Thermometry and refrigeration on nanostructures

Jukka Pekola
Low Temperature Laboratory
Aalto University, Finland

Outline:
1. Nanostructures, temperature, energy relaxation
2. Thermometry
3. Refrigeration
4. On-going research on refrigerators: information-powered refrigerators, quantum refrigerators...
LECTURE 1

Energy relaxation, temperature

(Quantum) heat conductance
Micro- and nanofabrication

1. Lithography and thin film deposition (structures in 2D)
   Single-electron transistor

2. Etching (structures in 3D)
   Antenna-coupled micro-bolometer
Temperature in an electronic device

\[ f(E) = \frac{1}{1 + e^{(E - \mu)/k_B T}} \]
Generic thermal model for an electronic conductor

- $f(E)$
- $k_B T$
- $\mu_L$
- $\mu_R$
- $G_{e-ph}$
- $Q_{e-ph}$
- $G_{ph-sub}$
- $Q_{ph-sub}$
- $G_0$
- $Q_0$
- $T_e$
- $T_{ph}$
- $T_{sub}$
- $T_0$
- Heat bath (sample holder)
Electron-electron and electron-phonon relaxation

$\text{e-e relaxation drives the system towards quasi-equilibrium}$

$(\tau_{ee} \sim 1 \text{ ns in metals at low } T)$

$\text{e-p relaxation drives the system towards equilibrium}$

$(\tau_{ep} \sim 10 \mu s \text{ in metals at low } T)$
Kinetic equation

\[
\frac{\partial f(E, t)}{\partial t} = I_{\text{inel}}[f] = I_{\text{e-e}}[f] + I_{\text{e-\text{ph}}}[f] + \ldots
\]

(in a spatially homogeneous case)

\[
I_{\text{e-e}}[f] = - \int d\omega dE' K_{ee}(E, E', \omega)[f(E)f(E')(1 - f(E + \hbar\omega))(1 - f(E' - \hbar\omega)) - (1 - f(E))(1 - f(E'))f(E + \hbar\omega)f(E' - \hbar\omega)]
\]

\[
I_{\text{e-\text{ph}}}[f] = - \int_0^\infty d\omega K_{e\text{ph}}(E, \omega)\{[f(E)(1 - f(E - \hbar\omega)) - f(E + \hbar\omega)(1 - f(E'))]\times [n(\hbar\omega) + 1] - [f(E - \hbar\omega)(1 - f(E)) - f(E)(1 - f(E + \hbar\omega))]n(\hbar\omega)\}.
\]
The energy distribution of electrons in a small metal conductor

The distribution is determined by energy relaxation:

**Equilibrium** – Thermometer measures the temperature of the "bath"

**Quasi-equilibrium** – Thermometer measures the temperature of the electron system which can be different from that of the "bath"

**Non-equilibrium** – There is no well defined temperature measured by the "thermometer"

Illustration: diffusive normal metal wire
H. Pothier et al. 1997
Analogy between "electronics" and "heattronics"

\[ P = CT \dot{T} + G_{th}(T - T_{bath}) \]

\[ I = CV \dot{V} + G(V - V_0) \]
Thermal response of an absorber

Steady-state heating ("bolometer")

Response to a heat pulse ("calorimeter")

\[ C, T \]

\[ G_{th} \]

\[ T_{bath} \]
Electron-phonon relaxation in metals at low $T$

Hot-electron effects in metals

F. C. Wellstood,* C. Urbina, † and John Clarke
Department of Physics, University of California, Berkeley, California 94720
and Materials Sciences Division, Lawrence Berkeley Laboratory, Berkeley, California 94720
(Received 21 July 1993)

\[ \dot{Q}_{\text{ep}} = \sum \Omega (T_e^5 - T_p^5) \]

FIG. 2. Emission and absorption of phonons of wave vector $q$ by an electron of wave vector $k$. 
Measurement of electron-phonon coupling of a normal metal wire

