# Thermometry and refrigeration on nanostructures

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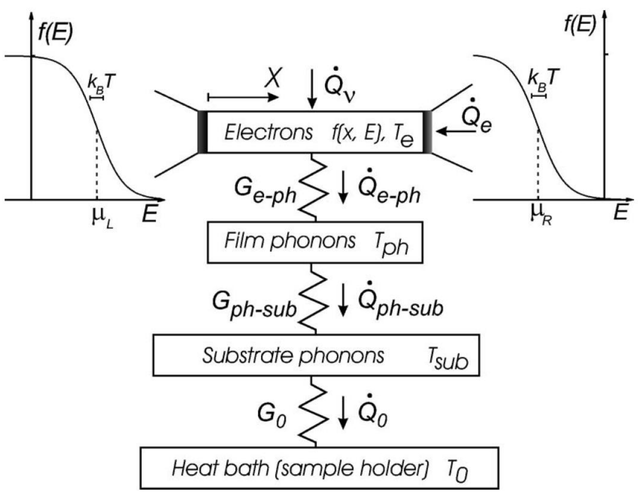
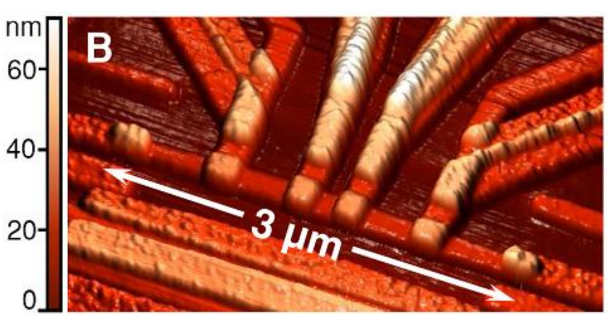
**Outline:**

1. **Nanostructures, temperature, energy relaxation**
2. **Thermometry**
3. **Refrigeration**
4. **On-going research on refrigerators: information-powered refrigerators, quantum refrigerators...**

