# Cavity Optomechanics

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### **Radiation forces**

### baryon-photon fluid: sound speed $\,c/\sqrt{3}$



#### **Radiation pressure**



(Comet Hale-Bopp; by Robert Allevo)

#### **Radiation pressure**

#### Nichols and Hull, 1901 Lebedev, 1901

#### A PRELIMINARY COMMUNICATION ON THE PRESSURE OF HEAT AND LIGHT RADIATION.

BY E. F. NICHOLS AND G. F. HULL.

MAXWELL,<sup>1</sup> dealing mathematically with the stresses in an electro-magnetic field, reached the conclusion that "in a medium in which waves are propagated there is a pressure normal to the waves and numerically equal to the energy in unit volume."



Nichols and Hull, Physical Review 13, 307 (1901)

Laser Interferometer for Gravitational Wave Detection

4 km

(LIGO Livingston)

Image: SXS project

#### **Observation of Gravitational Waves**



LIGO collaboration, Phys. Rev. Lett. 116, 061102 (2016)

Laser Interferometer for Gravitational Wave Detection

4 km

(LIGO Livingston)

#### How precise can one measure?



Laser power

#### How precise can one measure?



(Painter group, Caltech)

2 µm











### $\hat{H} = \hbar \omega_{\rm cav}(\hat{x})\hat{a}^{\dagger}\hat{a} + \hbar \omega_M \hat{b}^{\dagger}\hat{b} + \dots$

...any dielectric moving inside a cavity generates an optomechanical interaction!

## A bit of history

#### First cavity optomechanics experiments



Static bistability in an optical cavity experiment Dorsel, McCullen, Meystre, Vignes, Walther PRL 1983



### A zoo of devices

### The zoo of optomechanical systems (2005-now)



#### The zoo of optomechanical systems



#### The zoo of optomechanical systems





#### **Optomechanics:** general outlook



**Regal/Lehnert** 



# **Fundamental tests of quantum mechanics in a new regime:** entanglement with 'macroscopic' objects, unconventional decoherence?

[e.g.: gravitationally induced?]

Mechanics as a 'bus' for connecting hybrid components: superconducting qubits, spins, photons, cold atoms, ....

#### **Precision measurements**

small displacements, masses, forces, and accelerations



**Optomechanical circuits & arrays** Exploit nonlinearities for classical and quantum information processing, storage, and amplification; study collective dynamics in arrays

### Sensing mechanical motion at the quantum limit

Laser Interferometer for Gravitational Wave Detection

LIGO Livingston

#### **Optical displacement detection**



#### Thermal fluctuations of a harmonic oscillator



Classical equipartition theorem:

$$\frac{m\omega_M^2}{2} \langle x^2 \rangle = \frac{k_B T}{2} \Rightarrow \langle x^2 \rangle = \frac{k_B T}{m\omega_M^2}$$
 extract temperature!

Direct time-resolved detection

Analyze fluctuation spectrum of x

#### **Fluctuation spectrum**



#### **Fluctuation spectrum**



#### **Fluctuation spectrum**

$$\tilde{x}(\omega) = \frac{1}{\sqrt{\tau}} \int_{0}^{\tau} dt e^{i\omega t} x(t)$$
$$S_{xx}(\omega) \equiv \left\langle |\tilde{x}(\omega)|^{2} \right\rangle =$$
$$\approx \int_{-\infty}^{+\infty} dt e^{i\omega t} \left\langle x(t)x(0) \right\rangle$$

#### **Wiener-Khinchin theorem**

$$\langle |\tilde{x}(\omega)|^2 \rangle \equiv S_{xx}(\omega)$$
area yields
area yields
variance of x: 
$$\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} S_{xx}(\omega) = \langle x^2 \rangle$$

General relation between noise spectrum and linear response susceptibility

 $\begin{array}{l} \left< \delta x \right> (\omega) = \chi_{xx}(\omega) F(\omega) \\ \text{susceptibility} \end{array}$ 

$$S_{xx}(\omega) = \frac{2k_BT}{\omega} \operatorname{Im}\chi_{xx}(\omega)$$
 (classical limit)

