Cavity Optomechanics

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Radiation forces

baryon-photon fluid: sound speed $\,c/\sqrt{3}$



Radiation pressure



(Comet Hale-Bopp; by Robert Allevo)

Radiation pressure

Nichols and Hull, 1901 Lebedev, 1901

A PRELIMINARY COMMUNICATION ON THE PRESSURE OF HEAT AND LIGHT RADIATION.

BY E. F. NICHOLS AND G. F. HULL.

MAXWELL,¹ dealing mathematically with the stresses in an electro-magnetic field, reached the conclusion that "in a medium in which waves are propagated there is a pressure normal to the waves and numerically equal to the energy in unit volume."



Nichols and Hull, Physical Review 13, 307 (1901)

Laser Interferometer for Gravitational Wave Detection

4 km

(LIGO Livingston)

Image: SXS project

Observation of Gravitational Waves



LIGO collaboration, Phys. Rev. Lett. 116, 061102 (2016)

Laser Interferometer for Gravitational Wave Detection

4 km

(LIGO Livingston)

How precise can one measure?



Laser power

How precise can one measure?



(Painter group, Caltech)

2 µm











$\hat{H} = \hbar \omega_{\rm cav}(\hat{x})\hat{a}^{\dagger}\hat{a} + \hbar \omega_M \hat{b}^{\dagger}\hat{b} + \dots$

...any dielectric moving inside a cavity generates an optomechanical interaction!

A bit of history

First cavity optomechanics experiments



Static bistability in an optical cavity experiment Dorsel, McCullen, Meystre, Vignes, Walther PRL 1983



A zoo of devices

The zoo of optomechanical systems (2005-now)



The zoo of optomechanical systems



The zoo of optomechanical systems





Optomechanics: general outlook



Regal/Lehnert



Fundamental tests of quantum mechanics in a new regime: entanglement with 'macroscopic' objects, unconventional decoherence?

[e.g.: gravitationally induced?]

Mechanics as a 'bus' for connecting hybrid components: superconducting qubits, spins, photons, cold atoms,

Precision measurements

small displacements, masses, forces, and accelerations



Optomechanical circuits & arrays Exploit nonlinearities for classical and quantum information processing, storage, and amplification; study collective dynamics in arrays

Sensing mechanical motion at the quantum limit

Laser Interferometer for Gravitational Wave Detection

LIGO Livingston

Optical displacement detection



Thermal fluctuations of a harmonic oscillator



Classical equipartition theorem:

$$\frac{m\omega_M^2}{2} \langle x^2 \rangle = \frac{k_B T}{2} \Rightarrow \langle x^2 \rangle = \frac{k_B T}{m\omega_M^2}$$
 extract temperature!

Direct time-resolved detection

Analyze fluctuation spectrum of x

Fluctuation spectrum



Fluctuation spectrum



Fluctuation spectrum

$$\tilde{x}(\omega) = \frac{1}{\sqrt{\tau}} \int_{0}^{\tau} dt e^{i\omega t} x(t)$$
$$S_{xx}(\omega) \equiv \left\langle |\tilde{x}(\omega)|^{2} \right\rangle =$$
$$\approx \int_{-\infty}^{+\infty} dt e^{i\omega t} \left\langle x(t)x(0) \right\rangle$$

Wiener-Khinchin theorem

$$\langle |\tilde{x}(\omega)|^2 \rangle \equiv S_{xx}(\omega)$$
area yields
area yields
variance of x:
$$\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} S_{xx}(\omega) = \langle x^2 \rangle$$

General relation between noise spectrum and linear response susceptibility

 $\begin{array}{l} \left< \delta x \right> (\omega) = \chi_{xx}(\omega) F(\omega) \\ \text{susceptibility} \end{array}$

$$S_{xx}(\omega) = \frac{2k_BT}{\omega} \operatorname{Im}\chi_{xx}(\omega)$$
 (classical limit)

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 (classical limit)

 ω_M

for the damped oscillator:

$$m\ddot{x} + m\omega_{M}^{2}x + m\Gamma\dot{x} = F$$

$$x(\omega) = \frac{1}{m(\omega_{M}^{2} - \omega^{2}) - im\Gamma\omega}F(\omega)$$

$$\chi_{xx}(\omega)$$
Displacement spectrum





Measurement noise



Measurement noise



Two contributions to $x_{noise}(t)$

- I. measurement imprecision laser beam (shot noise limit!)
- 2. measurement back-action:
- fluctuating force on system
- noisy radiation pressure force

"Standard Quantum Limit"



"Standard Quantum Limit"



Best case allowed by quantum mechanics:

 $S_{xx}^{(\text{meas})}(\omega) \ge 2 \cdot S_{xx}^{T=0}(\omega) \qquad \text{``Standard quantum limit} \\ (SQL) \text{ of displacement} \\ \text{detection''}$

...as if adding the zero-point fluctuations a second time: "adding half a photon"

Notes on the SQL



- "weak measurement": integrating the signal over time to suppress the noise
- trying to detect slowly varying "quadratures of motion": $\hat{x}(t) = \hat{X}_1 \cos(\omega_M t) + \hat{X}_2 \sin(\omega_M t)$ $\left[\hat{X}_1, \hat{X}_2\right] = 2x_{\text{ZPF}}^2$ Heisenberg is the reason for SQL! no limit for instantaneous measurement of x(t)!
- SQL means: detect $\hat{X}_{1,2}$ down to x_{ZPF} on a time scale $1/\Gamma$ Impressive: $x_{\text{ZPF}} \sim 10^{-15} m$!