K. L. Viisanen and JP, arXiv:1606.02985

Copper and silver thin film wires measured
$G_{th}$ - electron-phonon coupling

\[ \dot{Q} = \Sigma V(T_e^5 - T_p^5) \]

\[ G_{th} = 5\Sigma VT^4 \]

(b) $V_b = 0.15 \text{ mV}$

(c) SINIS-thermometer, SINIS-heater

\[ T = 0.14, 0.16, 0.18, 0.20, 0.22, 0.24 \]

\[ \Sigma (\text{GW/m}^3\text{K}^5) \]
Formal treatment of e-p coupling at the end of the first lecture
Quasiparticle recombination in superconductor

Superconducting gap $2\Delta$

Recombination with $2\Delta$ phonon emission

$E_{ph} = 2\Delta$

\[
\frac{1}{\tau_{rec}} = \frac{1}{\tau_0} \sqrt{\frac{\pi}{kT_c}} \left( \frac{2\Delta}{kT_c} \right)^{5/2} \sqrt{\frac{T}{T_c}} e^{-\frac{\Delta}{kT}}
\]

Kaplan et al, 1976
Barends et al., 2008

This process represents electron-phonon relaxation in a superconductor at low $T$. The corresponding heat current is suppressed exponentially.

\[
P_{qp-ph} \approx \frac{64}{63\zeta(5)} \sum \mathcal{V} T^5 e^{-\Delta/k_BT}
\]
Measurement of recombination in a superconductor


$\tau^{-1} = 16$ kHz for a single qp pair in Al
Electronic heat conduction

1D heat diffusion along x-axis of a uniform wire with cross-sectional area $A$

$$\dot{Q} = -G_{th}A \frac{dT}{dx}$$

In a metal, diffusive heat transport is governed by the Wiedemann-Franz law:

$$G_{th}^N = S_0 G_N T$$

$$S_0 = \frac{\pi^2}{3} \left( \frac{k_B}{e} \right)^2$$ is the Lorenz number and

$G_N$ is the electrical conductivity.
Quasiparticle heat conduction in a superconductor

Bardeen et al. 1958

\[ \gamma(T) = \frac{G_{th}}{G_{th}^N} = \frac{3}{2\pi^2} \int_{\Delta/k_B T}^{\infty} \frac{x^2}{\text{sech}^2(x/2)} \, dx \approx \frac{3}{2\pi^2} \left( 8 + 8a + 4a^2 \right) e^{-a} \]

\[ a = \Delta/k_B T \]

Heat transport is exponentially suppressed at low temperatures in a superconductor!

\[ \frac{\Delta T_2}{\Delta T_1} = \frac{G_{th}}{G_{th} + G_{ep,2}} \]

(for small temperature differences)

J. Peltonen et al., PRL 2010
Recent measurement on Nb

A. Feshchenko et al. arXiv:1609.06519
Single-electron transistor (SET)

\[ R_{T1}, C_1 \quad R_{T2}, C_2 \]

Unit of charging energy:

\[ E_C = \frac{e^2}{2C_\Sigma} \]
Steady-state heat transport measurement through a single-electron transistor

\[ \dot{Q} = G_{th} (T_e - T_{bath}) \]

B. Dutta et al., in preparation
Quantum heat conductance

Electrons:

Electrical conductance in a ballistic contact:

\[ \sigma_Q = \frac{2e^2}{h} \]

Thermal conductance:

\[ G_Q = \frac{\pi k_B^2}{6h} T \]

\( G_Q \) and \( \sigma_Q \) related by Wiedemann-Franz law

More generally:

Quantum limit of heat flow across single electronic channels

Quantum thermal conductance by phonons in a nanobridge


\[ G = 4 \times 4 \times G_Q \]
Electromagnetic transfer of heat (photons)

Electron system

Electrical environment

Lattice

Schmidt et al., PRL 93, 045901 (2004)
Ojanen et al., PRB 76, 073414 (2007), PRL 100, 155902 (2008)
D. Segal, PRL 100, 105901 (2008)
L. Pascal et al., PRB 83, 125113 (2011)
Radiative heat transport in an electrical circuit