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 (classical limit)

 $\omega_M$ 

for the damped oscillator:  

$$m\ddot{x} + m\omega_{M}^{2}x + m\Gamma\dot{x} = F$$

$$x(\omega) = \frac{1}{m(\omega_{M}^{2} - \omega^{2}) - im\Gamma\omega}F(\omega)$$

$$\chi_{xx}(\omega)$$
#### **Displacement spectrum**





#### **Measurement noise**



#### **Measurement noise**



Two contributions to  $x_{noise}(t)$ 

- I. measurement imprecision laser beam (shot noise limit!)
- 2. measurement back-action:
- fluctuating force on system
- noisy radiation pressure force

#### "Standard Quantum Limit"



#### "Standard Quantum Limit"



#### Best case allowed by quantum mechanics:

 $S_{xx}^{(\text{meas})}(\omega) \ge 2 \cdot S_{xx}^{T=0}(\omega) \qquad \text{``Standard quantum limit} \\ (SQL) \text{ of displacement} \\ \text{detection''}$ 

...as if adding the zero-point fluctuations a second time: "adding half a photon"

#### Notes on the SQL



- "weak measurement": integrating the signal over time to suppress the noise
- trying to detect slowly varying "quadratures of motion":  $\hat{x}(t) = \hat{X}_1 \cos(\omega_M t) + \hat{X}_2 \sin(\omega_M t)$  $\left[\hat{X}_1, \hat{X}_2\right] = 2x_{\text{ZPF}}^2$  Heisenberg is the reason for SQL! no limit for instantaneous measurement of x(t)!
- SQL means: detect  $\hat{X}_{1,2}$  down to  $x_{\text{ZPF}}$  on a time scale  $1/\Gamma$  Impressive:  $x_{\text{ZPF}} \sim 10^{-15} m$ !

#### **Enforcing the SQL (Heisenberg)** in a weak optical measurement



reflection phase shift:  $\theta = 2kx$ (here: free space)

N photons arrive in time t

fluctuations:  $\delta N = \sqrt{VarN} = \sqrt{\bar{N}}$  a coherent laser beam

Poisson distribution for

I. Uncertainty in phase estimation:

$$\delta N \cdot \delta \theta \ge \frac{1}{2} \Rightarrow \delta \theta \ge \frac{1}{2\sqrt{\bar{N}}} \Rightarrow \delta x = \frac{\delta \theta}{2k} \sim \frac{1}{2\sqrt{\bar{N}}2k}$$
  
2. Fluctuating force: momentum transfer  $\Delta p = 2\hbar k \cdot N$   
 $\delta p = \sqrt{\operatorname{Var}\Delta p} = 2\hbar k \sqrt{\bar{N}}$   
Uncertainty product:  $\delta x \delta p \ge \frac{\hbar}{2}$  Heisenberg

 $\boldsymbol{\angle}$ 

## Quantum dynamics

#### **Optomechanical Hamiltonian**



#### **Optomechanical Hamiltonian**



#### **Converting photons into phonons**



#### **Converting photons into phonons**



#### **Optomechanical Hamiltonian**



#### **Optomechanical Interaction: Nonlinear**

## $\hat{a}^{\dagger}\hat{a}(\hat{b}^{\dagger}+\hat{b})$

#### "Linearized" Optomechanical Hamiltonian

### "laser-enhanced optomechanical coupling": $g=g_0\alpha$

#### $g_0 \sim \mathrm{Hz} - \mathrm{MHz}$

bare optomechanical coupling (geometry, etc.: fixed!) laser-driven cavity amplitude tuneable! **phase**!

 $\alpha$ 

After linearization: two linearly coupled harmonic oscillators!



#### **Photon-phonon polaritons**



#### **Photon-phonon polaritons**



After linearization: two linearly coupled harmonic oscillators!



After linearization: two linearly coupled harmonic oscillators!