Enforcing the SQL (Heisenberg) in a weak optical measurement



reflection phase shift: $\theta = 2kx$ (here: free space)

N photons arrive in time t

fluctuations: $\delta N = \sqrt{VarN} = \sqrt{\bar{N}}$ a coherent laser beam

Poisson distribution for

I. Uncertainty in phase estimation:

$$\delta N \cdot \delta \theta \ge \frac{1}{2} \Rightarrow \delta \theta \ge \frac{1}{2\sqrt{\bar{N}}} \Rightarrow \delta x = \frac{\delta \theta}{2k} \sim \frac{1}{2\sqrt{\bar{N}}2k}$$

2. Fluctuating force: momentum transfer $\Delta p = 2\hbar k \cdot N$
 $\delta p = \sqrt{\operatorname{Var}\Delta p} = 2\hbar k \sqrt{\bar{N}}$
Uncertainty product: $\delta x \delta p \ge \frac{\hbar}{2}$ Heisenberg

 $\boldsymbol{\angle}$

Quantum dynamics

Optomechanical Hamiltonian



Optomechanical Hamiltonian



Converting photons into phonons



Converting photons into phonons



Optomechanical Hamiltonian



Optomechanical Interaction: Nonlinear

$\hat{a}^{\dagger}\hat{a}(\hat{b}^{\dagger}+\hat{b})$

"Linearized" Optomechanical Hamiltonian

"laser-enhanced optomechanical coupling": $g=g_0\alpha$

$g_0 \sim \mathrm{Hz} - \mathrm{MHz}$

bare optomechanical coupling (geometry, etc.: fixed!) laser-driven cavity amplitude tuneable! **phase**!

 α

After linearization: two linearly coupled harmonic oscillators!



Photon-phonon polaritons



Photon-phonon polaritons



After linearization: two linearly coupled harmonic oscillators!



After linearization: two linearly coupled harmonic oscillators!



Aside: Quantum Heat Engine in Optomechanics



Keye Zhang, Francesco Bariani, Pierre Meystre; Phys. Rev. Lett. 2014

Different regimes



Effective Optomechanical Damping Rate



Effective Optomechanical Damping Rate



Laser-cooling towards the ground state



"The slopes of Optomechanics"



Linear Optomechanics

Displacement detection
 Optical Spring
 Cooling & Amplification
 Two-tone spectroscopy
 State transfer, pulsed operation
 Wavelength conversion
 Radiation Pressure Shot Noise
 Squeezing of Light
 Squeezing of Mechanics
 Entanglement
 Precision measurements

Optomechanical Arrays

Bandstructure in arrays
Synchronization in arrays
Transport of photons & phonons
Topological phases

Nonlinear Classical Optomechanics
Self-induced mechanical oscillations
Synchronization of oscillations
Chaos

Nonlinear Quantum Optomechanics

- Phonon number detection
- Phonon shot noise
- Photon blockade
- Optomechanical "which-way" expt.
- Nonclassical mechanical q. states
- Optomechanical matter-wave interference
- Nonlinear OMIT
- Noncl. via Conditional Detection
- Single-photon sources
- Coupling to two-level systems

Optomechanical wavelength conversion







optics to optics:



microwave/RF to optics:





Cleland 2013



Schliesser, Polzik 2014

Lehnert, Regal 2014

Optomechanical Arrays

Single-mode optomechanics



✓ displacement sensing
 ✓ cooling
 ✓ strong coupling
 ✓ self-oscillations (limit cycles)
Many modes











First realizations



Lipson group, Cornell arXiv: 1505.02009 (synchronization)

= free-standing photonic crystal structures (Painter group)

localized optical and vibrational (GHz) mode



advantages:

tight vibrational confinement: high frequencies, small mass (stronger quantum effects)

tight optical confinement: large optomechanical coupling (100 GHz/nm)

integrated on a chip

Safavi-Naeini et al PRL 2014 Eichenfield et al Nature 2009

Optomechanical arrays

Optomechanical array: Many coupled optomechanical cells



Possible design based on "snowflake" 2D optomechanical crystal (Painter group), here: with suitable defects forming a superlattice (array of cells)