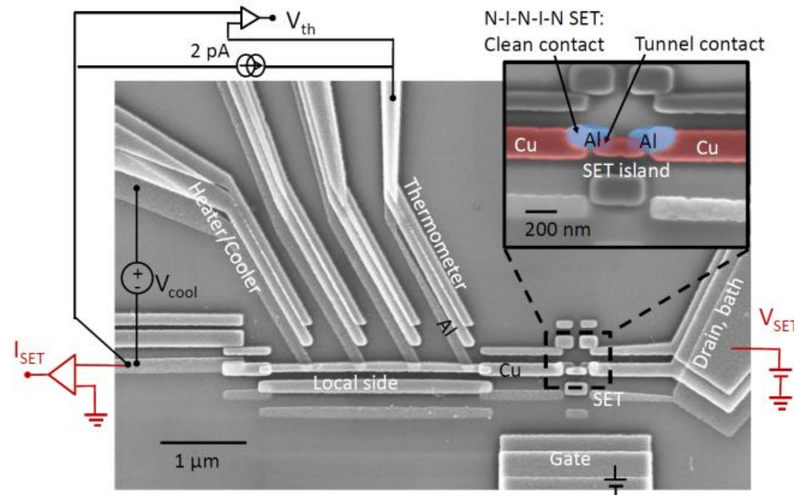
# *LECTURE 1*

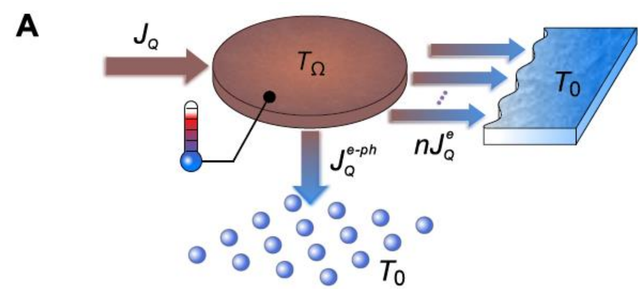
## Energy relaxation, temperature



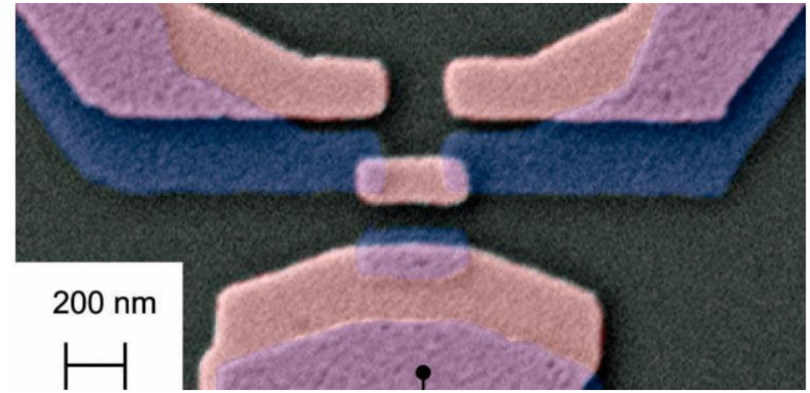

## (Quantum) heat conductance



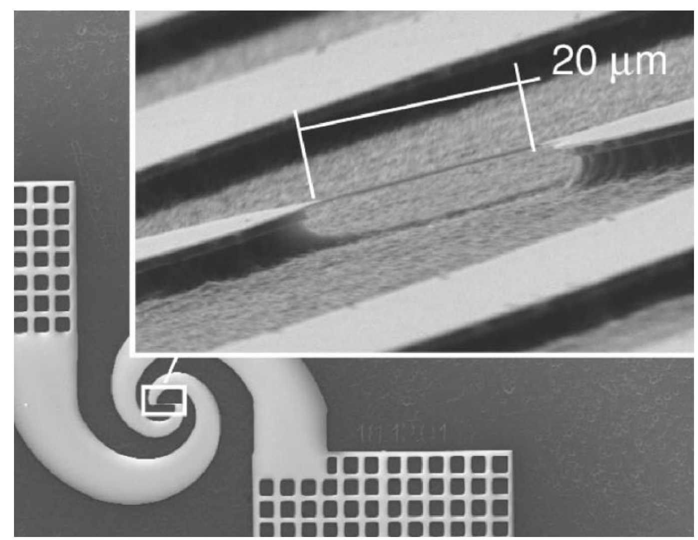



# Micro- and nanofabrication

**1. Lithography and thin film deposition (structures in 2D)**
Single-electron transistor

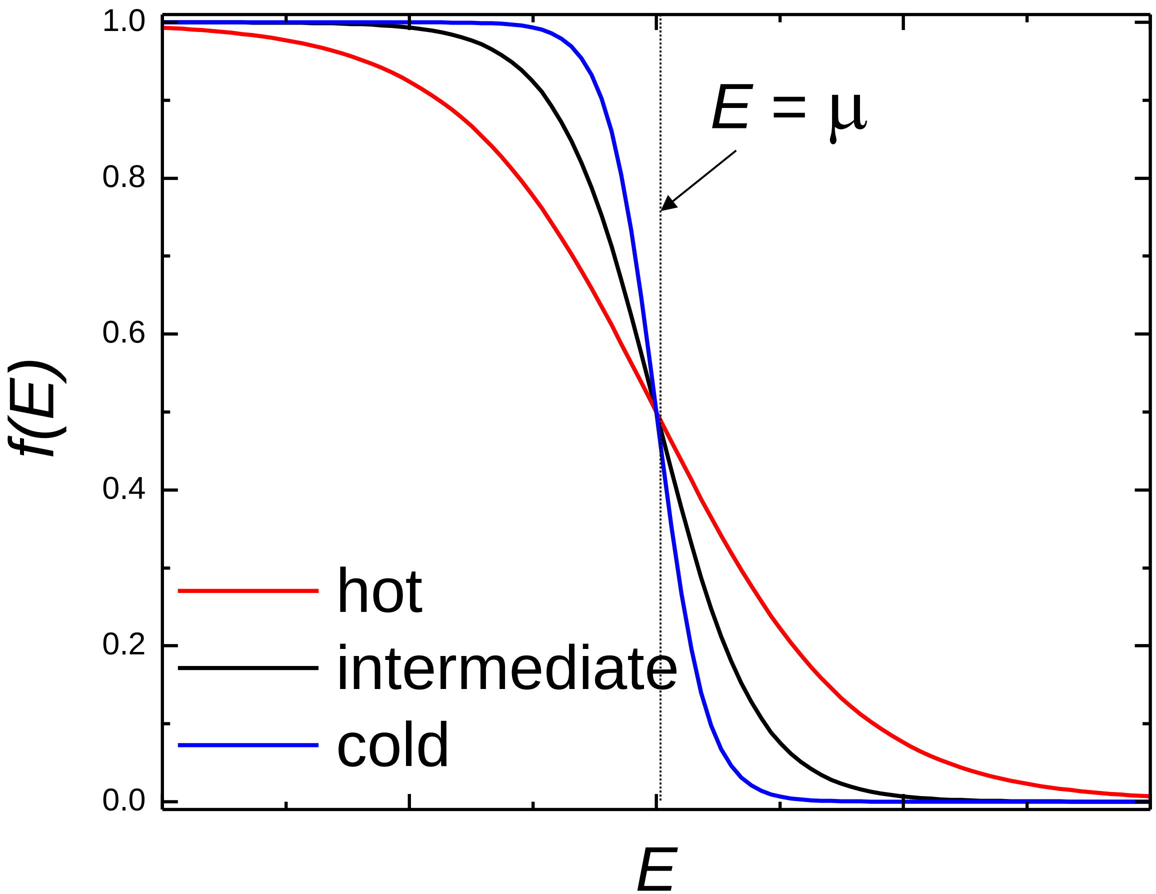


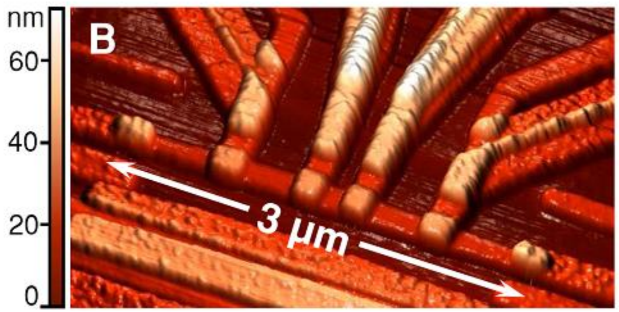

**2. Etching (structures in 3D)**
Antenna-coupled micro-bolometer



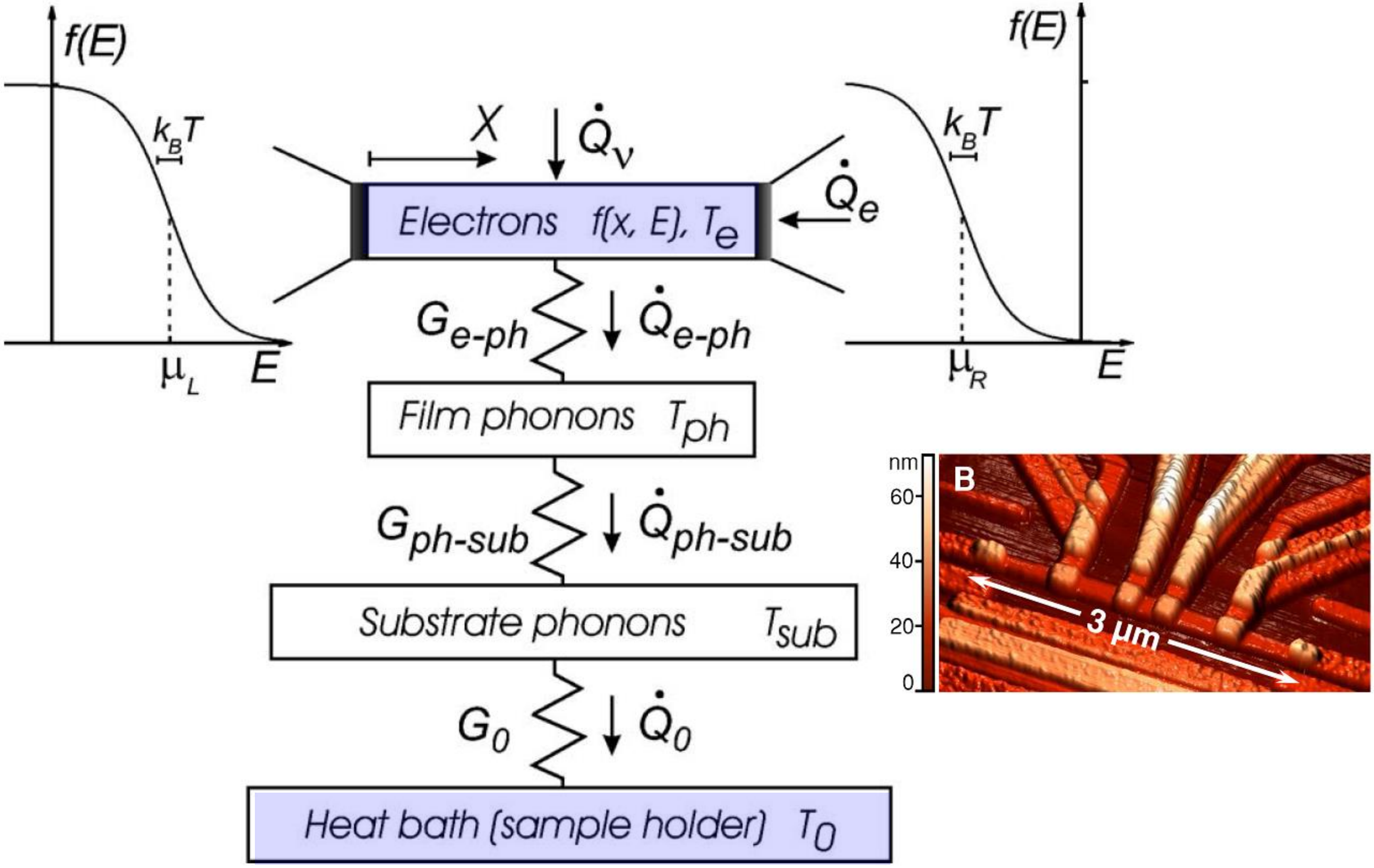
# Temperature in an electronic device

$$f(E) = \frac{1}{1 + e^{(E-\mu)/k_B T}}$$

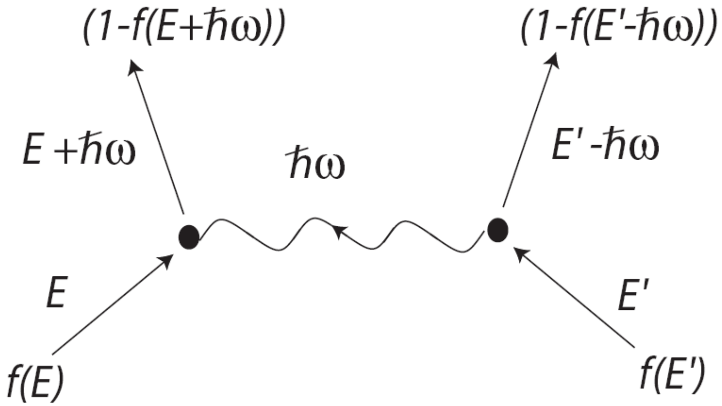


# Generic thermal model for an electronic conductor

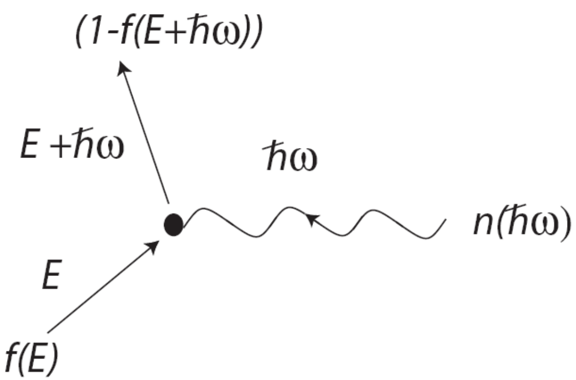


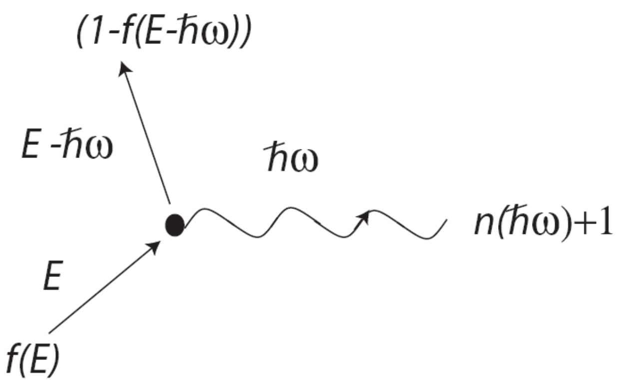
# Electron-electron and electron-phonon relaxation




e-e relaxation drives the system towards *quasi-equilibrium*

($\tau_{ee} \sim$ 1 ns in metals at low $T$)



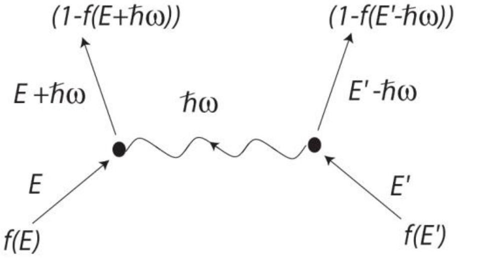

e-p relaxation drives the system towards *equilibrium*

($\tau_{ep} \sim$ 10 $\mu$s in metals at low $T$)

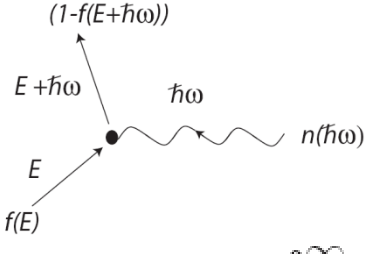
# Kinetic equation

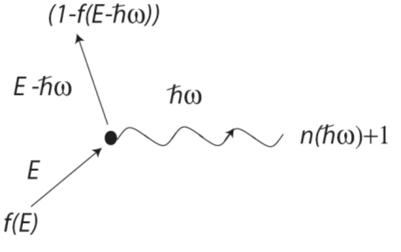
$$\frac{\partial f(E,t)}{\partial t} = I_{\mathrm{inel}}[f] = I_{\mathrm{e-e}}[f] + I_{\mathrm{e-ph}}[f] + \ldots$$

(in a spatially homogeneous case)




$$I_{\mathrm{e-e}}[f] = -\int d\omega dE' K_{ee}(E,E',\omega)[f(E)f(E')(1-f(E+\hbar\omega))(1-f(E'-\hbar\omega)) \\ -(1-f(E))(1-f(E'))f(E+\hbar\omega)f(E'-\hbar\omega)]$$




$$I_{\mathrm{e-ph}}[f] = -\int_0^\infty d\omega K_{eph}(E,\omega)\{[f(E)(1-f(E-\hbar\omega)) - f(E+\hbar\omega)(1-f(E))] \\ \times [n(\hbar\omega)+1] - [f(E-\hbar\omega)(1-f(E)) - f(E)(1-f(E+\hbar\omega))]\, n(\hbar\omega)\}.$$

# The energy distribution of electrons in a small metal conductor

The distribution is determined by energy relaxation:

**Equilibrium** –**Thermometer measures the temperature of the ”bath”**
**Quasi-equilibrium** –**Thermometer measures the temperature of the electron system which can be different from that of the ”bath”**
**Non-equilibrium** –**There is no well defined temperature measured by the ”thermometer”**

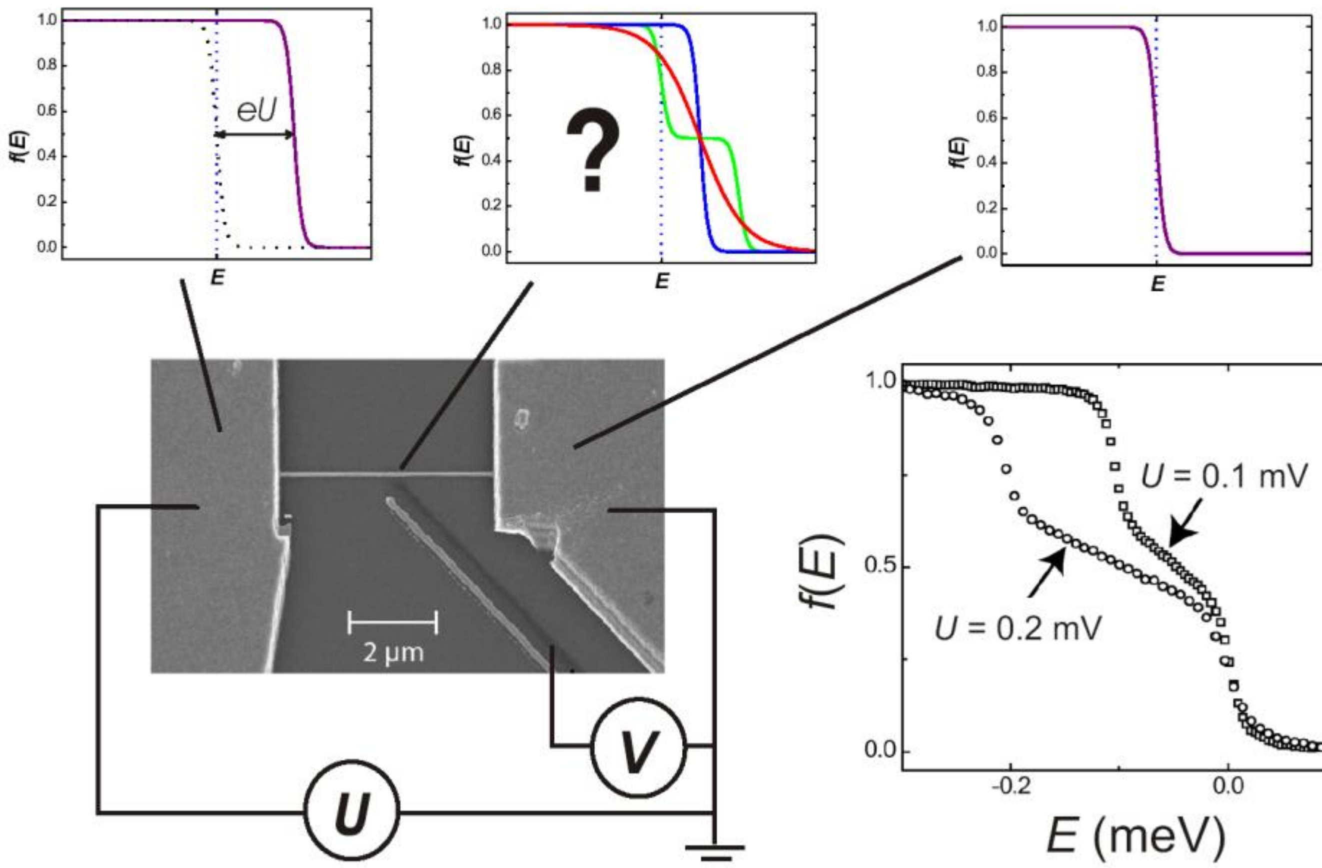
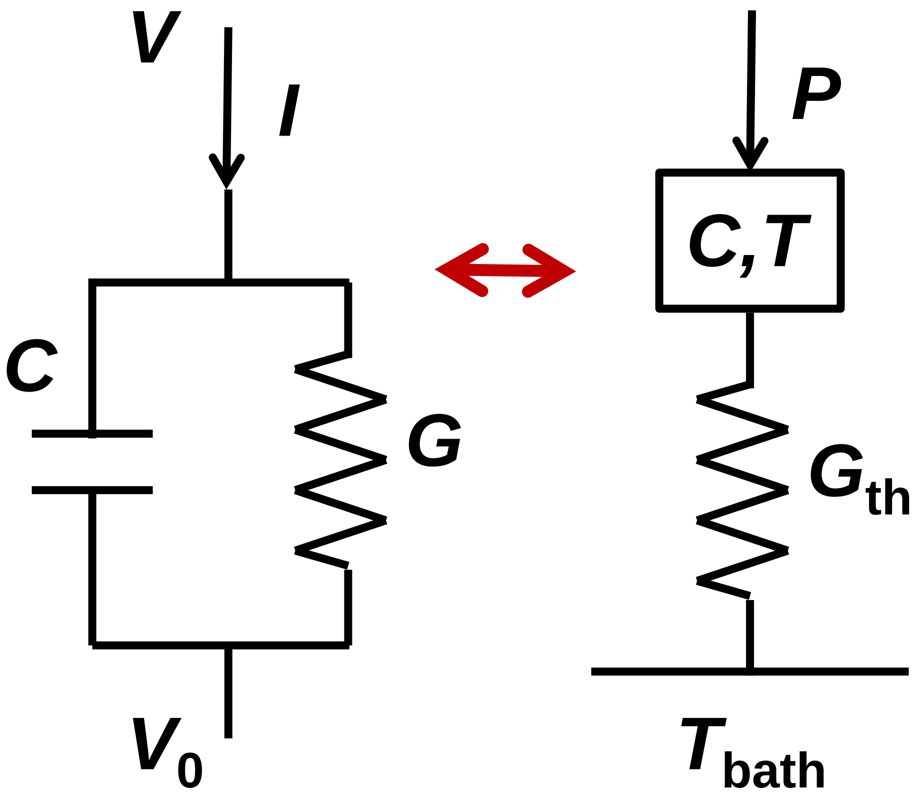



**Illustration: diffusive normal metal wire**
H. Pothier et al. 1997

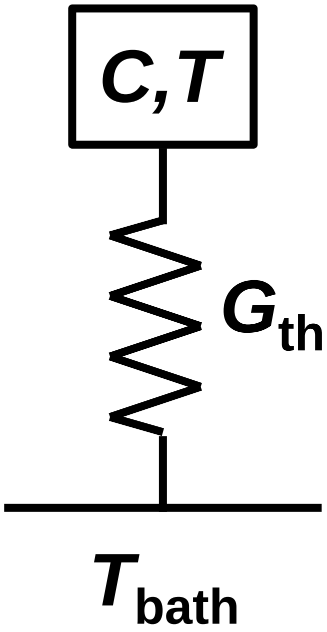
# Analogy between "electronics" and "heattronics"



