Josephson junction and SQUID based technology

Cryocourse 2016: Aalto School and Workshop in Cryogenics and Quantum Engineering

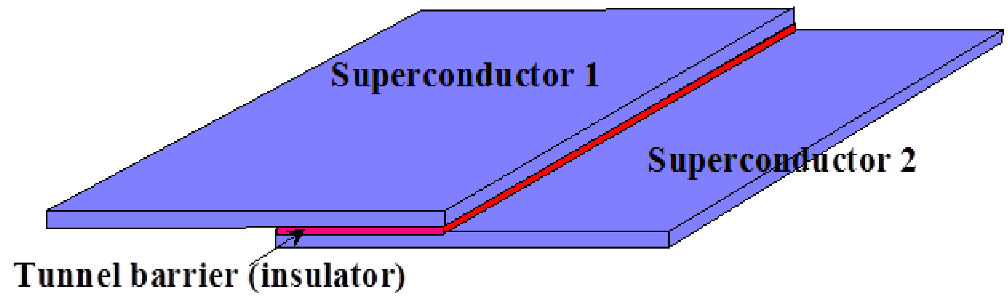
Juha Hassel 2.10.2016

Outline

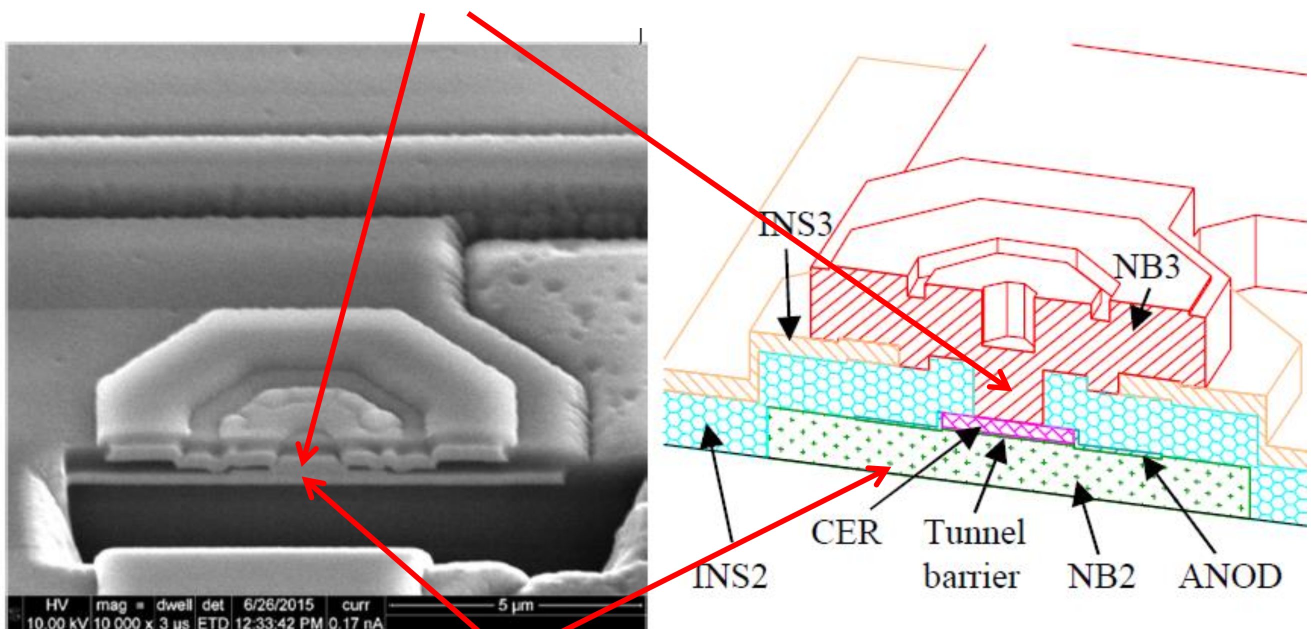
1. **Josephson junctions and SQUIDs; physics and technology**
   A) Physical models for systems of *classical* Josephson junctions.
   B) Junction fabrication technology.
2. **Appication examples**
   A) Josephson voltage standards.
   B) SQUIDs in (bio)magnetometry.
   C) SQUIDs as low(esh) frequency amplifiers
   D) SQUIDs as microwave amplifiers
   E) Rapid single flux quantum logic.

1A Physical models for systems of classical Josephson junctions (JJs).

- JJ: weak link (tunnel barrier) between two superconductors.
- Fabrication by micro and nanolithographic processes