#### Aside: Quantum Heat Engine in Optomechanics



Keye Zhang, Francesco Bariani, Pierre Meystre; Phys. Rev. Lett. 2014

#### **Different regimes**



#### **Effective Optomechanical Damping Rate**



#### **Effective Optomechanical Damping Rate**



#### Laser-cooling towards the ground state



# "The slopes of Optomechanics"



#### Linear Optomechanics

Displacement detection
 Optical Spring
 Cooling & Amplification
 Two-tone spectroscopy
 State transfer, pulsed operation
 Wavelength conversion
 Radiation Pressure Shot Noise
 Squeezing of Light
 Squeezing of Mechanics
 Entanglement
 Precision measurements

#### **Optomechanical Arrays**

Bandstructure in arrays
Synchronization in arrays
Transport of photons & phonons
Topological phases

Nonlinear Classical Optomechanics
Self-induced mechanical oscillations
Synchronization of oscillations
Chaos

#### Nonlinear Quantum Optomechanics

- Phonon number detection
- Phonon shot noise
- Photon blockade
- Optomechanical "which-way" expt.
- Nonclassical mechanical q. states
- Optomechanical matter-wave interference
- Nonlinear OMIT
- Noncl. via Conditional Detection
- Single-photon sources
- Coupling to two-level systems

## Optomechanical wavelength conversion

![](_page_65_Figure_0.jpeg)

![](_page_66_Picture_0.jpeg)

![](_page_67_Picture_0.jpeg)

#### optics to optics:

![](_page_68_Figure_1.jpeg)

#### microwave/RF to optics:

![](_page_69_Picture_1.jpeg)

![](_page_69_Picture_2.jpeg)

#### Cleland 2013

![](_page_69_Figure_4.jpeg)

#### Schliesser, Polzik 2014

Lehnert, Regal 2014

## Optomechanical Arrays

#### **Single-mode optomechanics**

![](_page_71_Figure_1.jpeg)

✓ displacement sensing
 ✓ cooling
 ✓ strong coupling
 ✓ self-oscillations (limit cycles)
# Many modes











#### **First realizations**



Lipson group, Cornell arXiv: 1505.02009 (synchronization)

= free-standing photonic crystal structures (Painter group)

# localized optical and vibrational (GHz) mode



# advantages:

tight vibrational confinement: high frequencies, small mass (stronger quantum effects)

tight optical confinement: large optomechanical coupling (100 GHz/nm)

integrated on a chip

# Safavi-Naeini et al PRL 2014 Eichenfield et al Nature 2009

#### **Optomechanical arrays**

# Optomechanical array: Many coupled optomechanical cells



Possible design based on "snowflake" 2D optomechanical crystal (Painter group), here: with suitable defects forming a superlattice (array of cells)

### Modeling an optomechanical array

 $\hat{a}_{j}$ 

strengt

0

Tight-binding model for photons & phonons hopping and interacting on a lattice

 $\Delta = \omega_L - \omega_{\rm opt}$ 

optical coupling: 0 optomech. interaction laser drive each cell:  $\hat{H}_{\text{om},j} = -\Delta \hat{a}_j^{\dagger} \hat{a}_j + \Omega \hat{b}_j^{\dagger} \hat{b}_j - g_0 (\hat{b}_j^{\dagger} + \hat{b}_j) \hat{a}_j^{\dagger} \hat{a}_j + \alpha_L (\hat{a}_j^{\dagger} + \hat{a}_j)$  $\hat{H}_{\text{int}} = - \mathbf{J} \sum \left( \hat{a}_i^{\dagger} \hat{a}_j + \hat{a}_i \hat{a}_j^{\dagger} \right) - \mathbf{K} \sum \left( \hat{b}_i^{\dagger} \hat{b}_j + \hat{b}_i \hat{b}_j^{\dagger} \right)$  $\langle i,j \rangle$  optical coupling  $\langle i,j \rangle$  mechanical coupling (photon tunneling) (phonon tunneling)

Max Ludwig, FM, Phys. Rev. Lett. 111, 073602 (2013)

#### **Optomechanical Arrays**

# global view: light-tunable metamaterial for photons & phonons



similar in spirit: optical lattices nonlinear optical materials

#### conceptually simple: one material, with holes

laser drive

#### Synthetic magnetic fields for photons/phonons

Dirac Physics

Synchronization and Pattern Formation Topological Phases

Transport (edge states/wires)

Nonequilibrium dynamics/Quench physics/Thermalization

Quantum Information Processing

Strongly Correlated Quantum Physics?