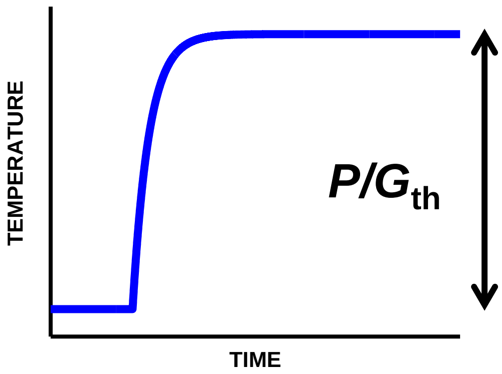

$$P = C\dot{T} + G_{\mathrm{th}}(T - T_{\mathrm{bath}})$$

$$I = C\dot{V} + G(V - V_0)$$

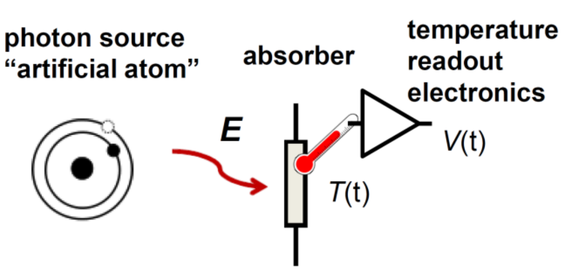
# Thermal response of an absorber

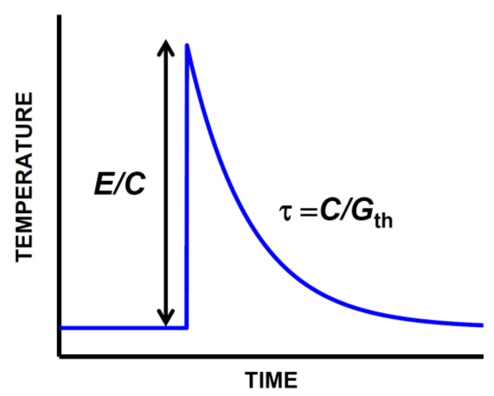



## Steady-state heating ("bolometer")




## Response to a heat pulse ("calorimeter")



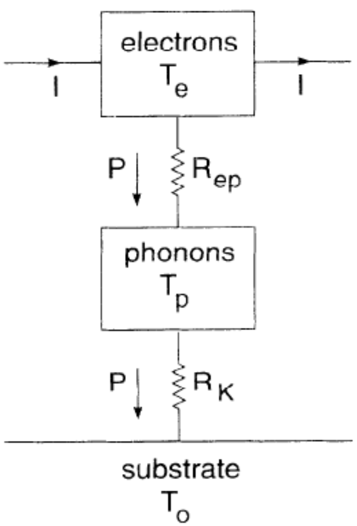



# Electron-phonon relaxation in metals at low *T*

PHYSICAL REVIEW B VOLUME 49, NUMBER 9 1 MARCH 1994-I

## Hot-electron effects in metals

F. C. Wellstood,* C. Urbina,† and John Clarke
*Department of Physics, University of California, Berkeley, California 94720*
*and Materials Sciences Division, Lawrence Berkeley Laboratory, Berkeley, California 94720*
(Received 21 July 1993)



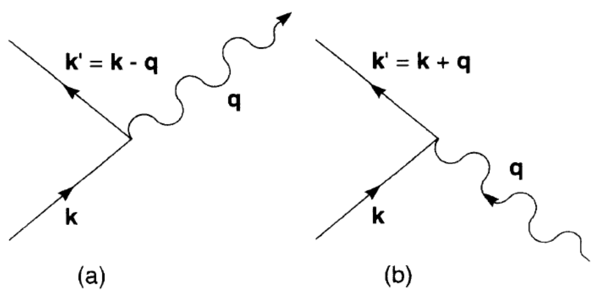
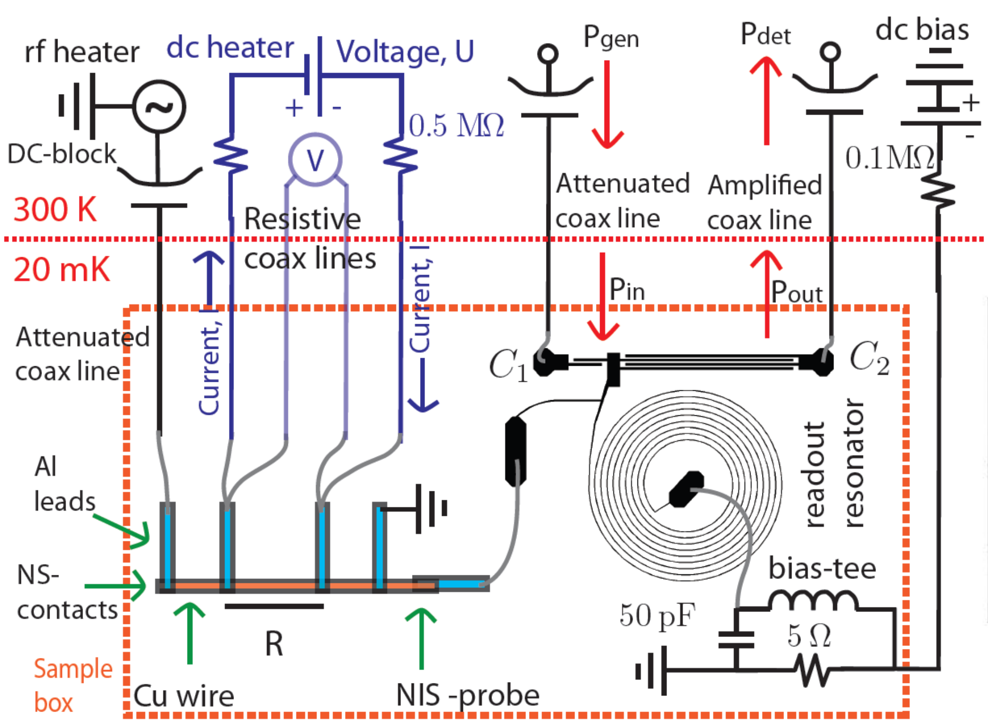



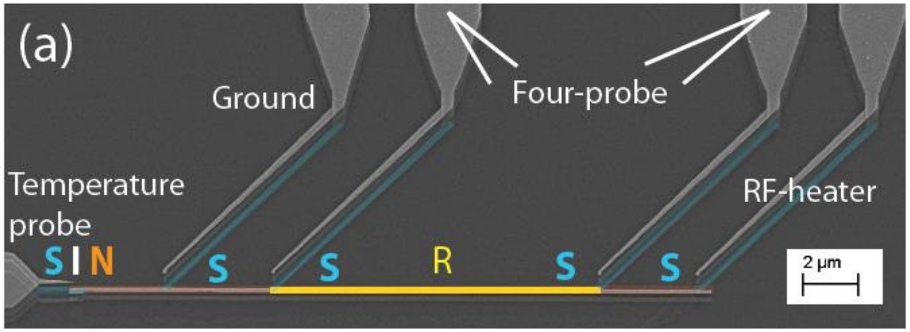

FIG. 2. Emission and absorption of phonons of wave vector **q** by an electron of wave vector **k**.

$$\boxed{\dot{Q}_{\rm ep} = \Sigma\Omega(T_{\rm e}^5 - T_{\rm p}^5)}$$

# Measurement of electron-phonon coupling of a normal metal wire

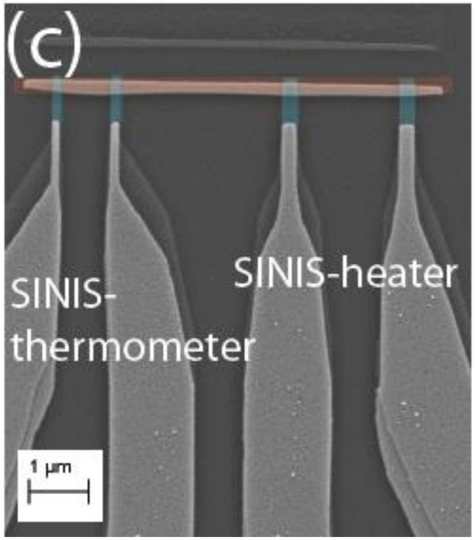
K. L. Viisanen and JP, arXiv:1606.02985

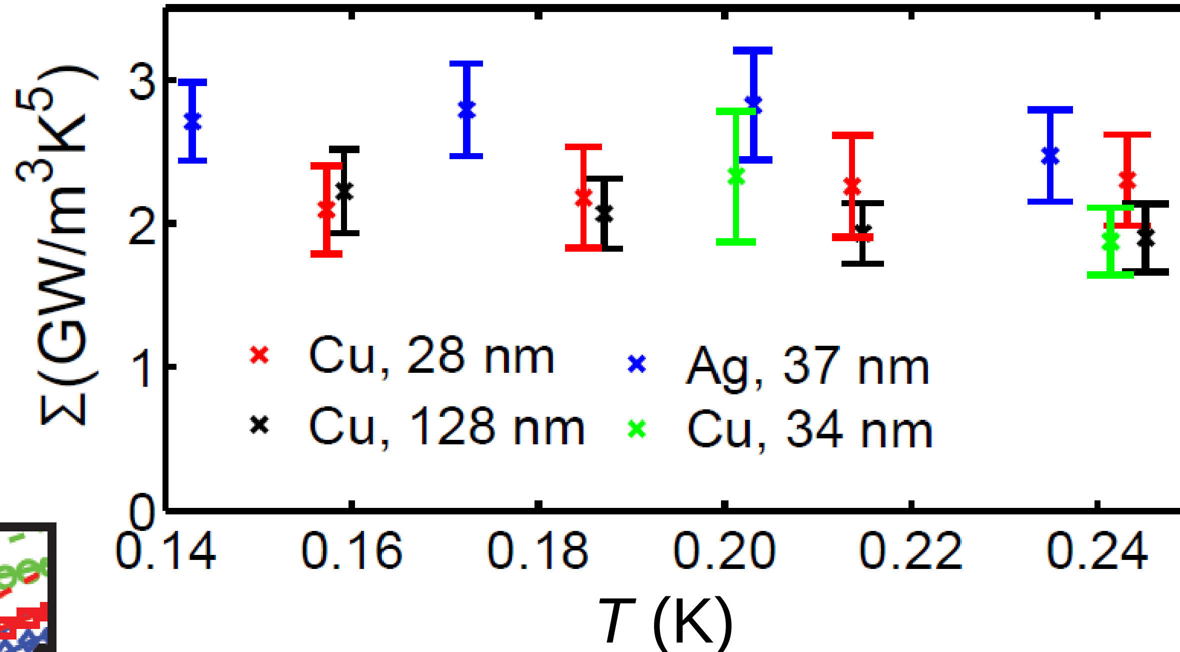


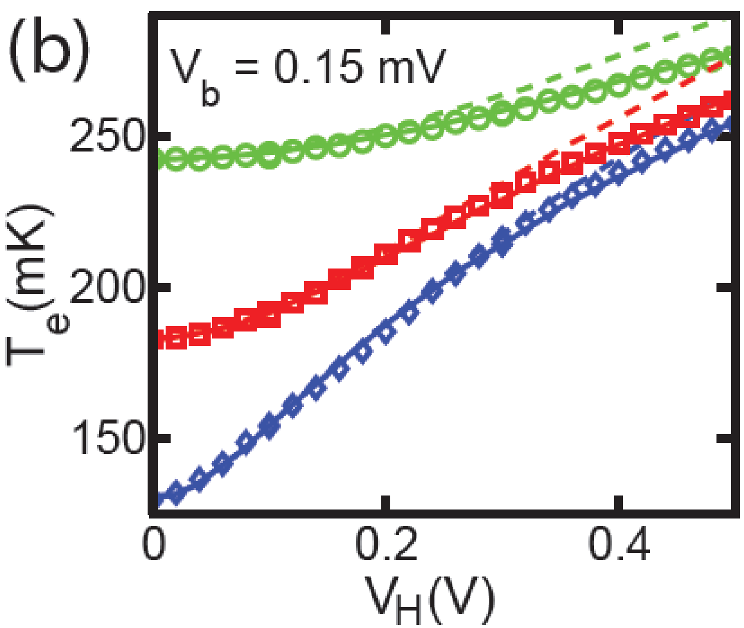




Copper and silver thin film wires measured

# $G_{th}$ - electron-phonon coupling



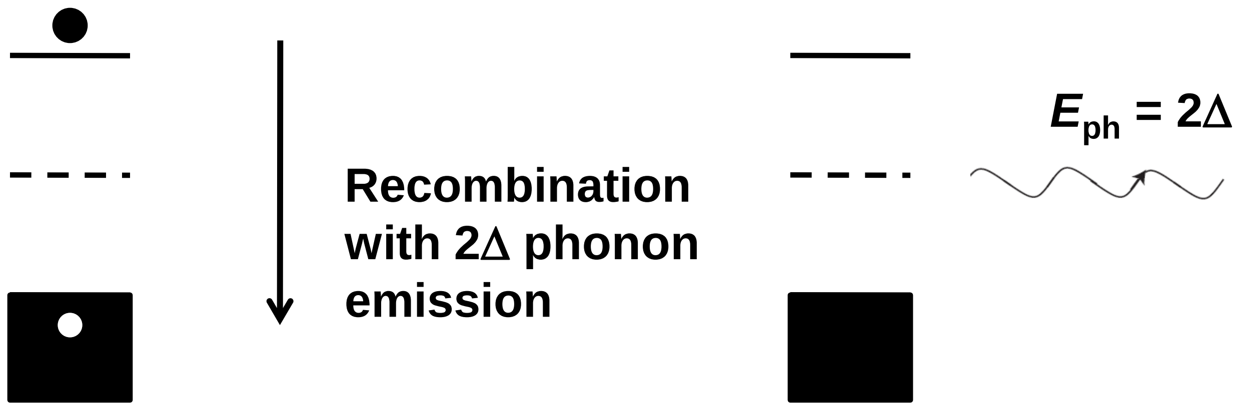

$$\dot{Q} = \Sigma\mathcal{V}(T_e^5 - T_p^5)$$

$$G_{\mathrm{th}} = 5\Sigma\mathcal{V}T^4$$

# Formal treatment of e-p coupling at the end of the first lecture

# Quasiparticle recombination in superconductor

**Superconducting gap 2Δ**



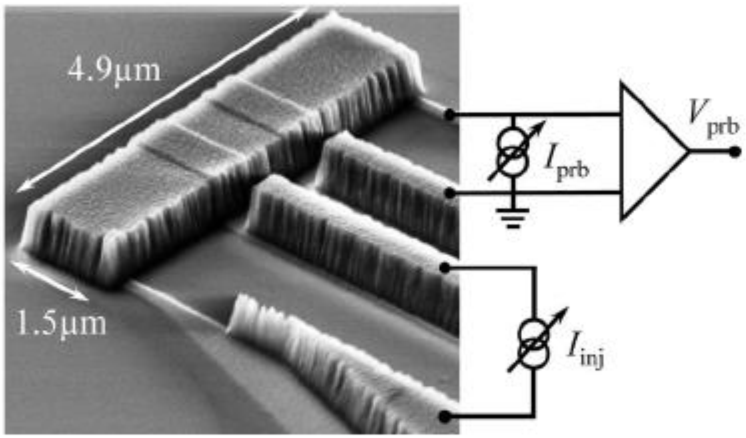

$$\frac{1}{\tau_{rec}} = \frac{1}{\tau_0}\sqrt{\pi}\left(\frac{2\Delta}{kT_c}\right)^{5/2}\sqrt{\frac{T}{T_c}}e^{-\frac{\Delta}{kT}}$$