Superconductor 2

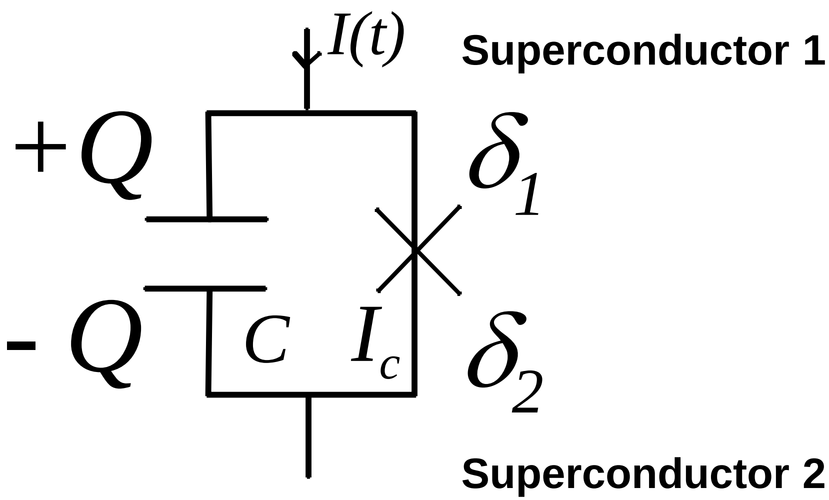


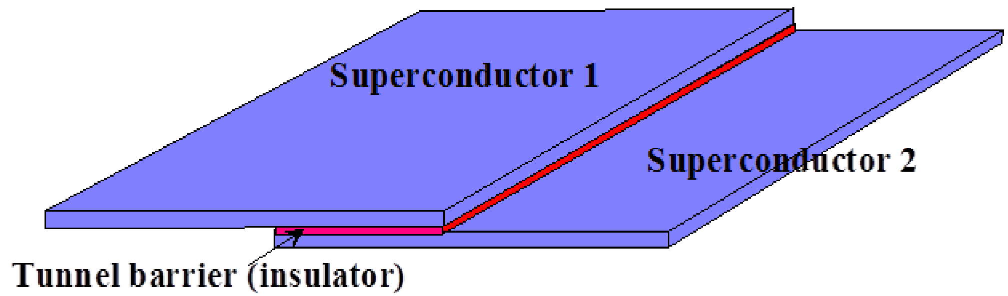
Superconductor 1

M. Kiviranta, et al, IEEE Trans. Appl Supercond, in print

- Josephson junction dynamics [1]:
  - Dynamical quantities phase difference $\phi = \delta_1 - \delta_2$ and charge Q
  - The system is described by the total energy:

$$H = \frac{Q^2}{2C} - \left(\frac{\Phi_0}{2\pi} I(t)\phi + E_J \cos\phi \right) \qquad \left(E_J = \frac{\Phi_0 I_c}{2\pi} \right)$$





[1] Literature, see e.g.
- P. Orlando, K. A. Delin, Foundations of Applied Superconductivity
- T. van Duzer, Principles of Superconducting Devices and Circuits

Kinetic energy

Potential energy

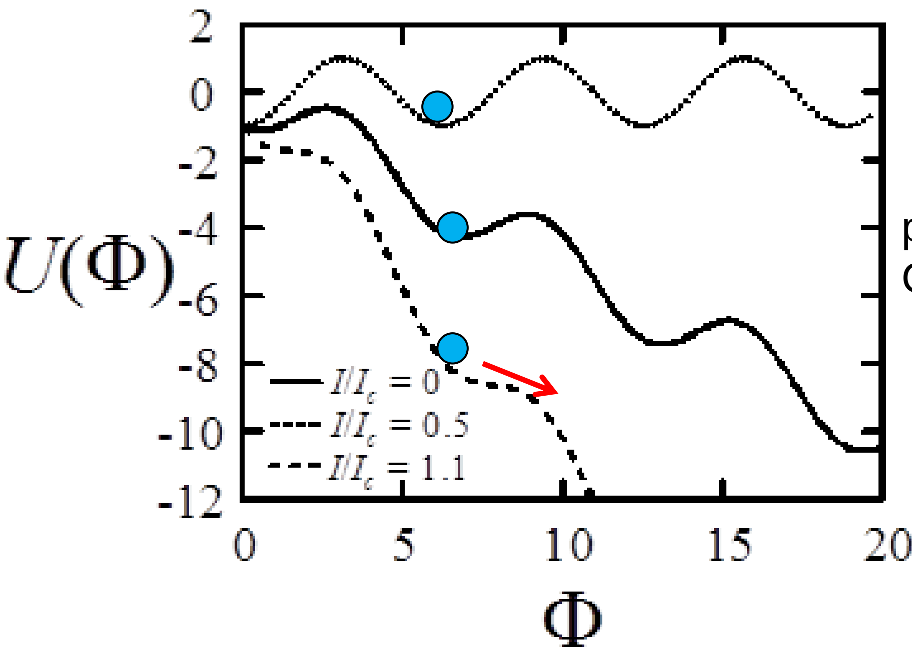
$$H = \boxed{\frac{Q^2}{2C}} - \boxed{\left(\frac{\Phi_0}{2\pi}I(t)\phi + E_J\cos\phi\right),}$$

where $E_J = \frac{\Phi_0 I_c}{2\pi}$ and $\Phi_0 = \frac{h}{2e}$ is the flux quantum

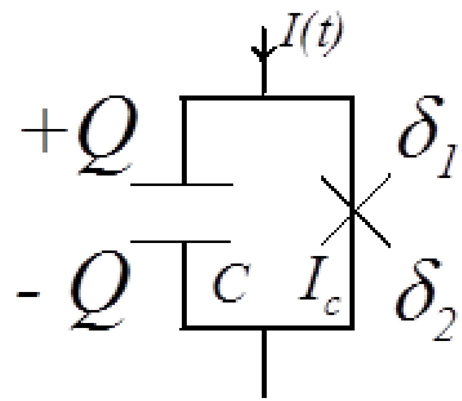
$$Q = -2ei\frac{\partial}{\partial\phi}$$

$$H = \boxed{\frac{p^2}{2m}} - \boxed{\left(\frac{E_0}{x_0}(\tan\alpha)x + E_1\cos\left(\frac{2\pi x}{x_1}\right)\right)}$$

$$p = -i\hbar\frac{\partial}{dx}$$



charge ~ momentum
voltage ~ speed
phase diff. ~ position
Capacitance~ mass
Current ~ potential tilting



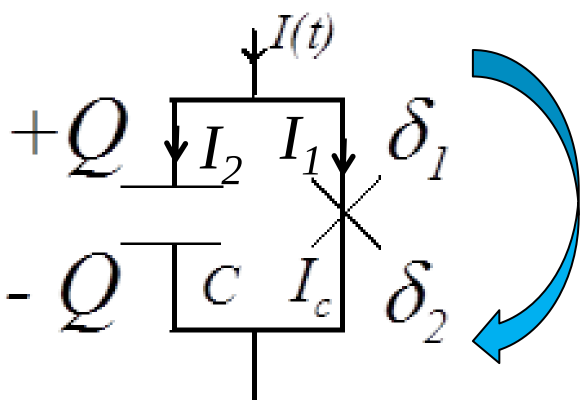
- Solution of classical Hamiltonian equations of motion => Josephson relations

$$H = \frac{Q^2}{2C} - \left(\frac{\Phi_0}{2\pi} I(t)\phi + E_J \cos\phi \right),$$