Tuneable/reconfigurable in-situ

#### All-optical control/readout

# Nonlinear Optomechanics (Classical Regime)

#### **Nonlinear dynamics**

# blue-detuned laser: anti-damping! < 0 $\Gamma = \Gamma_M + \Gamma_{\rm opt}$

# **Nonlinear Dynamics**

#### **Nonlinear Dynamics**

# Beyond some laser input power threshold: instability



Self-sustained mechanical oscillations!













#### An optomechanical cell as a Hopf oscillator



#### Amplitude fixed, phase undetermined!

#### An optomechanical cell as a Hopf oscillator



### Amplitude fixed, phase undetermined!



Collective dynamics in an array of coupled cells? Phase-locking: **synchronization**!

# Synchronization: Huygens' observation



(Huygens' original drawing!)

Coupled pendula synchronize... ...even though intrinsic frequencies slightly different important in physics, chemistry, biology, ... Josephson arrays, laser arrays, ...

#### The Kuramoto model



Kuramoto model:

$$\dot{\varphi}_1 = \Omega_1 + K \sin(\varphi_2 - \varphi_1)$$
$$\dot{\varphi}_2 = \Omega_2 + K \sin(\varphi_1 - \varphi_2)$$

captures essential features
often found as limiting model

Kuramoto 1975, 1984 Acebron et al., Rev. Mod. Phys. 77, 137 (2005)

#### The Kuramoto model

$$\varphi_1 \\ K \\ \varphi_2$$

Synchronization:  
$$\dot{\varphi}_1 = \dot{\varphi}_2 \quad \Rightarrow$$

$$\dot{\varphi}_{1} = \Omega_{1} + K \sin(\varphi_{2} - \varphi_{1})$$
$$\dot{\varphi}_{2} = \Omega_{2} + K \sin(\varphi_{1} - \varphi_{2})$$
$$\sin(\varphi_{2} - \varphi_{1}) = \frac{\Omega_{2} - \Omega_{1}}{2K}$$
phase lag

#### The Kuramoto model



# **Frequency Locking**



# **Frequency Locking**



#### The washboard potential



#### Synchronization of two optomechanical oscillators?

- limit cycle (blue-detuned drive)
  - two coupled cells



intrisic frequencies

#### Synchronization of two optomechanical oscillators!



G. Heinrich et al., Phys. Rev. Lett. 107, 043603 (2011)

# Experiments (two cells, joint optical mode)

# Michal Lipson lab, Cornell



laser detuning



mechanical frequency (Zhang et al., PRL 2012)

# Hong Tang lab, Yale



(Bagheri, Poot, FM, Tang; PRL 2013)

#### 7-disk array



#### Lipson group PRL 2015 (synchronization)

### 7-disk array



Lipson group PRL 2015 (synchronization)

#### **Optomechanical Kuramoto model**



G. Heinrich et al., Phys. Rev. Lett. 107, 043603 (2011)

#### **Optomechanical Kuramoto model**



G. Heinrich et al., Phys. Rev. Lett. 107, 043603 (2011)

#### **Optomechanical Kuramoto model**



#### Pattern formation in optomechanical arrays

Phase field



#### Questions...

"Phase Diagram" of this stochastic field theory? What about the quantum regime? Can the phase evolution show quantum coherence? Can one couple this to topological transport?

See our publications: www.mpl.mpg.de/en/institute/marquardt-division.html


## www.mpl.mpg.de

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