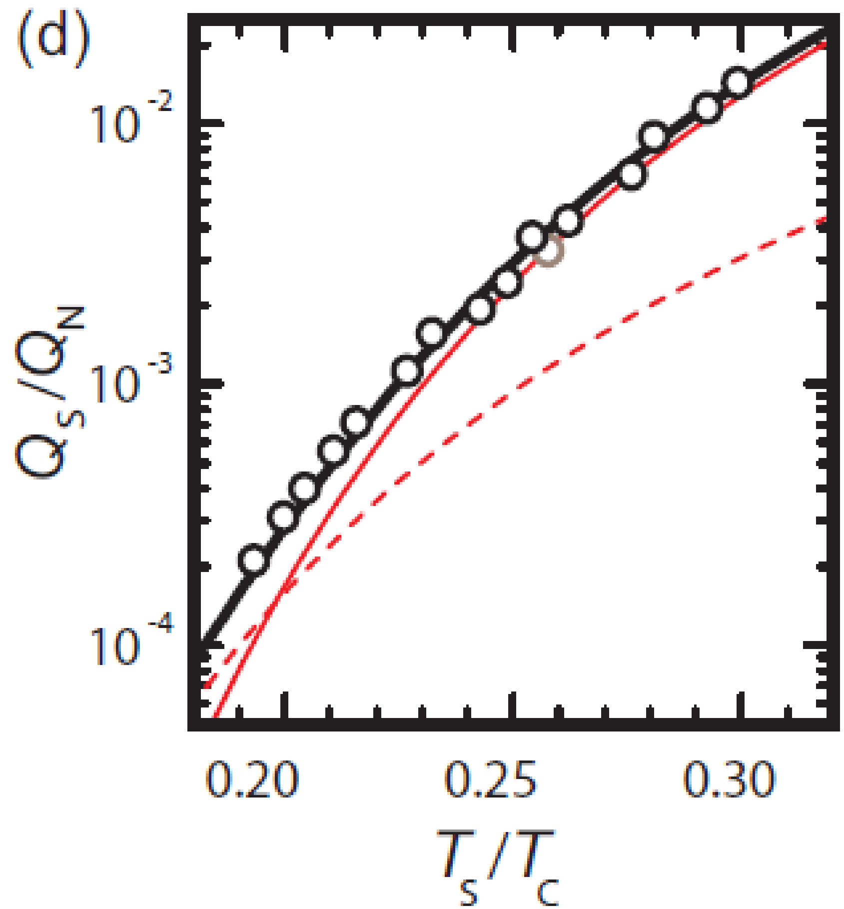
Kaplan et al, 1976
Barends et al., 2008

This process represents electron-phonon relaxation in a superconductor at low *T*. The corresponding heat current is suppressed exponentially.

$$P_{\text{qp-ph}} \simeq \frac{64}{63\zeta(5)}\Sigma\mathcal{V}T^5\; e^{-\Delta/k_B T}$$

# Measurement of recombination in a superconductor




Measurement of relaxation in an aluminium bar, A. Timofeev et al, 2009, V. Maisi et al. 2013

$\tau^{-1}$ = 16 kHz for a single qp pair in Al

# Electronic heat conduction

1D heat diffusion along x-axis of a uniform wire with cross-sectional area *A*

$$\dot{Q} = -G_{\mathrm{th}} A \frac{dT}{dx}$$

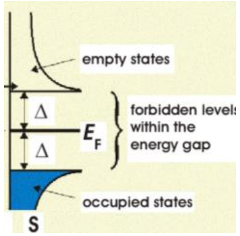
In a metal, diffusive heat transport is governed by the Wiedemann-Franz law:

$$G_{\mathrm{th}}^{\mathrm{N}} = \mathcal{L}_0 G_{\mathrm{N}} T$$

$\mathcal{L}_0 = \pi^2 (k_{\mathrm{B}}/e)^2/3$ is the Lorenz number and

$G_N$ is the electrical conductivity.

# Quasiparticle heat conduction in a superconductor

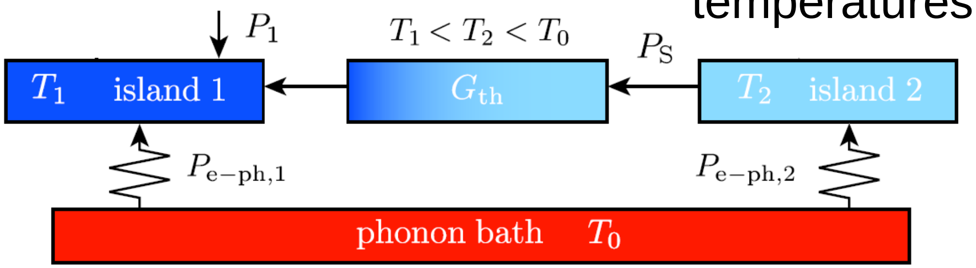



Bardeen et al. 1958

$$\gamma(T) = \frac{G_{\mathrm{th}}}{G_{\mathrm{th}}^{\mathrm{N}}} = \frac{3}{2\pi^2}\int_{\Delta/k_{\mathrm{B}}T}^{\infty} dx \frac{x^2}{\mathrm{sech}^2(x/2)} \simeq \frac{3}{2\pi^2}\left(8+8a+4a^2\right)e^{-a}$$

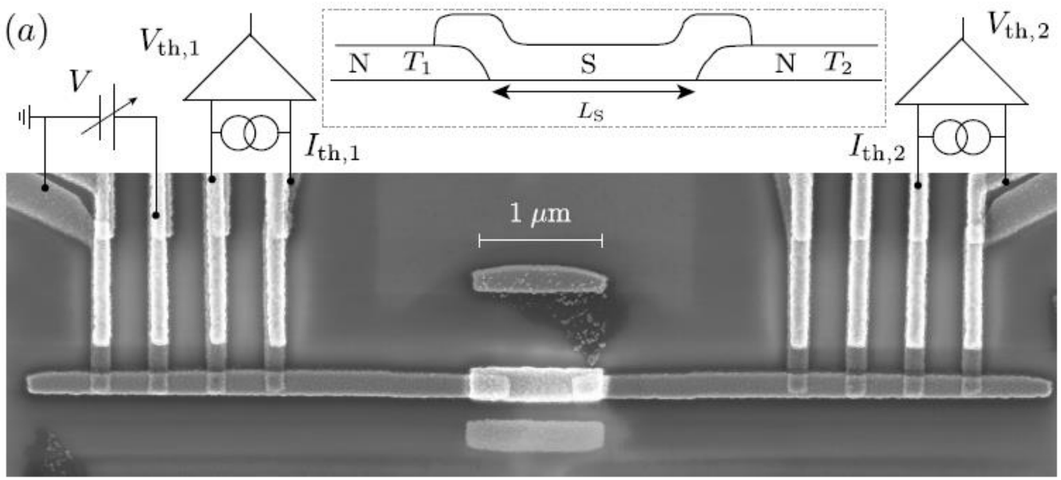
$$a = \Delta/k_{\mathrm{B}}T$$

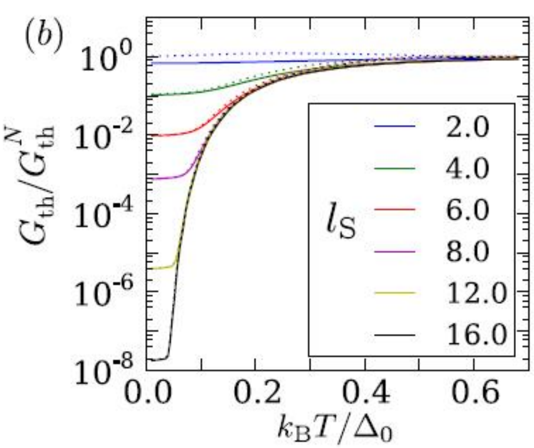
Heat transport is exponentially suppressed at low temperatures in a superconductor!




$$\frac{\Delta T_2}{\Delta T_1} = \frac{G_{\mathrm{th}}}{G_{\mathrm{th}} + G_{\mathrm{ep},2}}$$

(for small temperature differences)