$$\frac{\partial H}{\partial Q} = \frac{\Phi_0}{2\pi}\frac{\partial \phi}{\partial t}$$

$$\frac{2\pi}{\Phi_0}\frac{\partial H}{\partial \phi} = -\frac{\partial Q}{\partial t}$$

$$\Leftrightarrow$$

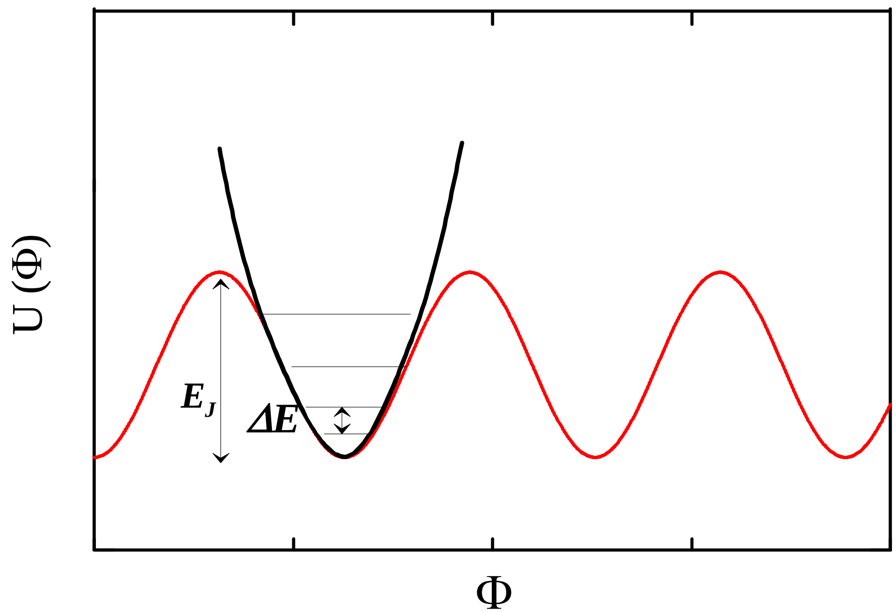


V=Q/C

$$\overbrace{C\frac{\partial V}{dt}}^{I_2} + \overbrace{I_c \sin\phi}^{I_1} = I\,(t)$$

$$V = \frac{\Phi_0}{2\pi}\frac{\partial \phi}{\partial t}$$

- **’Classical’ limit (today’s subject)**
  - The number of energy states corresponding to a potential well minimum large (ΔE << E_J)
  - Thermal energy corresponding to energy state separation large (kT>>ΔE)
- **Quantum limit (beyond today’s scope)**
  - Opposite limit: ΔE >~ E_J, kT<<ΔE
  - Single charge dynamics, quantum computing applications …



Exercise :

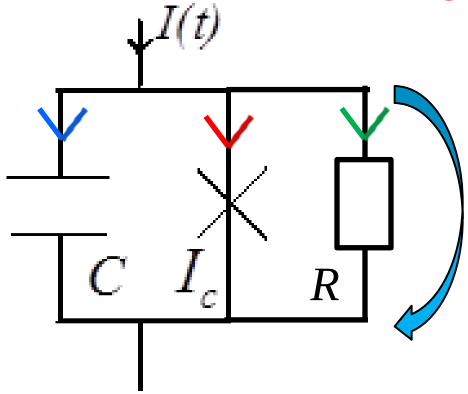
a) Show that $\Delta E << E_J$ implies $E_J >> E_c$ with $E_c = e^2/2C$ single charge energy.

- Building models for the systems of classical Josephson junctions:
    - Write differential forms of Kirchoffs equations supplemented by Josephson relations.

Add resistive damping (RCSJ model*):

Josephson relations:

$$I_d = C(dV/dt)$$



$$I_J = I_c \sin\phi$$

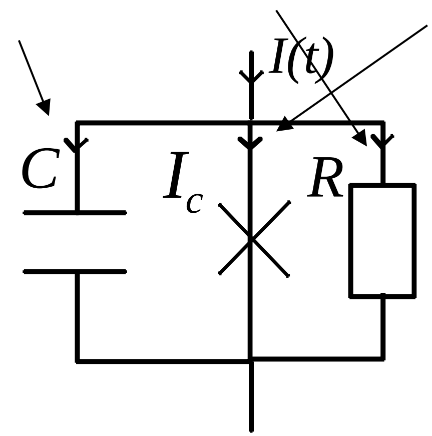
$$V = (\Phi_0/2\pi)(d\phi/dt)$$

$$I_R = V/R$$

*Resistively and capacitively shunted junction'

- Introducing dimensionless unit system:
  - Avoid redundant degrees of freedom.
  - Basic equation:

$$\beta_c \frac{d^2\phi}{\partial\tau} + \frac{d\phi}{d\tau} + \sin\phi = i(t)$$



$$\beta_c \frac{d^2\phi}{\partial\tau} + \frac{d\phi}{d\tau} + \sin\phi = i(t),$$