Voltage noise of a resistor:

$$S_{V_i}(\omega) = 4\hbar \omega R_i n_i(\omega)$$

Bose distribution:

$$n_i(\omega) = \frac{1}{e^{\hbar \omega / k_B T_i} - 1}$$

Current noise created by resistor 1:

$$S_{I_1}(\omega) = S_{V_1}(\omega) / |Z_{tot}|^2$$

$$Z_{tot} = R_1 + R_2$$

Spectrum of dissipation of energy created by resistor 1 and absorbed by resistor 2:

$$S_{P_{12}}(\omega) = R_2 S_{I_1}(\omega)$$
Heat transported between two resistors

Radiative contribution to net heat flow between electrons of 1 and 2:

\[
P_\nu = \int_0^\infty \frac{d\omega}{2\pi} \left[ S_{P12}(\omega) - S_{P21}(\omega) \right] = r \frac{\pi k_B^2}{12\hbar} (T_1^2 - T_2^2)
\]

Coupling constant:

\[
r \equiv \frac{4R_1R_2}{(R_1 + R_2)^2}
\]

Linearized expression for small temperature difference \(\Delta T = T_1 - T_2\):

\[
P_\nu = rG_Q \Delta T
\]

\[
G_Q = \frac{\pi k_B^2}{6\hbar} T
\]

\[
G_\nu = rG_Q
\]
Classical or quantum heat transport?

\[ P_\nu = \int_0^\infty \frac{d\omega}{2\pi} \frac{4R_1 R_2 \hbar \omega}{|Z_t(\omega)|^2} \left( \frac{1}{e^{\hbar \omega / k_B T_1} - 1} - \frac{1}{e^{\hbar \omega / k_B T_2} - 1} \right) \]

\[ \frac{4R_1 R_2}{|Z_t(\omega)|^2} \]

\( \omega_c \) \hspace{1cm} k_B T / \hbar \]

"Classical"

\[ G_\nu \sim r k_B \omega_C \]

"Quantum"

\[ G_\nu = r G_Q \]
Classical or quantum heat transport?

Classical:
\[ \frac{\hbar}{k_BT} \frac{1}{RC} \ll 1 \]
\[ \frac{\hbar}{k_BT} \frac{R}{L} \ll 1 \]

Quantum limited:
\[ \frac{\hbar}{k_BT} \frac{1}{RC} \gg 1 \]
\[ \frac{\hbar}{k_BT} \frac{R}{L} \gg 1 \]

Johnson, Nyquist 1928

\[ T = 300 \text{ K}, \ell = 1 \text{ cm}: \quad \frac{\hbar}{k_BT} \frac{1}{RC} \sim \frac{\hbar}{k_BT} \frac{R}{L} < 10^{-3} \ll 1 \]

\[ T = 100 \text{ mK}, \ell = 100 \text{ \mu m}: \quad \frac{\hbar}{k_BT} \frac{1}{RC} \sim \frac{\hbar}{k_BT} \frac{R}{L} \sim 10^2 \gg 1 \]
Demonstration of photonic heat conduction

\[ L_J = \frac{\hbar}{2eI_{C,0} |\cos(\pi \Phi / \Phi_0)|} \]

Tunable impedance matching using DC-SQUIDs

2nd experiment

SAMPLE A in a loop ("matched")
[SAMPLE B without loop ("not matched")]

Heat transport in different set-ups

Loop geometry (Sample A)

Linear geometry (Sample B)

\[ P_A^A = G_Q \Delta T \]

for small temperature difference

\[ \frac{P_B^B}{P_A^A} = \frac{2}{5} \left( \frac{k_B T R C}{\hbar} \right)^2 \]

\[ \simeq 10^{-3} \]

in that experiment
Results in the two sample geometries

Heat transported by residual quasiparticles at $T > 0.3$ K and by photons (in the loop sample) at $T < 0.3$ K

$$\frac{\Delta T_2}{\Delta T_1} = \frac{G_v + G_s}{G_v + G_s + G_{ep,2}}$$

Quasiparticles
Photon transport over a macroscopic distance

Electron-phonon heat flux in metals at low $T$

Hamiltonian of the electron-phonon system:

$$H = H_e + H_p + H_{ep}$$

$$H_e = \sum_k \epsilon_k a_k^\dagger a_k$$

$$H_p = \sum_q \hbar \omega_q c_q^\dagger c_q$$

$$H_{ep} = \gamma \sum_{k,q} \omega_q^{1/2} (a_k^\dagger a_{k-q} c_q + a_k^\dagger a_{k+q} c_q^\dagger)$$