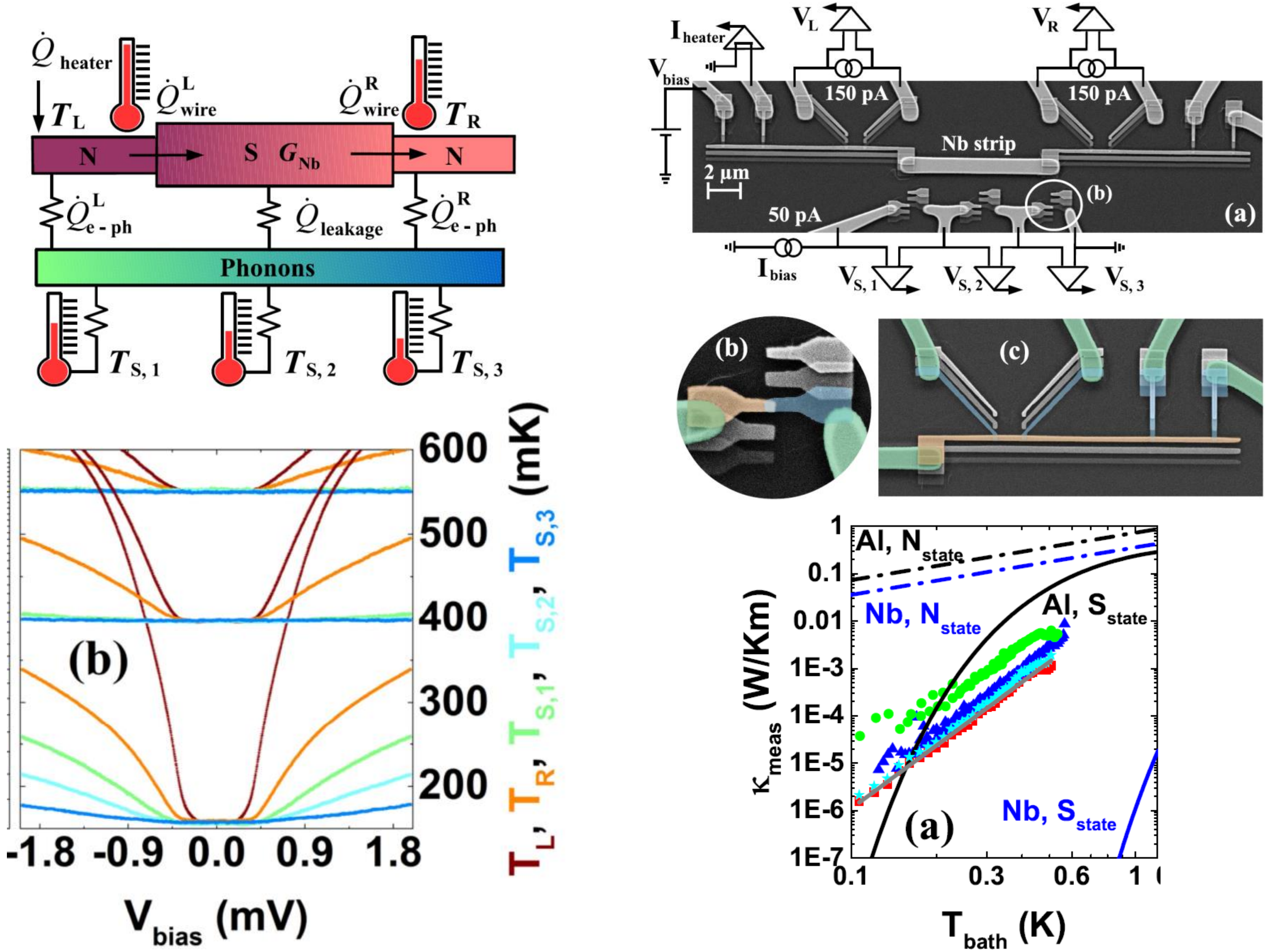


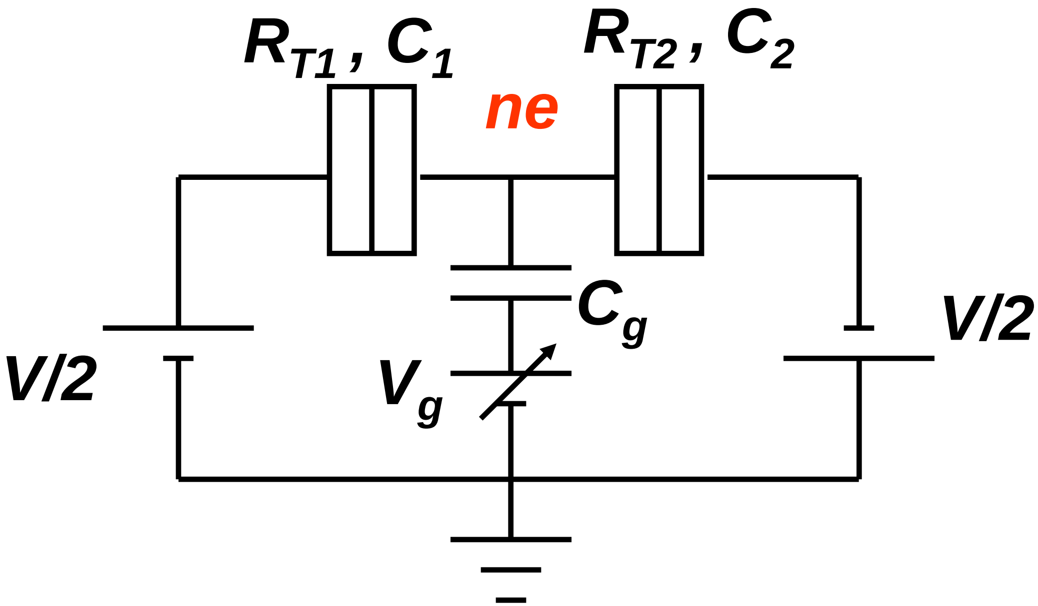

J. Peltonen et al., PRL 2010

# Recent measurement on Nb

A. Feshchenko et al. arXiv:1609.06519

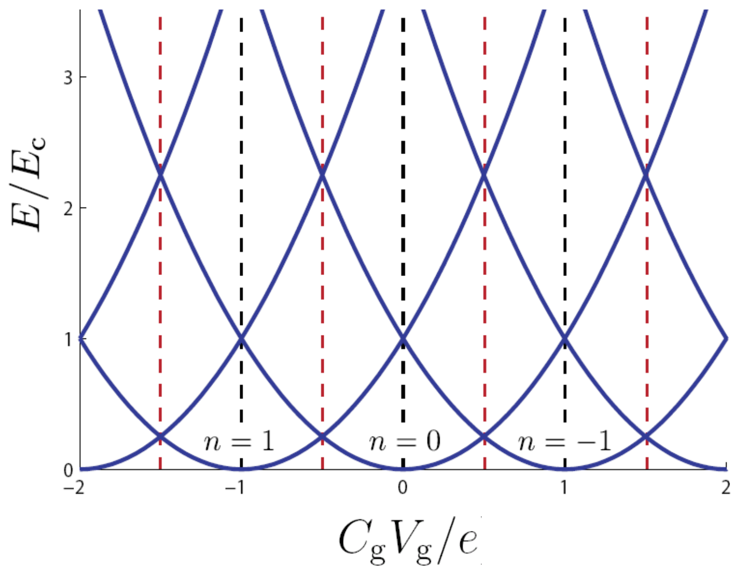


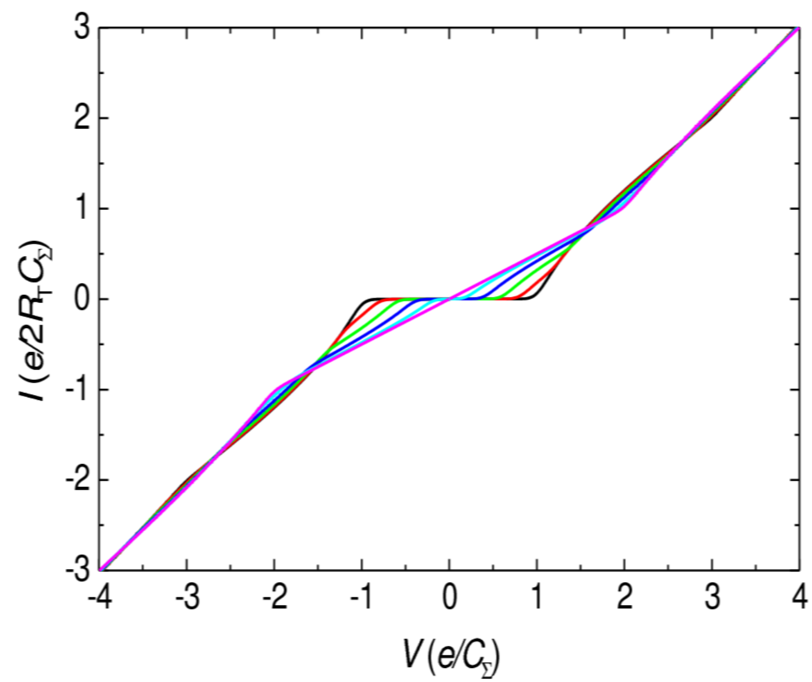
# Single-electron transistor (SET)



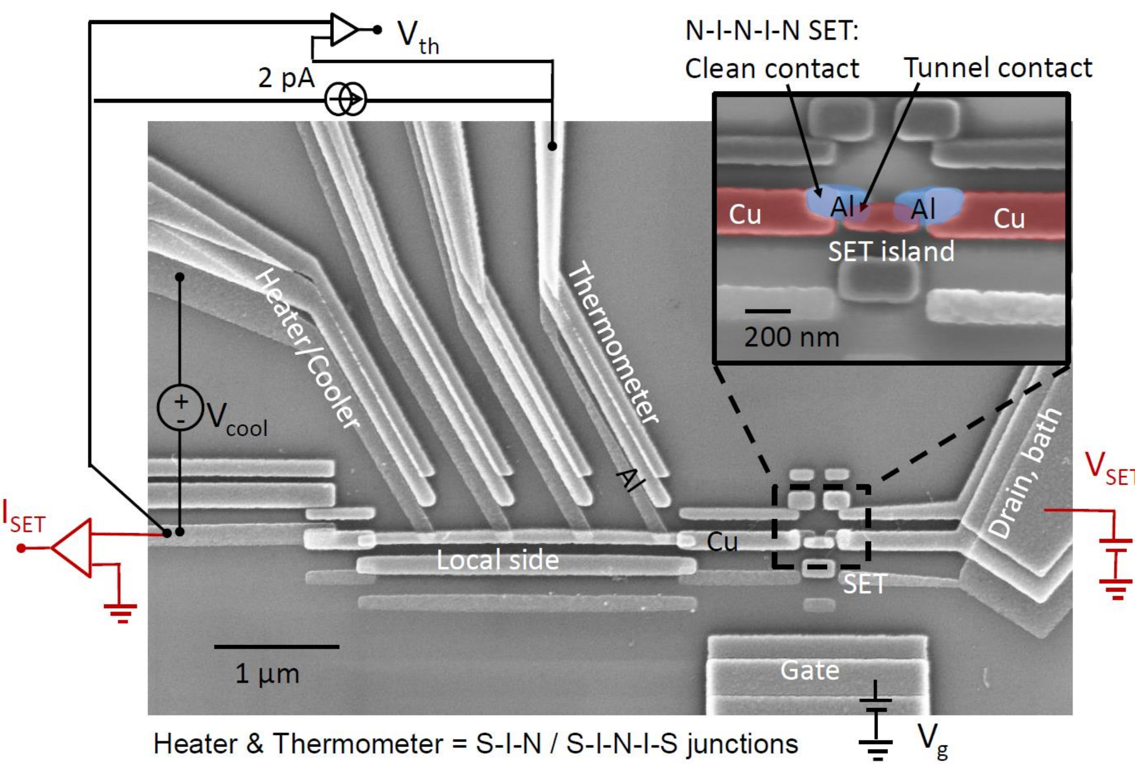

**Unit of charging energy:**

$$E_C = \frac{e^2}{2C_\Sigma}$$

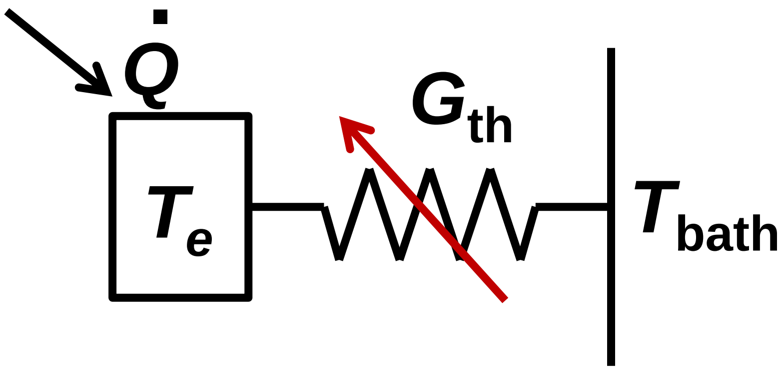


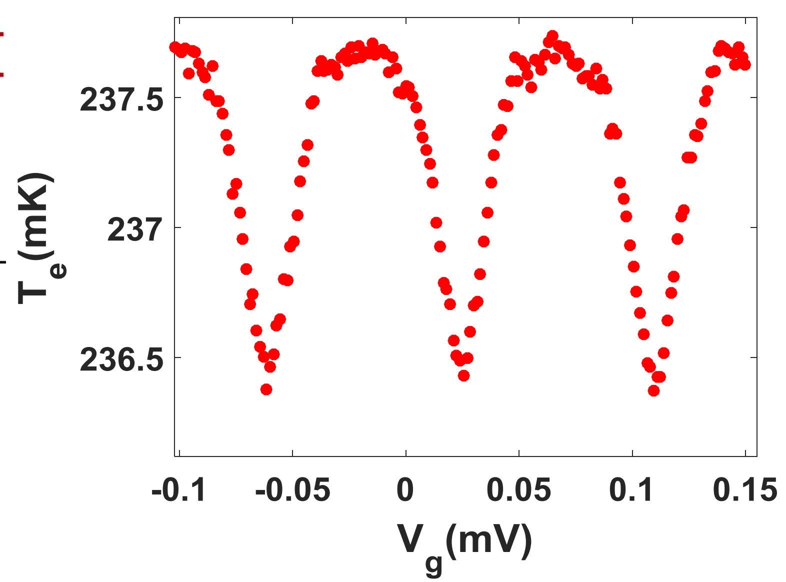
# Steady-state heat transport measurement through a single-electron transistor




Heater & Thermometer = S-I-N / S-I-N-I-S junctions



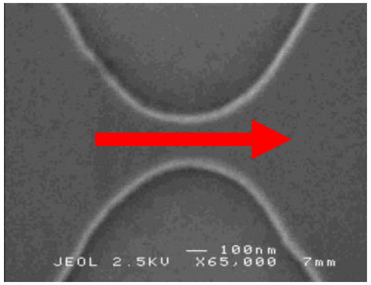

$$\dot{Q} = G_{\mathrm{th}}(T_e - T_{\mathrm{bath}})$$




**B. Dutta et al., in preparation**

# Quantum heat conductance

**Electrons:**




**Electrical conductance in a ballistic contact:**

$$\sigma_{\mathrm{Q}} = 2e^2/h$$

**Thermal conductance:**

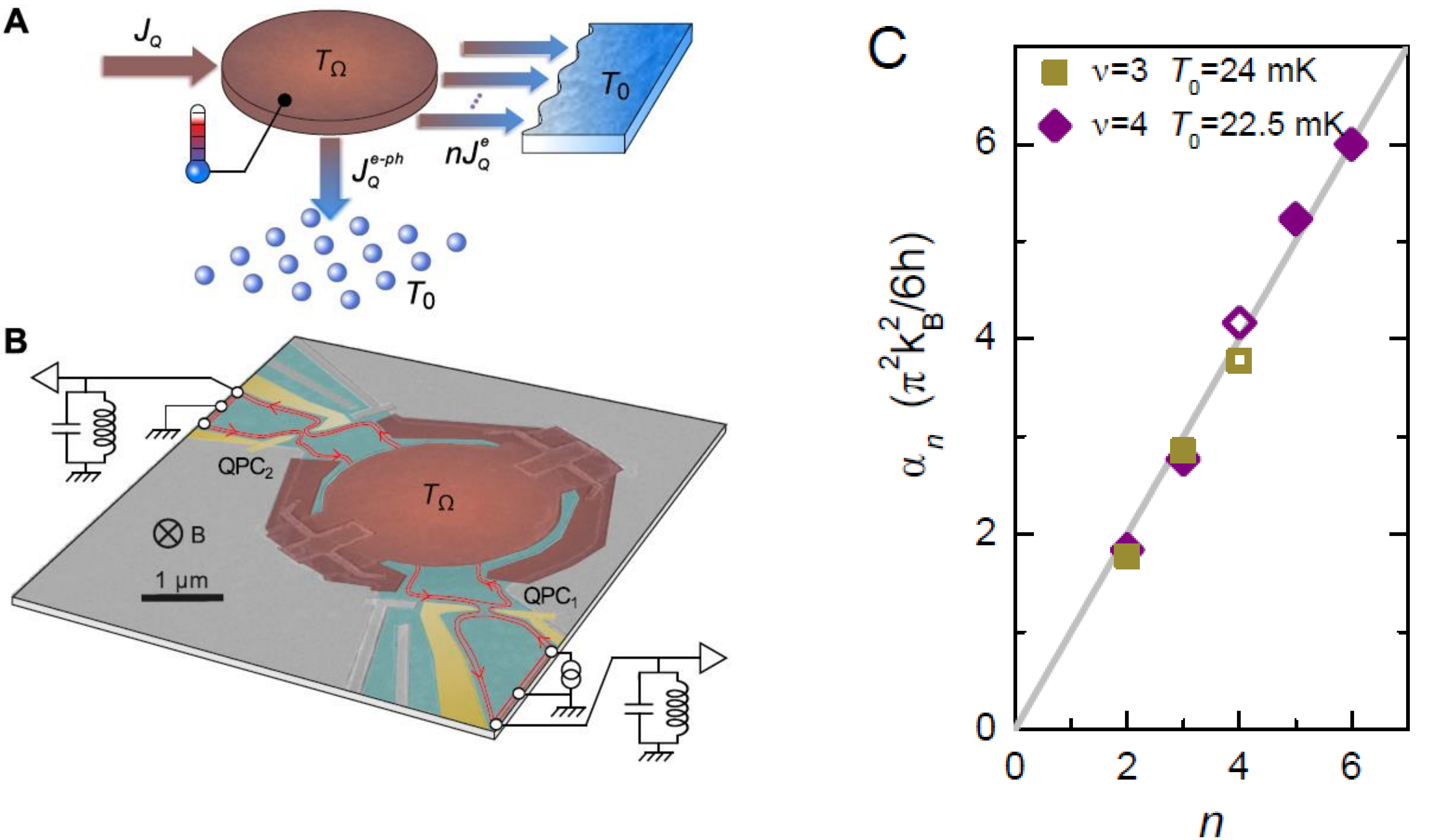
$$G_{\mathrm{Q}} = \frac{\pi k_{\mathrm{B}}^2}{6\hbar} T$$