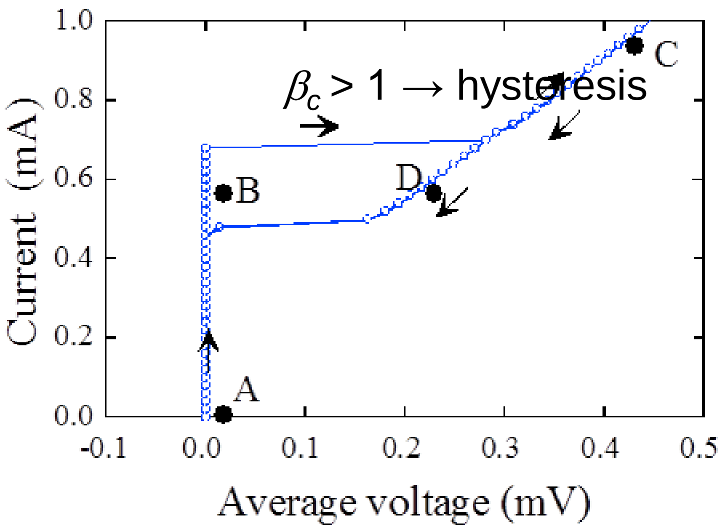
where

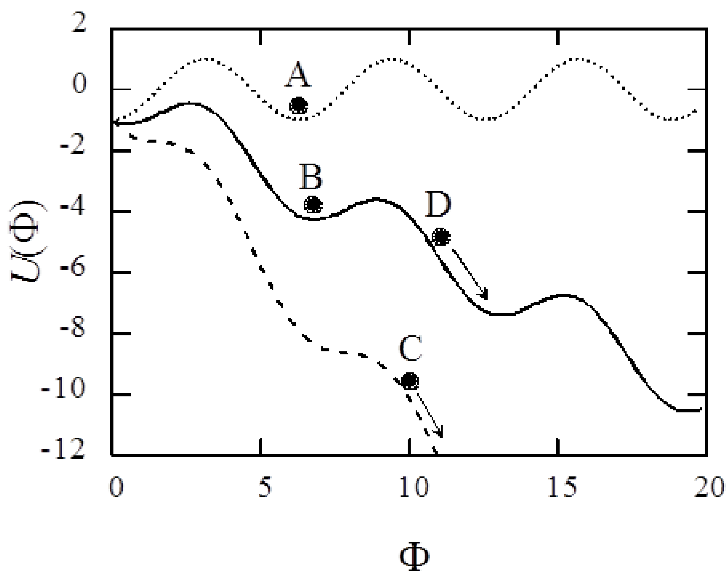
$$\beta_c = \frac{2\pi I_c R^2 C}{\Phi_0}$$

and the dimensionless unit system has been adopted:

$$i = \frac{I}{I_c}$$

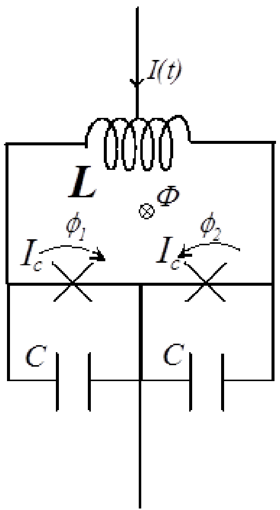
$$\tau = \frac{2\pi I_c R}{\Phi_0} t$$





- SQUID:
  - superconducting loops cut by JJ:s
  - Example: DC SQUID (two identical junctions)
  - Analysis similar to the single JJ
    - though "new physics": relationship between the magnetic flux quantization
    - Consequence of superconductor electrodynamics and requirement of single valuedness of wavefunction phase

$$\Phi = \frac{\Phi_0}{2\pi}(\phi_1 - \phi_2)$$

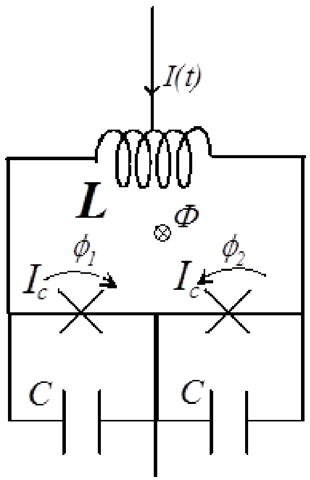


- Dynamical equations of DC SQUID.

$$\beta_c \frac{d^2\nu}{dt^{*2}} + \frac{d\nu}{dt^*} + \sin\nu\cos\varphi = i$$

$$\beta_c \frac{d^2\varphi}{dt^{*2}} + \frac{d\varphi}{dt^*} + \cos\nu\sin\varphi + \frac{2}{\beta_L}(\varphi - \varphi_a) = 0$$

External flux



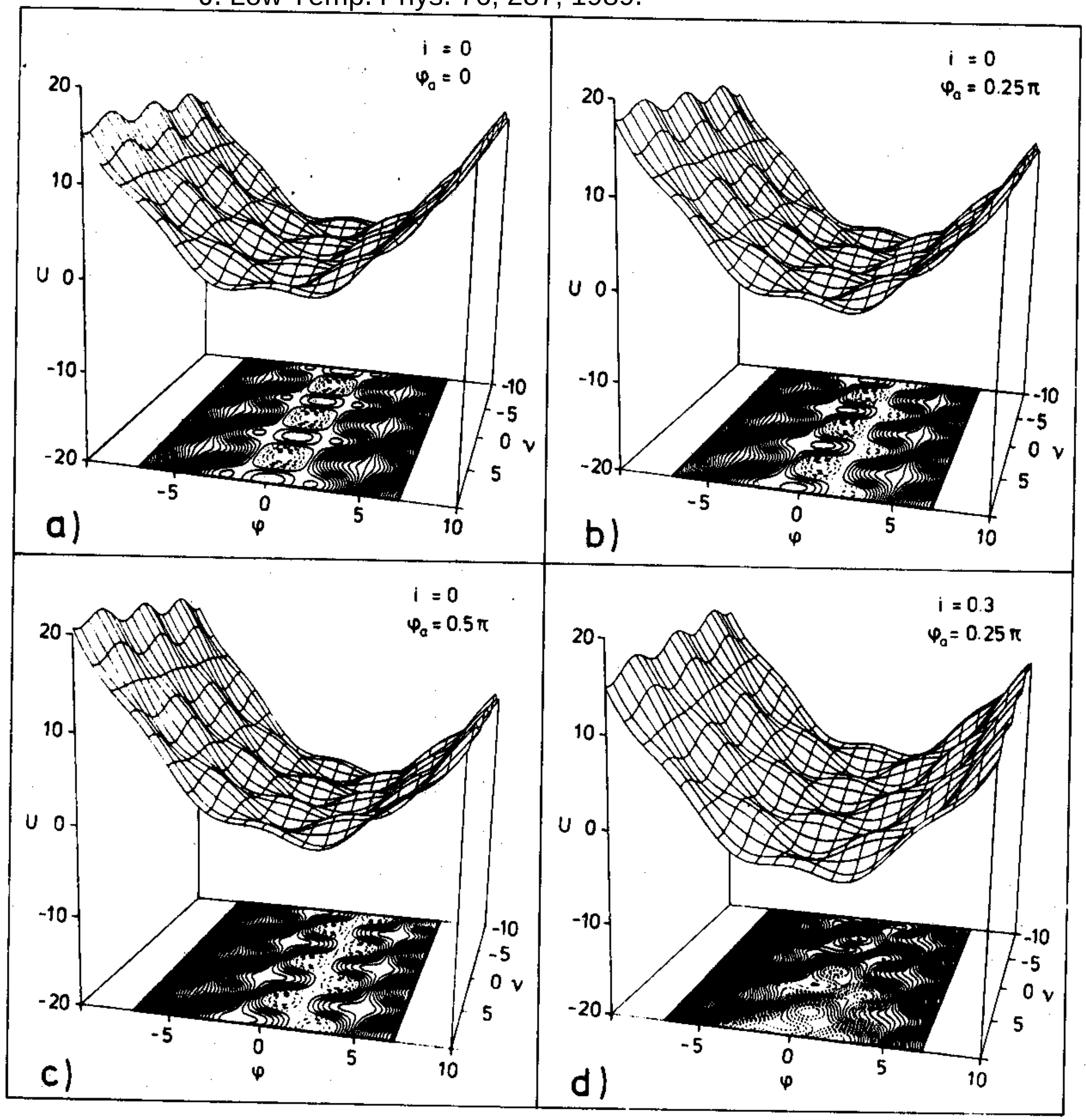
$$\nu = \frac{\phi_1+\phi_2}{2}, \phi = \frac{\phi_1-\phi_2}{2}$$