- Phonon absorption
- Phonon emission
Electron-phonon heat flux

\[ \dot{H}_p = \frac{i}{\hbar} [H, H_p] = \frac{i}{\hbar} [H_{ep}, H_p] \]

\[ \dot{H}_p = i \gamma \sum_{k,q} \omega_q^{3/2} (a_{k-q}^\dagger a_k c_q - a_k^\dagger a_{k+q} c_q^\dagger) \]

Kubo formula (linear response):

\[ \dot{Q}_{ep} = \langle \dot{H}_p \rangle = -i \int_0^t dt' \langle [\dot{H}_p(t), H_{ep}(t')] \rangle_0 \]

\[ = \gamma^2 \left( \int dE_k N(E_k) \int d^3q D(q) \omega_q^2 f(E_k) [1 - f(E_{k-q})][1 + n(\omega_q)] \delta(E_{k-q} - E_k + \hbar \omega_q) \right. \]

\[ - \left. \int dE_k N(E_k) \int d^3q D(q) \omega_q^2 f(E_k) [1 - f(E_{k+q})] n(\omega_q) \delta(E_{k+q} - E_k - \hbar \omega_q) \right) \]

\[ = P_e - P_a \]
Electron-phonon heat flux in metals at low $T$

The rate at which an electron at wave vector $\mathbf{k}$ is scattered to $\mathbf{k}' = \mathbf{k} - \mathbf{q}$ with a phonon $\mathbf{q}$ emitted is given by

$$
\tau_{k,k-q}^{-1} = \frac{2\pi}{\hbar} \mathcal{M}^2 \delta(E_k - E_{k-q} - \epsilon_q)[1 - f(E_{k-q})][n_p(q) + 1].
$$

(1)

Here $\mathcal{M}^2$ is the square of the matrix element of electron-phonon coupling, and

$$
n_p(q) = \frac{1}{e^{\beta q} - 1}
$$

(2)

the phonon occupation number. We use $\beta_i = (k_B T_i)^{-1}$.

The electrons emit energy to phonons at the rate $P_e = \int dE_k N(E_F) f(E_k) \int d^3 q D_p(q) \epsilon_q \tau_{k,k-q}^{-1}$, i.e.,

$$
P_e = \frac{2\pi}{\hbar} \int dE_k N(E_F) f(E_k) \int d^3 q D_p(q) \mathcal{M}^2 \epsilon_q \delta(E_k - E_{k-q} - \epsilon_q)[1 - f(E_{k-q})][n_p(q) + 1].
$$

(3)

Here $D_p(q)$ is the phonon density of states, and $N(E_F)$ is the density of states for electrons at the Fermi level.

Correspondingly, electrons absorb energy from phonons at the rate

$$
P_a = \frac{2\pi}{\hbar} \int dE_k N(E_F) f(E_k) \int d^3 q D_p(q) \mathcal{M}^2 \epsilon_q \delta(E_k - E_{k+q} + \epsilon_q)[1 - f(E_{k+q})]n_p(q).
$$

(4)