**$G_Q$ and $\sigma_Q$ related by Wiedemann-Franz law**

**More generally:**

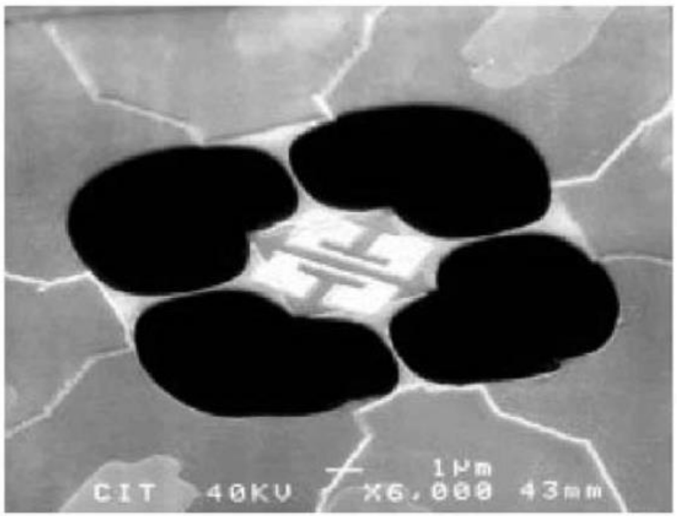
**Expression of $G_Q$ is expected to hold for carriers obeying arbitrary statistics, in particular for electrons, phonons, photons (Pendry 1983, Greiner et al. 1997, Rego and Kirczenow 1999, Blencowe and Vitelli 1999).**

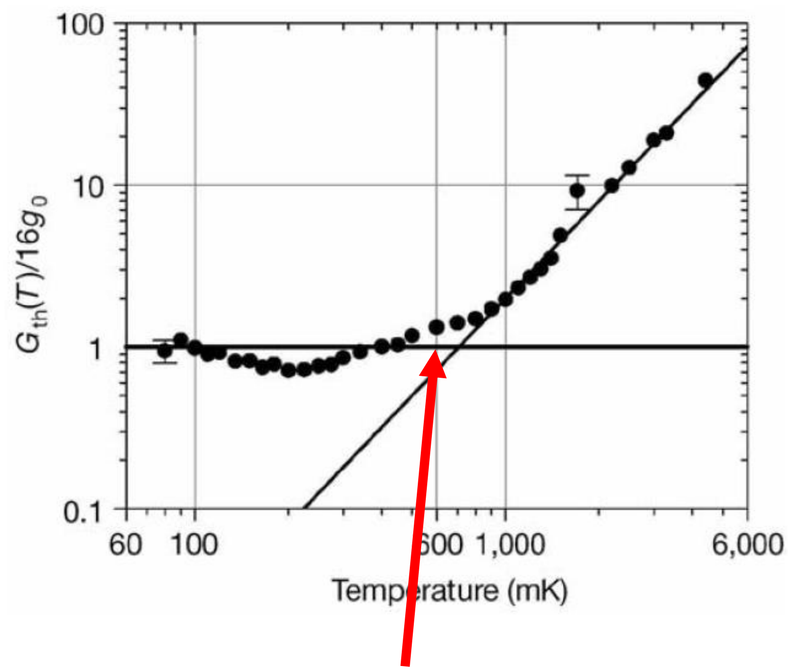
# Quantum limit of heat flow across single <u>electronic</u> channels




S. Jezouin et al., Science 342, 601 (2013)

# Quantum thermal conductance by phonons in a nanobridge



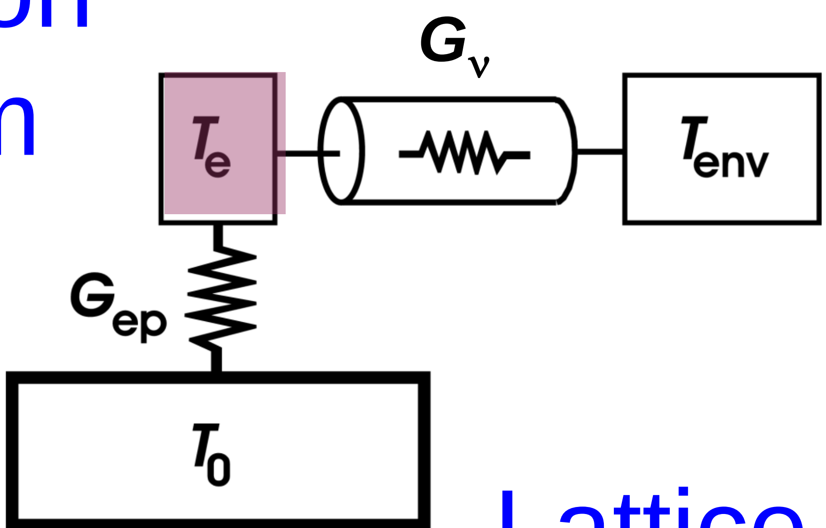




$$G = 4 \times 4 \times G_{\mathrm{Q}}$$

K. Schwab et al., Nature 404, 974 (2000)
C. Yung, D. Schmidt and A. Cleland, Appl. Phys. Lett. 81 31 (2002)

# Electromagnetic transfer of heat (photons)

Electron system

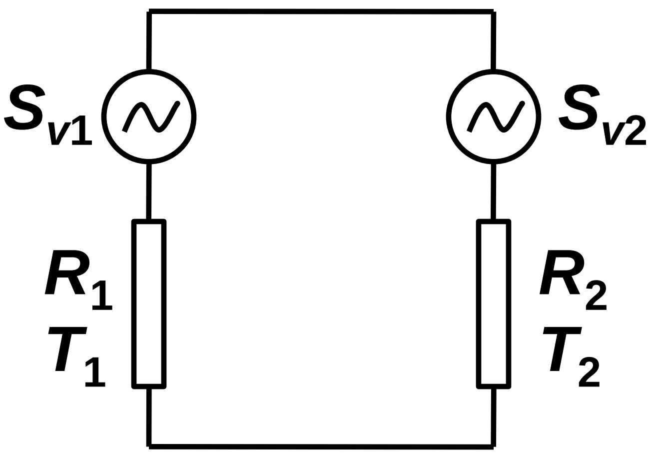



Electrical environment

Lattice

**Schmidt et al., PRL 93, 045901 (2004)**
**Meschke et al., Nature 444, 187 (2006)**
**Ojanen et al., PRB 76, 073414 (2007), PRL 100, 155902 (2008)**
**D. Segal, PRL 100, 105901 (2008)**
**L. Pascal et al., PRB 83, 125113 (2011)**

# Radiative heat transport in an electrical circuit




Voltage noise of a resistor:

$$S_{Vi}(\omega) = 4\hbar\omega R_i n_i(\omega)$$

Bose distribution:

$$n_i(\omega) = \frac{1}{e^{\hbar\omega/k_B T_i} - 1}$$

Current noise created by resistor 1:

$$S_{I1}(\omega) = S_{V1}(\omega)/|Z_{\text{tot}}|^2$$

$$Z_{\text{tot}} = R_1 + R_2$$

Spectrum of dissipation of energy created by resistor 1 and absorbed by resistor 2:

$$S_{P12}(\omega) = R_2 S_{I1}(\omega)$$

# Heat transported between two resistors


**Radiative contribution to net heat flow between electrons of 1 and 2:**

$$P_\nu = \int_0^\infty \frac{d\omega}{2\pi}\left[S_{P12}(\omega) - S_{P21}(\omega)\right] = r\frac{\pi k_B^2}{12\hbar}(T_1^2 - T_2^2)$$

**Coupling constant:**

$$r \equiv \frac{4R_1R_2}{(R_1+R_2)^2}$$

**Linearized expression for small temperature difference $\Delta T = T_1 - T_2$:**

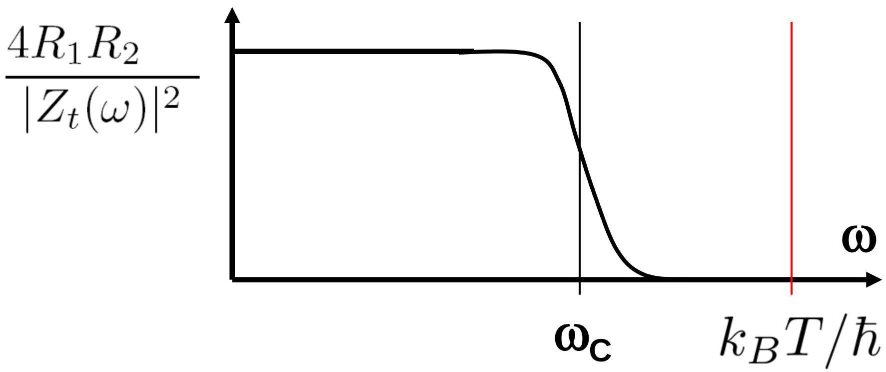
$$P_\nu = rG_Q\Delta T$$

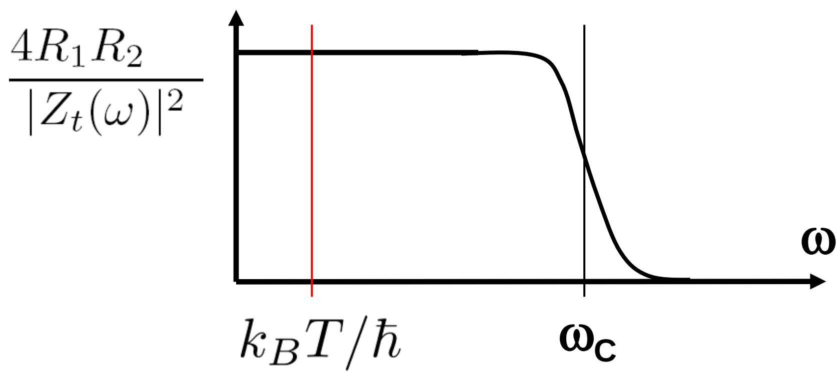
$$\boxed{G_Q = \frac{\pi k_B^2}{6\hbar}T}$$

$$G_\nu = rG_Q$$

# Classical or quantum heat transport?

$$P_\nu = \int_0^\infty \frac{d\omega}{2\pi}\frac{4R_1R_2\hbar\omega}{|Z_t(\omega)|^2}\left(\frac{1}{e^{\hbar\omega/k_BT_1}-1} - \frac{1}{e^{\hbar\omega/k_BT_2}-1}\right)$$



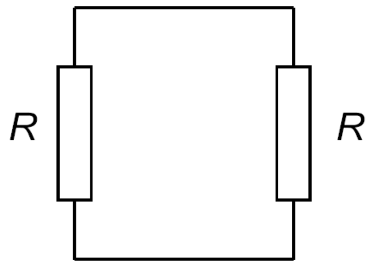

”Classical”

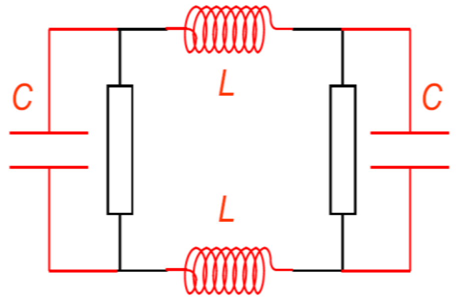
$$G_\nu \sim r k_B \omega_C$$

”Quantum”

$$G_\nu = r G_Q$$

# Classical or quantum heat transport?




$$C \sim \epsilon \ell$$

$$L \sim \mu \ell$$

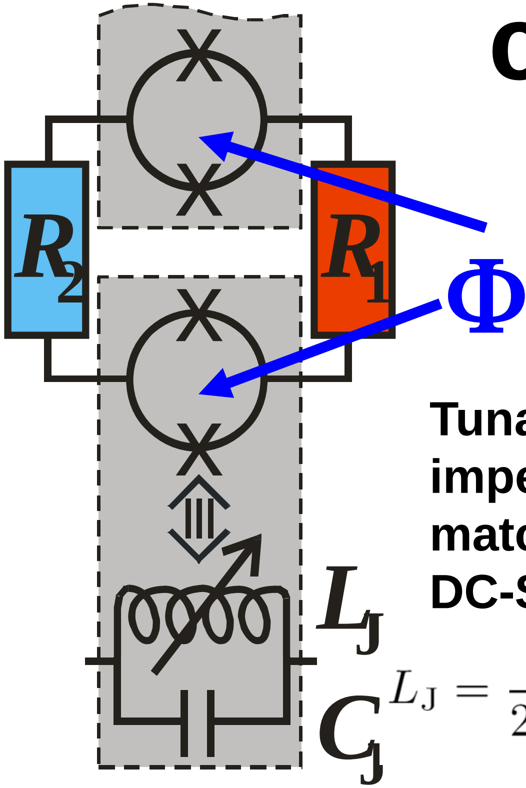
Classical: $$\frac{\hbar}{k_B T} \frac{1}{RC} \ll 1 \qquad \frac{\hbar}{k_B T} \frac{R}{L} \ll 1$$ Johnson, Nyquist 1928