$$\beta_L = \frac{2\pi L I_c}{\Phi_0}$$

$$U(\nu,\phi) = \frac{1}{\beta_L}(\phi - \phi_{ext})^2 - i\nu - \cos\nu\cos\phi$$

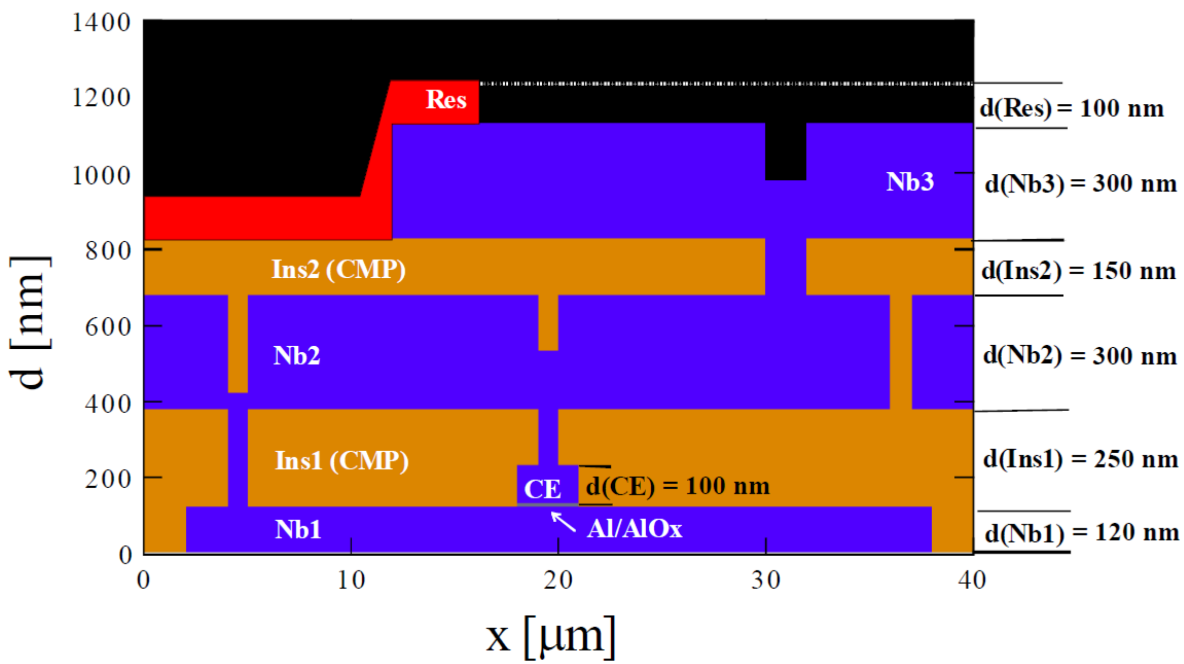
320 **Tapani Ryhänen, Heikki Seppä, Risto Ilmoniemi, and Jukka Knuutila**