The net heat flux between electrons and phonons is then

$$
\dot{Q} = P_e - P_a.
$$

(5)
We can first integrate over the angle \( \theta \) between electron and phonon wave vectors. In general for 3D phonons we have 
\[ D_p(q) = \mathcal{V} / (2\pi)^3, \] 
where \( \mathcal{V} \) is the volume of the system. In spherical coordinates, 
\[ \int d^3q \rightarrow 2\pi \int_0^\infty dq q^2 \int_{-1}^1 \ d(\cos \theta). \]
Further, \( E_k = \frac{\hbar^2 k^2}{2m} \) and \( E_{k\mp q} = \frac{\hbar^2 (k \mp q)^2}{2m} \approx E_k \mp \frac{\hbar^2 k_F}{m} q \cos \theta \), where the last approximation is due to \( k \approx k_F \) and \( q \ll k_F \). Then, collecting the angle dependent terms and integrating over \( \cos \theta \), we have
\[
\int d^3q D_p(q) \delta(E_k - E_{k\mp q} + \epsilon_q)[1 - f(E_{k\mp q})]
\]
\[
= \frac{\mathcal{V}}{(2\pi)^2} \int_0^\infty dq q^2 \int_{-1}^1 d(\cos \theta) \delta(\pm \frac{\hbar^2 k_F}{m} q \cos \theta + \epsilon_q)[1 - f(E_k \mp \frac{\hbar^2 k_F}{m} q \cos \theta)]
\]
\[
= \frac{m \mathcal{V}}{(2\pi)^2 \hbar^2 k_F} \int_0^\infty dq [1 - f(E_k + \epsilon_q)]. \tag{6}
\]

Inserting this result into Eqs. (3) and (4), using the standard coupling with \( \mathcal{M}^2 = \mathcal{M}_0^2 q \) (scalar deformation potential, longitudinal phonons), writing \( \epsilon \equiv \epsilon_q = \hbar \epsilon_\ell q \), and dropping the index \( k \) we obtain
\[
P_e = \frac{m \mathcal{V} N(E_F) \mathcal{M}_0^2}{2\pi \hbar^6 k_F c_\ell^3} \int_0^\infty d\epsilon \epsilon^3 [n_p(q) + 1] \int_{-\infty}^\infty dE f(E)[1 - f(E - \epsilon)] \tag{7}
\]
and
\[
P_a = \frac{m \mathcal{V} N(E_F) \mathcal{M}_0^2}{2\pi \hbar^6 k_F c_\ell^3} \int_0^\infty d\epsilon \epsilon^3 n_p(q) \int_{-\infty}^\infty dE f(E)[1 - f(E + \epsilon)]. \tag{8}
\]

Using (5), we then have
\[
\dot{Q} = \frac{m \mathcal{V} N(E_F) \mathcal{M}_0^3}{2\pi \hbar^6 k_F c_\ell^3} \int_0^\infty d\epsilon \epsilon^3 \left\{ \int_{-\infty}^\infty dE f(E)[1 - f(E - \epsilon)] + n_p(q) \left[ \int_{-\infty}^\infty dE f(E)[1 - f(E - \epsilon)] - \int_{-\infty}^\infty dE f(E)[1 - f(E + \epsilon)] \right] \right\}. \tag{9}
\]
Using the identity
\[
f(E)[1 - f(E + x)] = \frac{f(E) - f(E + x)}{1 - e^{-\beta x}},
\]
and the symmetry \(f(-E) = 1 - f(E)\), and \(\int_{-\infty}^{\infty} dE[f(E) - f(E + \epsilon)] = \epsilon\), we can simplify (9) into
\[
\dot{Q} = \frac{m\mathcal{V}N(E_F)M_0^2}{2\pi\hbar^6k_Fc_\ell^3} \int_{0}^{\infty} d\epsilon \epsilon^4 \left[ \frac{1}{e^{\beta_c\epsilon} - 1} - \frac{1}{e^{\beta_p\epsilon} - 1} \right].
\]
This can be readily integrated to yield
\[
\dot{Q} = \frac{12\zeta(5)m\mathcal{V}N(E_F)M_0^2}{\pi\hbar^6k_Fc_\ell^3} (\beta_c^{-5} - \beta_p^{-5}),
\]
which is equivalent to the well known form
\[
\dot{Q} = \Sigma \mathcal{V}(T_c^5 - T_p^5),
\]
by identifying
\[
\Sigma \equiv \frac{12M_0^2\zeta(5)N(E_F)k_B^5}{\pi\hbar^5v_Fc_\ell^3}.
\]