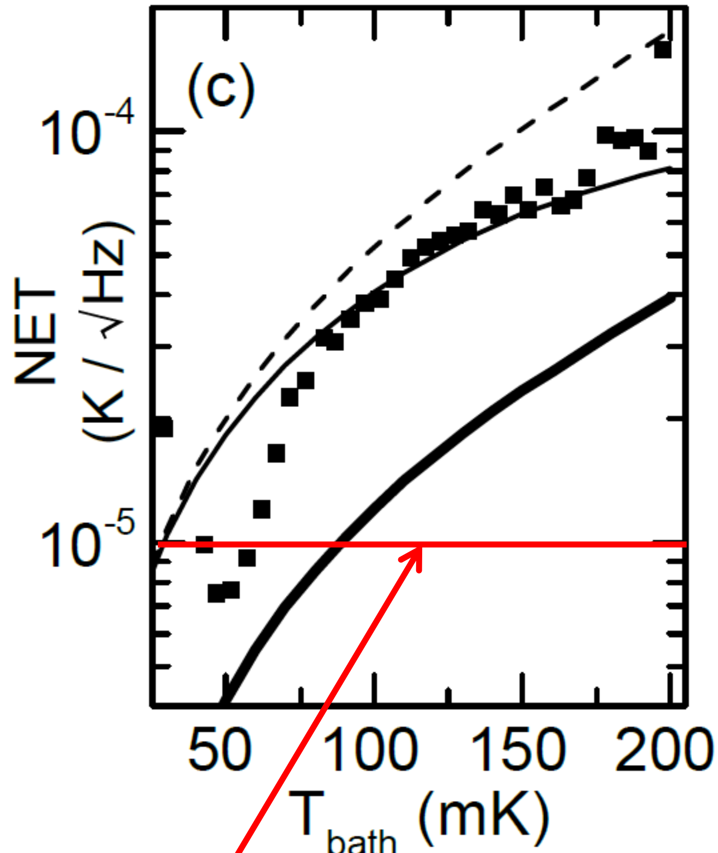
$$f = f_r + \Delta f_T \left(\frac{1}{1 + V^2/V_T^2} - \frac{1}{1 + \hbar\gamma_0/2eV_T} \right) \qquad \boxed{V_T = \frac{4\pi R_S k_B T}{e R_q}}$$

Only one fit parameter: R_S = 57.4 Ω.

O.-P. Saira, M. Zgirski, D. Golubev, K. Viisanen and JP, Phys. Rev. Applied **6**, 024005 (2016).

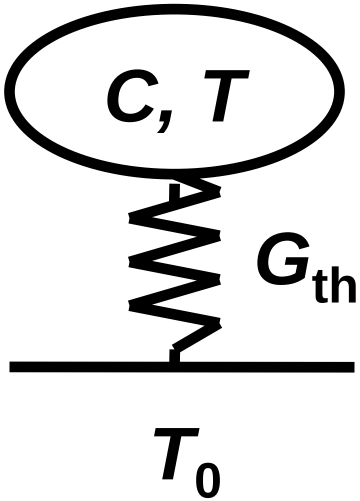
Josephson thermometer (at 5 GHz)

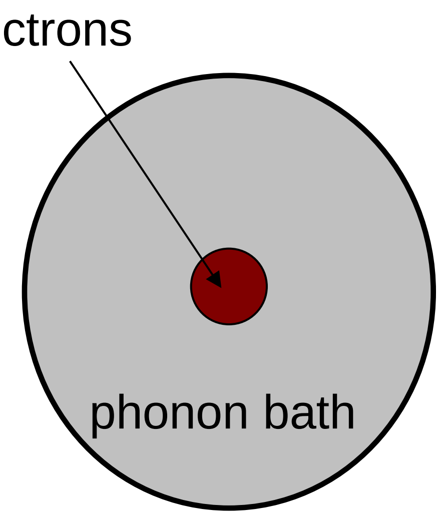


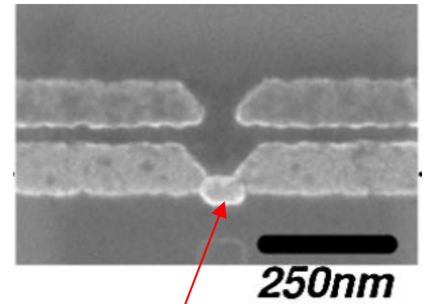


Expected 1 K photon resolution

Classical temperature fluctuations – ultimate lower bound for NET



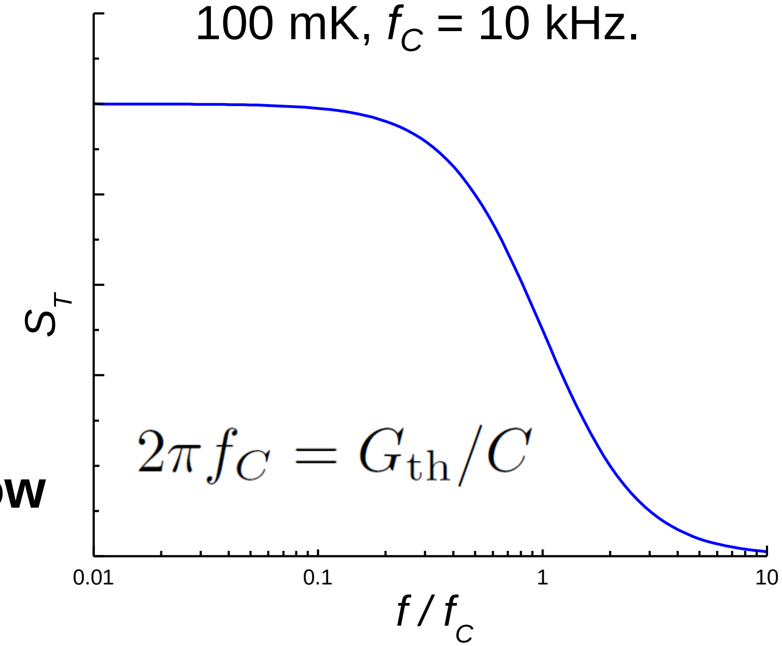




In this grain, $\langle \Delta T^2 \rangle$ is expected to be several mK at 100 mK, f_C = 10 kHz.

$$\langle \delta T^2 \rangle = k_B T^2 / C$$

$$S_T(\omega) = \frac{2 k_B T^2}{G_{\mathrm{th}}} \frac{1}{1 + \omega^2 C^2 / G_{\mathrm{th}}^2}$$



Leads to 1 μK/(Hz)$^{1/2}$ or somewhat below

van den Berg et al., NJP 17 075012 (2015)