J. Low Temp. Phys. 76, 287, 1989.

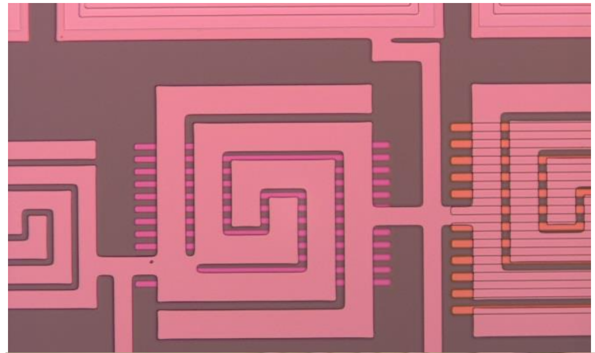


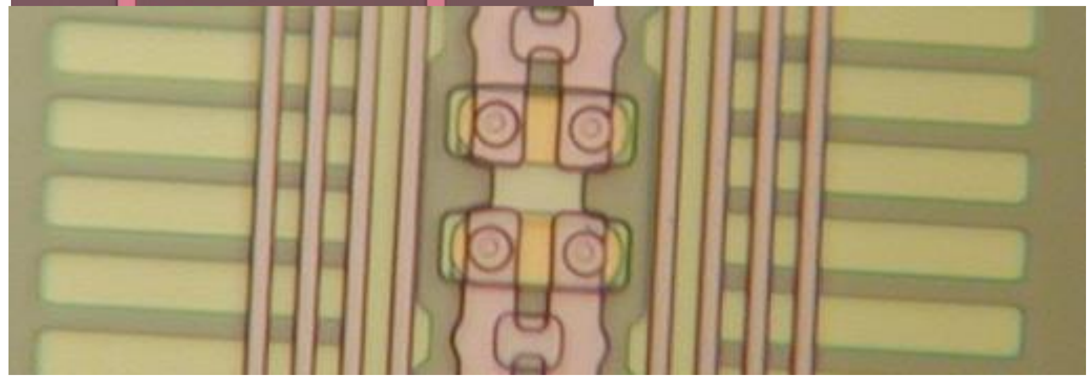
1B Junction fabrication technology

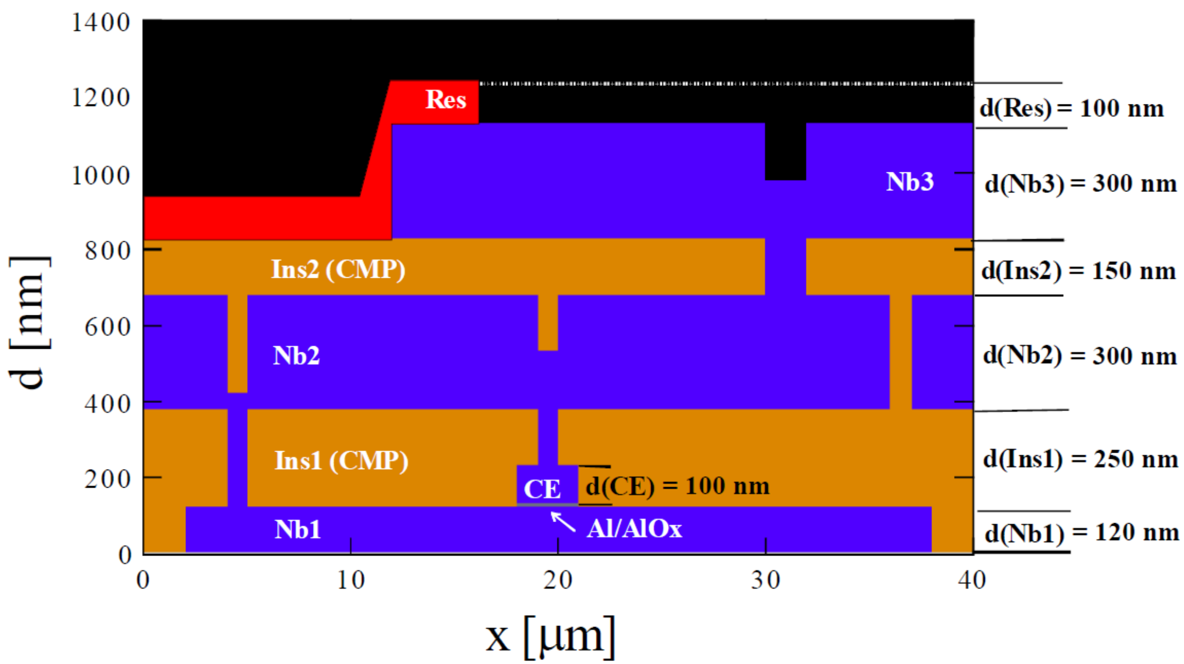
- Industrially, the most established technology is so called Niobium Trilayer technology
  - Nb/Al-AlOx/Nb junction stack grown in-situ.
  - Nb critical temperature Tc~9 K
  - A few nm of Al (Tc ~1 K) superconducting by proximity e.g. at T = 4.2 K
  - In-situ oxidation of aluminium with controlled exposure to tune the critical current



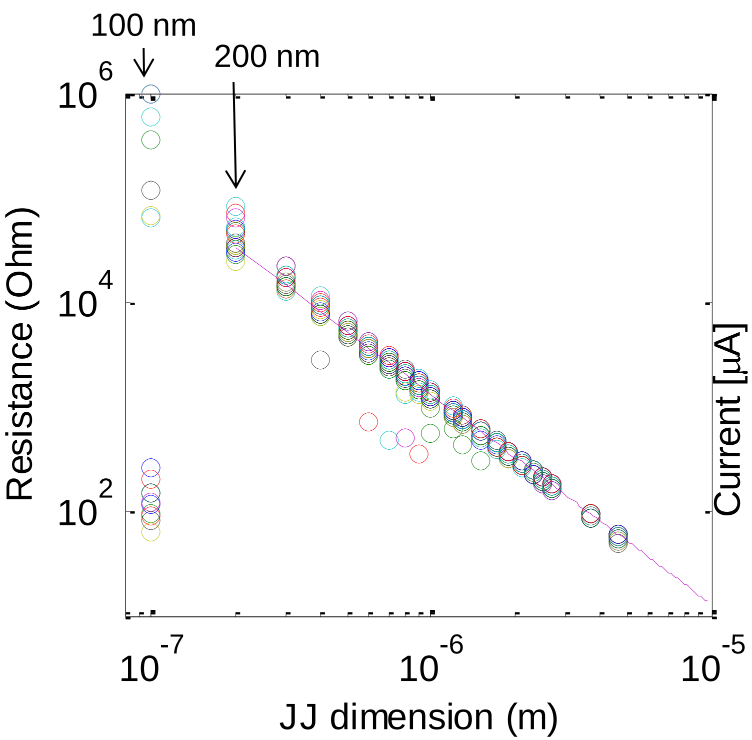
- Multilayer structures for flexible designs
    - Superconducting layers for jump wiring, flux coupling, …
    - Normal conducting resistive layers for shunts, terminations…
    - Insulating layers for capacior dielectrics and insulation
- Optical lithography and material-selective etching methods for patterning
    - Reactive ion etching in reactive plasma.
    - Wet etching with selected chemicals.
- Planarisation techniques for enabling smooth surfaces in layer crossings.