Quantum limited: $$\frac{\hbar}{k_B T} \frac{1}{RC} \gg 1 \qquad \frac{\hbar}{k_B T} \frac{R}{L} \gg 1$$

$$T = 300 \text{ K}, \ \ell = 1 \text{ cm:} \ \frac{\hbar}{k_B T} \frac{1}{RC} \sim \frac{\hbar}{k_B T} \frac{R}{L} < 10^{-3} \ll 1$$

$$T = 100 \text{ mK}, \ \ell = 100 \ \mu\text{m:} \ \frac{\hbar}{k_B T} \frac{1}{RC} \sim \frac{\hbar}{k_B T} \frac{R}{L} \sim 10^{2} \gg 1$$

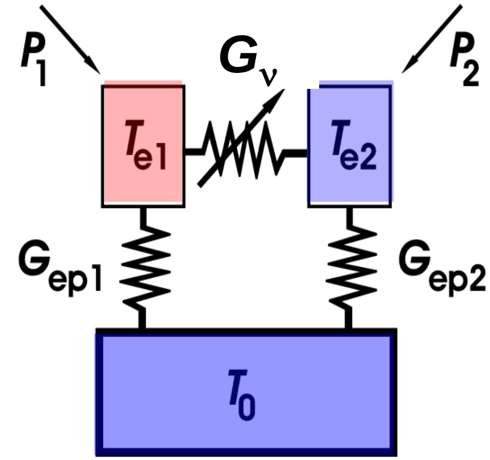
# Demonstration of photonic heat conduction

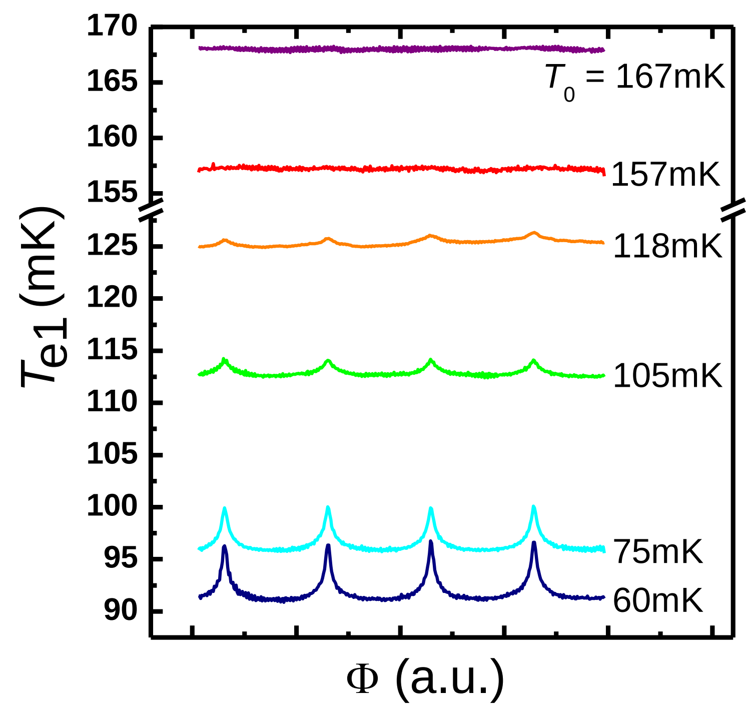


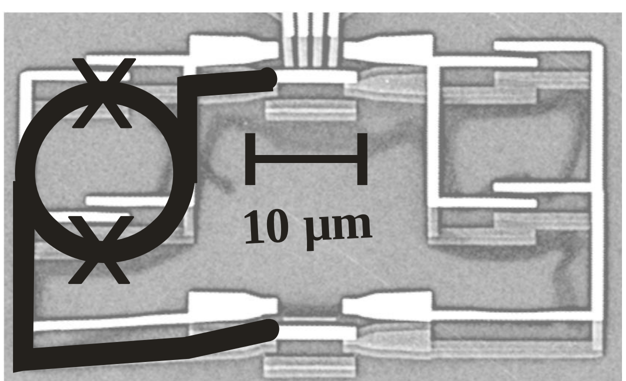

**Tunable impedance matching using DC-SQUIDs**

$$L_{\mathrm{J}} = \frac{\hbar}{2eI_{\mathrm{C},0}|\cos(\pi\Phi/\Phi_0)|}$$

**Thermal model**

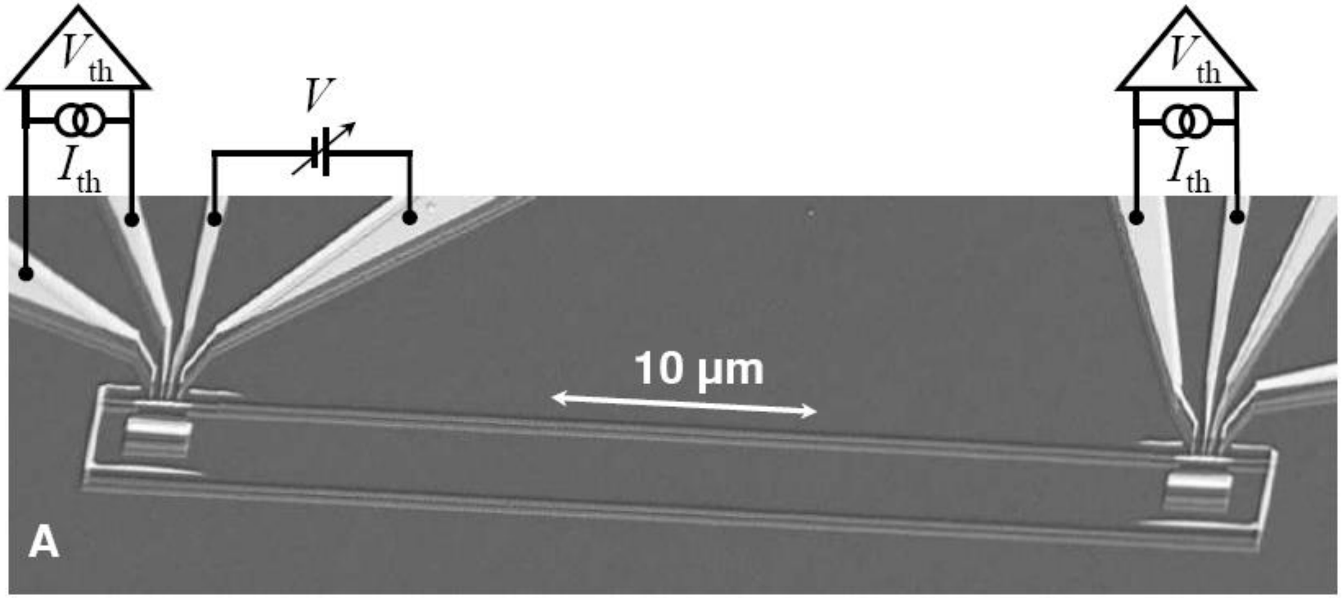


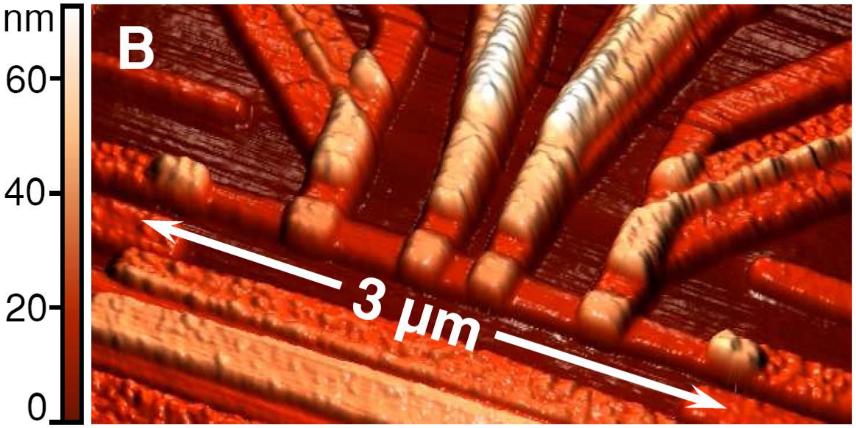




M. Meschke, W. Guichard and J.P., Nature 444, 187 (2006)

# 2nd experiment



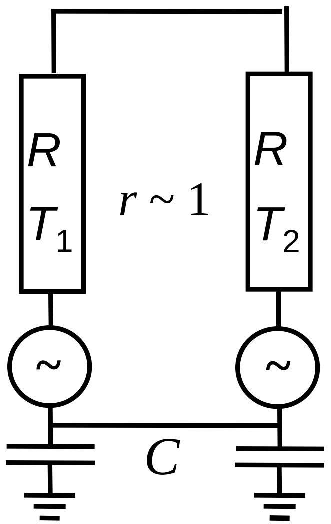




**SAMPLE A in a loop ("matched")**
**[SAMPLE B without loop ("not matched")]**

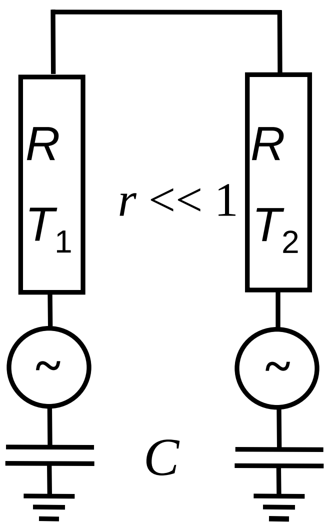
**A. Timofeev et al., Phys. Rev. Lett. 102, 200801 (2009).**

# Heat transport in different set-ups

**Loop geometry (Sample A)**




**Linear geometry (Sample B)**



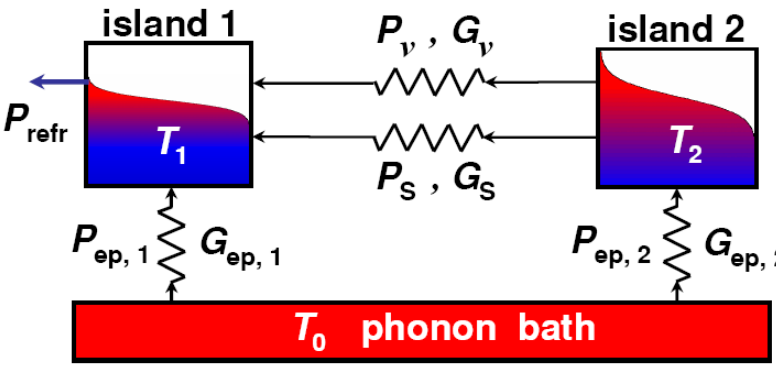

$$P_\nu^{\mathrm{A}} = G_{\mathrm{Q}} \Delta T$$

**for small temperature difference**

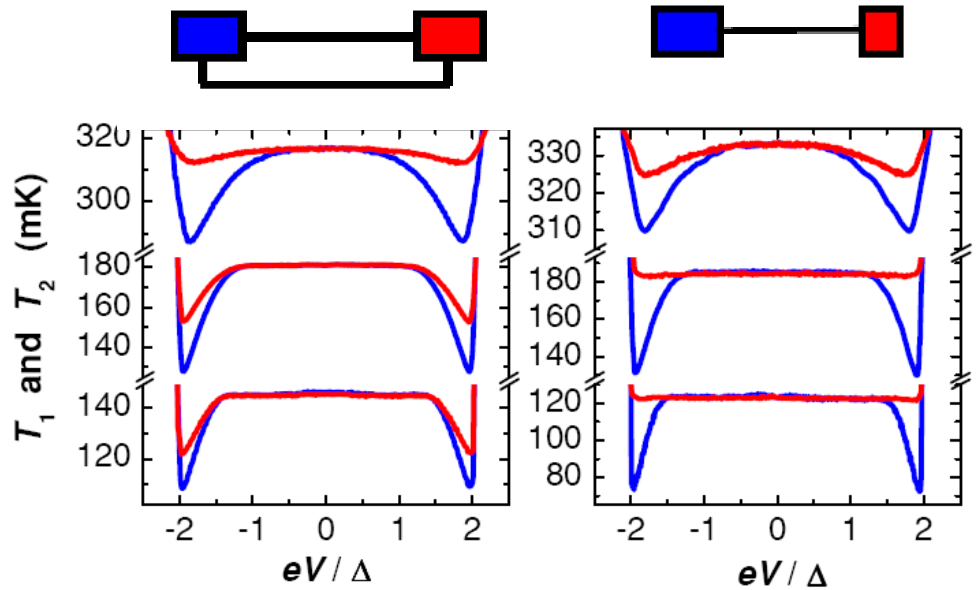
$$P_\nu^{\mathrm{B}} / P_\nu^{\mathrm{A}} = \frac{2}{5} (k_{\mathrm{B}} T R C / \hbar)^2 \simeq 10^{-3}$$