Modeling an optomechanical array

 \hat{a}_{j}

strengt

0

Tight-binding model for photons & phonons hopping and interacting on a lattice

 $\Delta = \omega_L - \omega_{\rm opt}$

optical coupling: 0 optomech. interaction laser drive each cell: $\hat{H}_{\text{om},j} = -\Delta \hat{a}_j^{\dagger} \hat{a}_j + \Omega \hat{b}_j^{\dagger} \hat{b}_j - g_0 (\hat{b}_j^{\dagger} + \hat{b}_j) \hat{a}_j^{\dagger} \hat{a}_j + \alpha_L (\hat{a}_j^{\dagger} + \hat{a}_j)$ $\hat{H}_{\text{int}} = - \mathbf{J} \sum \left(\hat{a}_i^{\dagger} \hat{a}_j + \hat{a}_i \hat{a}_j^{\dagger} \right) - \mathbf{K} \sum \left(\hat{b}_i^{\dagger} \hat{b}_j + \hat{b}_i \hat{b}_j^{\dagger} \right)$ $\langle i,j \rangle$ optical coupling $\langle i,j \rangle$ mechanical coupling (photon tunneling) (phonon tunneling)

Max Ludwig, FM, Phys. Rev. Lett. 111, 073602 (2013)

Optomechanical Arrays

global view: light-tunable metamaterial for photons & phonons



similar in spirit: optical lattices nonlinear optical materials

conceptually simple: one material, with holes

laser drive

Synthetic magnetic fields for photons/phonons

Dirac Physics

Synchronization and Pattern Formation Topological Phases

Transport (edge states/wires)

Nonequilibrium dynamics/Quench physics/Thermalization

Quantum Information Processing

Strongly Correlated Quantum Physics?

Tuneable/reconfigurable in-situ

All-optical control/readout

Nonlinear Optomechanics (Classical Regime)

Nonlinear dynamics

blue-detuned laser: anti-damping! < 0 $\Gamma = \Gamma_M + \Gamma_{\rm opt}$

Nonlinear Dynamics

Nonlinear Dynamics

Beyond some laser input power threshold: instability



Self-sustained mechanical oscillations!













An optomechanical cell as a Hopf oscillator



Amplitude fixed, phase undetermined!

An optomechanical cell as a Hopf oscillator



Amplitude fixed, phase undetermined!



Collective dynamics in an array of coupled cells? Phase-locking: **synchronization**!

Synchronization: Huygens' observation



(Huygens' original drawing!)

Coupled pendula synchronize... ...even though intrinsic frequencies slightly different important in physics, chemistry, biology, ... Josephson arrays, laser arrays, ...

The Kuramoto model



Kuramoto model:

$$\dot{\varphi}_1 = \Omega_1 + K \sin(\varphi_2 - \varphi_1)$$
$$\dot{\varphi}_2 = \Omega_2 + K \sin(\varphi_1 - \varphi_2)$$

captures essential features
often found as limiting model

Kuramoto 1975, 1984 Acebron et al., Rev. Mod. Phys. 77, 137 (2005)

The Kuramoto model

$$\varphi_1 \\ K \\ \varphi_2$$

Synchronization:
$$\dot{\varphi}_1 = \dot{\varphi}_2 \quad \Rightarrow$$

$$\dot{\varphi}_{1} = \Omega_{1} + K \sin(\varphi_{2} - \varphi_{1})$$
$$\dot{\varphi}_{2} = \Omega_{2} + K \sin(\varphi_{1} - \varphi_{2})$$
$$\sin(\varphi_{2} - \varphi_{1}) = \frac{\Omega_{2} - \Omega_{1}}{2K}$$
phase lag

The Kuramoto model



Frequency Locking



Frequency Locking



The washboard potential



Synchronization of two optomechanical oscillators?

- limit cycle (blue-detuned drive)
 - two coupled cells



intrisic frequencies

Synchronization of two optomechanical oscillators!



G. Heinrich et al., Phys. Rev. Lett. 107, 043603 (2011)

Experiments (two cells, joint optical mode)

Michal Lipson lab, Cornell



laser detuning



mechanical frequency (Zhang et al., PRL 2012)

Hong Tang lab, Yale



(Bagheri, Poot, FM, Tang; PRL 2013)

7-disk array



Lipson group PRL 2015 (synchronization)

7-disk array



Lipson group PRL 2015 (synchronization)

Optomechanical Kuramoto model



G. Heinrich et al., Phys. Rev. Lett. 107, 043603 (2011)

Optomechanical Kuramoto model



G. Heinrich et al., Phys. Rev. Lett. 107, 043603 (2011)

Optomechanical Kuramoto model



Pattern formation in optomechanical arrays

Phase field



Questions...

"Phase Diagram" of this stochastic field theory? What about the quantum regime? Can the phase evolution show quantum coherence? Can one couple this to topological transport?

See our publications: www.mpl.mpg.de/en/institute/marquardt-division.html


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