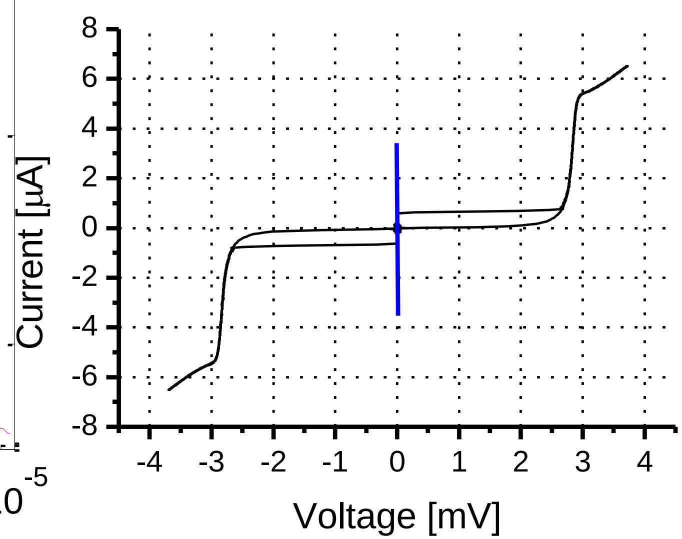




- 150 mm wafer automated processes.
- A variety of characterisation methods
  - On-wafer automated probe stations
  - Microscopy (optics, SEM, AFM…).



Room temperature probed resistances of Josephson junctions showing good yield down to 200 nm.

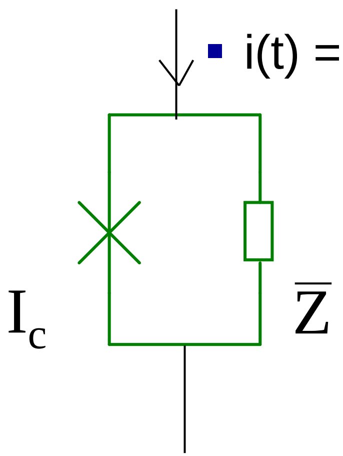


4 Kelvin IV of a Josephson junction.

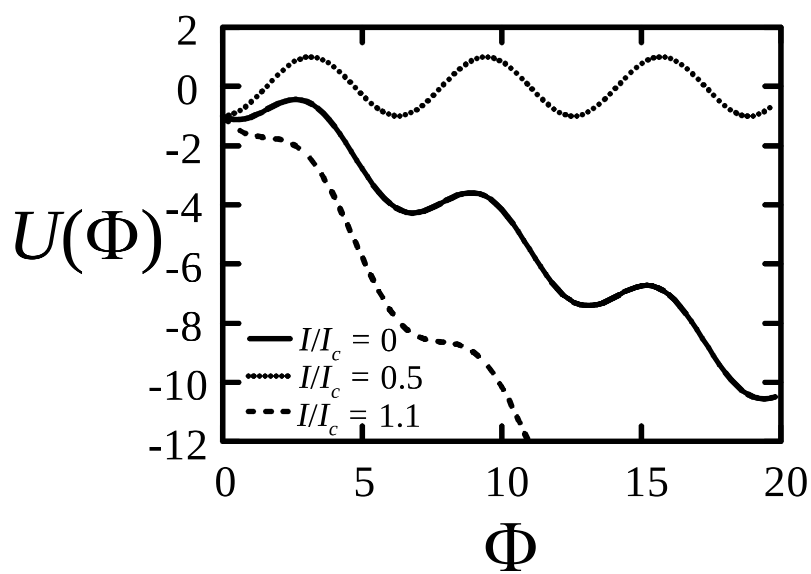
Automated (25 wafers) cassette to cassette Nb, NbN and tunnel junction sputter system.

2A Josephson voltage standards

- Under RF bias $i(t) = i_0 + i_1 \sin(\omega t)$
- In a desirable mode, during one pump period the phase propagates over integer *n* minima of washboard potential
  - Average voltage accross the junction then $V = n\Phi_0 f$
  - Criterion for ’clean’ phase locking is that the tunnel element is effectively biased by RF *voltage* Zi_1
    - Sufficiently small shunt impedance *Z*