**in that experiment**

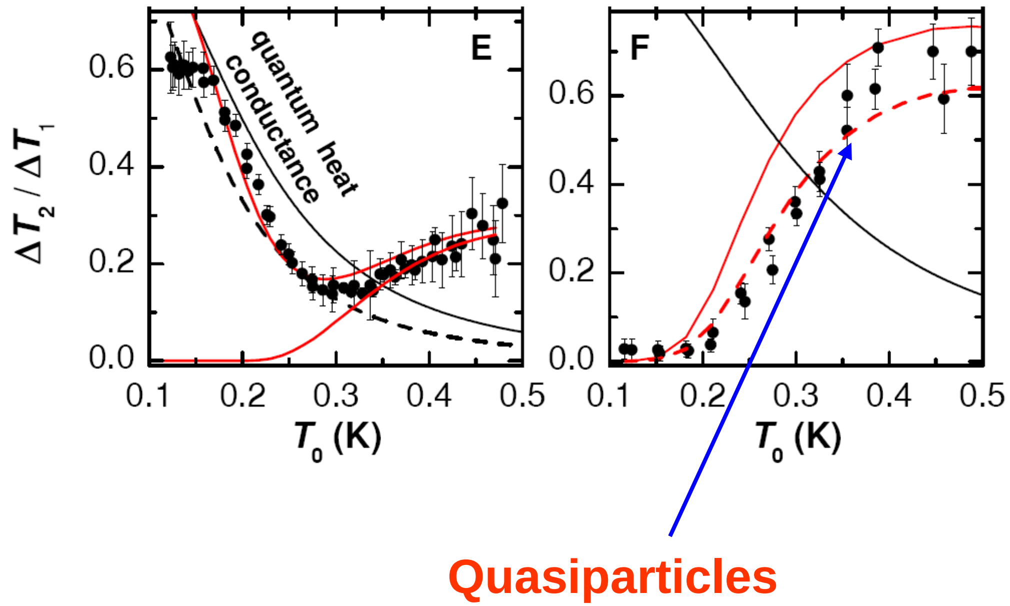
# Results in the two sample geometries



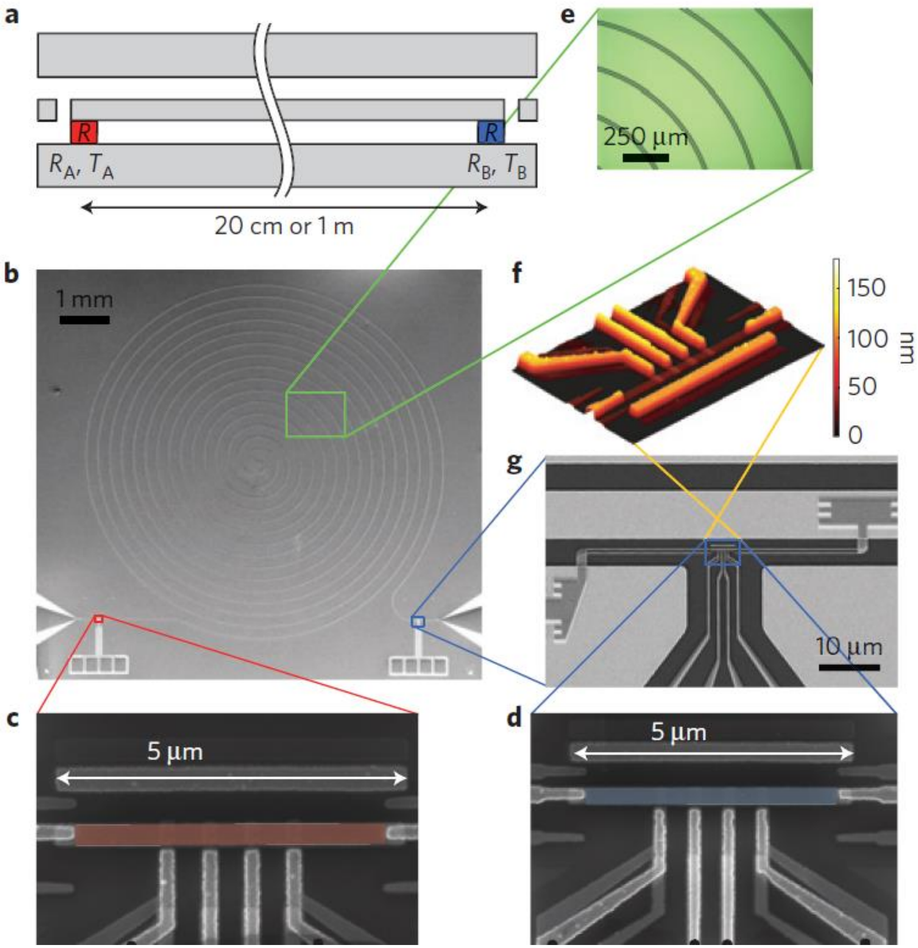

$$\frac{\Delta T_2}{\Delta T_1} = \frac{G_\nu + G_s}{G_\nu + G_s + G_{ep,2}}$$




**Heat transported by residual quasiparticles at $T$ > 0.3 K and by photons (in the loop sample) at $T$ < 0.3 K**



# Photon transport over a macroscopic distance




M. Partanen et al., Nature Physics 2016.

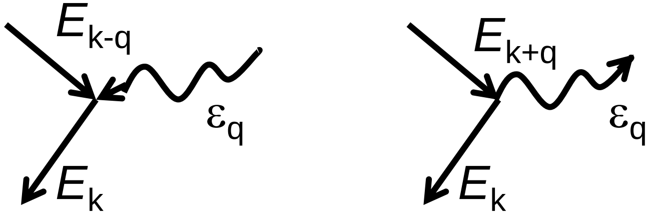
# Electron-phonon heat flux in metals at low *T*

**Hamiltonian of the electron-phonon system:**

$$H = H_e + H_p + H_{\text{ep}}$$

$$H_e = \sum_k \epsilon_k a_k^\dagger a_k \qquad H_p = \sum_q \hbar\omega_q c_q^\dagger c_q$$

$$H_{\text{ep}} = \gamma \sum_{k,q} \omega_q^{1/2} (a_k^\dagger a_{k-q} c_q + a_k^\dagger a_{k+q} c_q^\dagger)$$




Phonon absorption

Phonon emission

# Electron-phonon heat flux

$$\dot{H}_p = \frac{i}{\hbar}[H, H_p] = \frac{i}{\hbar}[H_{\mathrm{ep}}, H_p]$$

$$\dot{H}_p = i\gamma \sum_{k,q} \omega_q^{3/2} (a_k^\dagger a_{k-q} c_q - a_k^\dagger a_{k+q} c_q^\dagger)$$

Kubo formula (linear response):

$$\dot{Q}_{\mathrm{ep}} = \langle \dot{H}_p \rangle = -\frac{i}{\hbar} \int_0^t dt' \langle [\dot{H}_p(t), H_{\mathrm{ep}}(t')] \rangle_0$$

$$= \gamma^2 \Big( \int dE_k N(E_k) \int d^3q D(q) \omega_q^2 f(E_k)[1 - f(E_{k-q})][1 + n(\omega_q)]\delta(E_{k-q} - E_k + \hbar\omega_q)$$

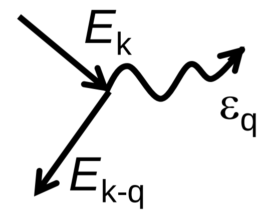
$$- \int dE_k N(E_k) \int d^3q D(q) \omega_q^2 f(E_k)[1 - f(E_{k+q})] n(\omega_q) \delta(E_{k+q} - E_k - \hbar\omega_q) \Big)$$

$= P_e - P_a$

# Electron-phonon heat flux in metals at low *T*

The rate at which an electron at wave vector $\mathbf{k}$ is scattered to $\mathbf{k}' = \mathbf{k} - \mathbf{q}$ with a phonon $\mathbf{q}$ emitted is given by

$$\tau^{-1}_{\mathbf{k},\mathbf{k}-\mathbf{q}} = \frac{2\pi}{\hbar}\mathcal{M}^2\delta(E_{\mathbf{k}} - E_{\mathbf{k}-\mathbf{q}} - \epsilon_{\mathbf{q}})[1 - f(E_{\mathbf{k}-\mathbf{q}})][n_p(\mathbf{q}) + 1]. \tag{1}$$




Here $\mathcal{M}^2$ is the square of the matrix element of electron-phonon coupling, and

$$n_p(\mathbf{q}) = \frac{1}{e^{\beta_p \epsilon_{\mathbf{q}}} - 1} \tag{2}$$

the phonon occupation number. We use $\beta_i \equiv (k_B T_i)^{-1}$.

The electrons emit energy to phonons at the rate $P_e = \int dE_{\mathbf{k}} N(E_F) f(E_{\mathbf{k}}) \int d^3q D_p(\mathbf{q})\epsilon_{\mathbf{q}}\tau^{-1}_{\mathbf{k},\mathbf{k}-\mathbf{q}}$, i.e.,

$$P_e = \frac{2\pi}{\hbar}\int dE_{\mathbf{k}} N(E_F) f(E_{\mathbf{k}}) \int d^3q D_p(\mathbf{q})\mathcal{M}^2\epsilon_{\mathbf{q}}\delta(E_{\mathbf{k}} - E_{\mathbf{k}-\mathbf{q}} - \epsilon_q)[1 - f(E_{\mathbf{k}-\mathbf{q}})][n_p(\mathbf{q}) + 1]. \tag{3}$$

Here $D_p(\mathbf{q})$ is the phonon density of states, and $N(E_F)$ is the density of states for electrons at the Fermi level.

Correspondingly, electrons absorb energy from phonons at the rate

$$P_a = \frac{2\pi}{\hbar}\int dE_{\mathbf{k}} N(E_F) f(E_{\mathbf{k}}) \int d^3q D_p(\mathbf{q})\mathcal{M}^2\epsilon_{\mathbf{q}}\delta(E_{\mathbf{k}} - E_{\mathbf{k}+\mathbf{q}} + \epsilon_{\mathbf{q}})[1 - f(E_{\mathbf{k}+\mathbf{q}})]n_p(\mathbf{q}). \tag{4}$$

The net heat flux between electrons and phonons is then

$$\dot{Q} = P_e - P_a. \tag{5}$$

We can first integrate over the angle $\theta$ between electron and phonon wave vectors. In general for 3D phonons we have $D_p(\mathbf{q}) = \mathcal{V}/(2\pi)^3$, where $\mathcal{V}$ is the volume of the system. In spherical coordinates, $\int d^3q \to 2\pi \int_0^\infty dqq^2 \int_{-1}^1 d(\cos\theta)$. Further, $E_\mathbf{k} = \frac{\hbar^2 k^2}{2m}$ and $E_{\mathbf{k}\mp\mathbf{q}} = \frac{\hbar^2(\mathbf{k}\mp\mathbf{q})^2}{2m} \simeq E_\mathbf{k} \mp \frac{\hbar^2 k_F}{m} q\cos\theta$, where the last approximation is due to $k \simeq k_F$ and $q \ll k_F$. Then, collecting the angle dependent terms and integrating over $\cos\theta$, we have

$$\begin{aligned}
&\int d^3q D_p(\mathbf{q})\delta(E_\mathbf{k} - E_{\mathbf{k}\mp\mathbf{q}} \mp \epsilon_\mathbf{q})[1 - f(E_{\mathbf{k}\mp\mathbf{q}})] \\
&= \frac{\mathcal{V}}{(2\pi)^2}\int_0^\infty dqq^2 \int_{-1}^1 d(\cos\theta)\delta(\pm\frac{\hbar^2 k_F}{m} q\cos\theta \mp \epsilon_\mathbf{q})[1 - f(E_\mathbf{k} \mp \frac{\hbar^2 k_F}{m} q\cos\theta)] \\
&= \frac{m\mathcal{V}}{(2\pi)^2\hbar^2 k_F}\int_0^\infty dqq[1 - f(E_\mathbf{k} \mp \epsilon_\mathbf{q})].
\end{aligned} \tag{6}$$

Inserting this result into Eqs. (3) and (4), using the standard coupling with $\mathcal{M}^2 = \mathcal{M}_0^2 q$ (scalar deformation potential, longitudinal phonons), writing $\epsilon \equiv \epsilon_\mathbf{q} = \hbar c_\ell q$, and dropping the index $\mathbf{k}$ we obtain

$$P_e = \frac{m\mathcal{V}N(E_F)\mathcal{M}_0^2}{2\pi\hbar^6 k_F c_\ell^3}\int_0^\infty d\epsilon\epsilon^3[n_p(\mathbf{q}) + 1]\int_{-\infty}^\infty dEf(E)[1 - f(E - \epsilon)] \tag{7}$$

and

$$P_a = \frac{m\mathcal{V}N(E_F)\mathcal{M}_0^2}{2\pi\hbar^6 k_F c_\ell^3}\int_0^\infty d\epsilon\epsilon^3 n_p(\mathbf{q})\int_{-\infty}^\infty dEf(E)[1 - f(E + \epsilon)]. \tag{8}$$

Using (5), we then have

$$\begin{aligned}
\dot{Q} = \frac{m\mathcal{V}N(E_F)\mathcal{M}_0^2}{2\pi\hbar^6 k_F c_\ell^3}\int_0^\infty d\epsilon\epsilon^3 \{ &\int_{-\infty}^\infty dEf(E)[1 - f(E - \epsilon)] + \\
n_p(\mathbf{q})[&\int_{-\infty}^\infty dEf(E)[1 - f(E - \epsilon)] - \int_{-\infty}^\infty dEf(E)[1 - f(E + \epsilon)]]\}.
\end{aligned} \tag{9}$$

Using the identity

$$f(E)[1 - f(E+x)] = \frac{f(E) - f(E+x)}{1 - e^{-\beta_e x}}, \tag{10}$$

and the symmetry $f(-E) = 1 - f(E)$, and $\int_{-\infty}^{\infty} dE[f(E) - f(E+\epsilon)] = \epsilon$, we can simplify (9) into

$$\dot{Q} = \frac{m\mathcal{V}N(E_F)\mathcal{M}_0^2}{2\pi\hbar^6 k_F c_\ell^3} \int_0^\infty d\epsilon \epsilon^4 [\frac{1}{e^{\beta_e \epsilon} - 1} - \frac{1}{e^{\beta_p \epsilon} - 1}]. \tag{11}$$

This can be readily integrated to yield

$$\dot{Q} = \frac{12\zeta(5) m\mathcal{V}N(E_F)\mathcal{M}_0^2}{\pi\hbar^6 k_F c_\ell^3}(\beta_e^{-5} - \beta_p^{-5}), \tag{12}$$

which is equivalent to the well known form

$$\dot{Q} = \Sigma\mathcal{V}(T_e^5 - T_p^5), \tag{13}$$

by identifying

$$\Sigma \equiv \frac{12\mathcal{M}_0^2 \zeta(5) N(E_F) k_B^5}{\pi\hbar^5 v_F c_\ell^3}. \tag{14}$$