- $i(t) = i_0 + i_1 \sin(\omega t)$



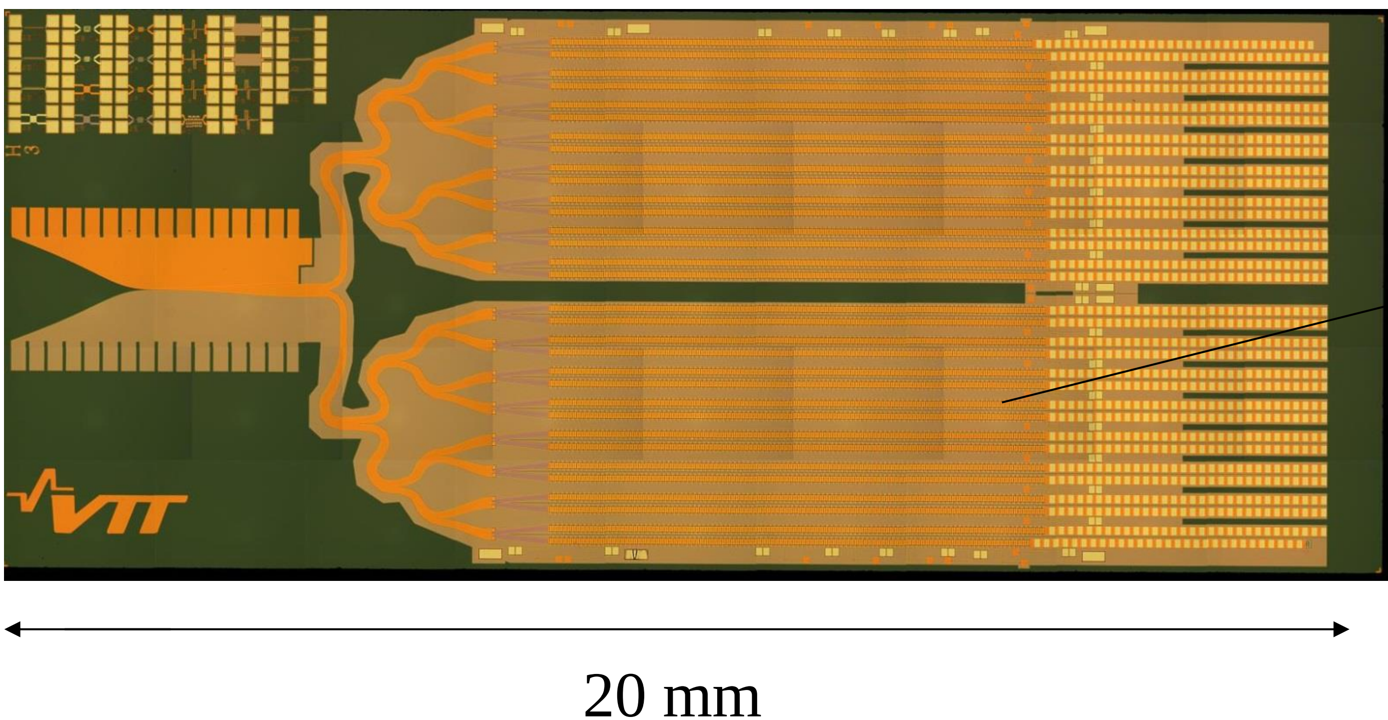
- $V = n\Phi_0 f$ depends only on constants of nature and the frequency f
- Frequency f can be accurately produced by an atomic clock

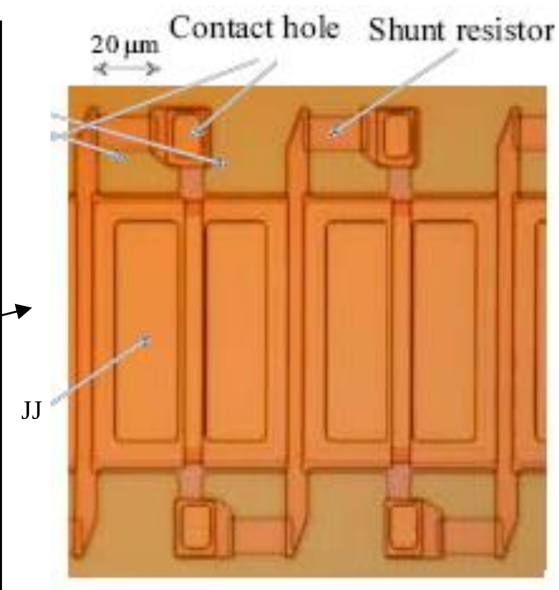
  $\Rightarrow$ Operates as an exact voltage reference

  $\Rightarrow$ Today the practical realisation of the unit of Voltage in all national standards laboratories
- Practical realisation
  - Voltage produced by a single junction small (typical frequencies 10-80 GHz, Φ_0 = 2.07 Vs $\Rightarrow$ voltage from tens to hundreds of microvolts
  - In practice voltage levels from 1 V to 10 V needed

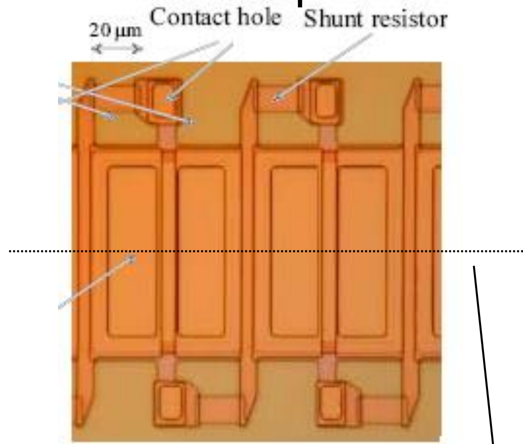
    $\Rightarrow$ arrays of 2000 - 20000 JJ:s needed

- Example:
  - A voltage standard designed and fabricated by VTT
    - 2x3577 JJ:s, driven at about 70 GHz
    - Produces a maximum voltage of about 1.7 V

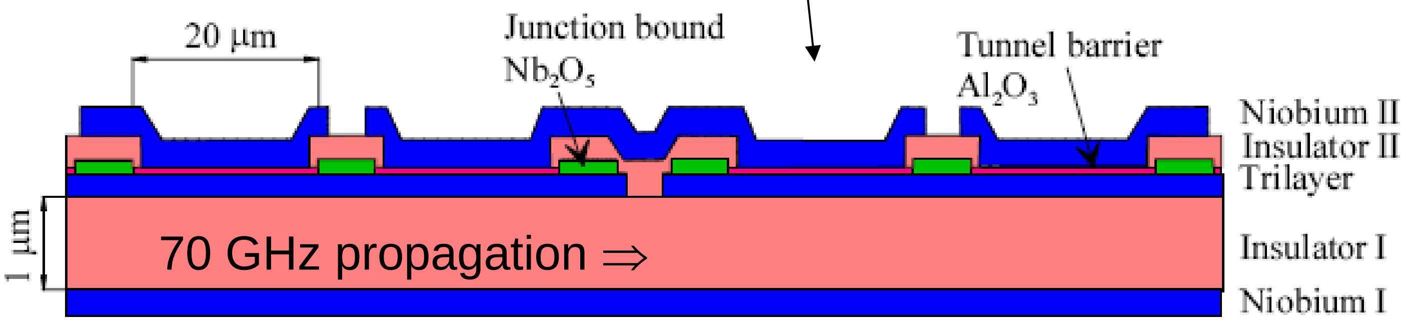




- Physical structure of the chip

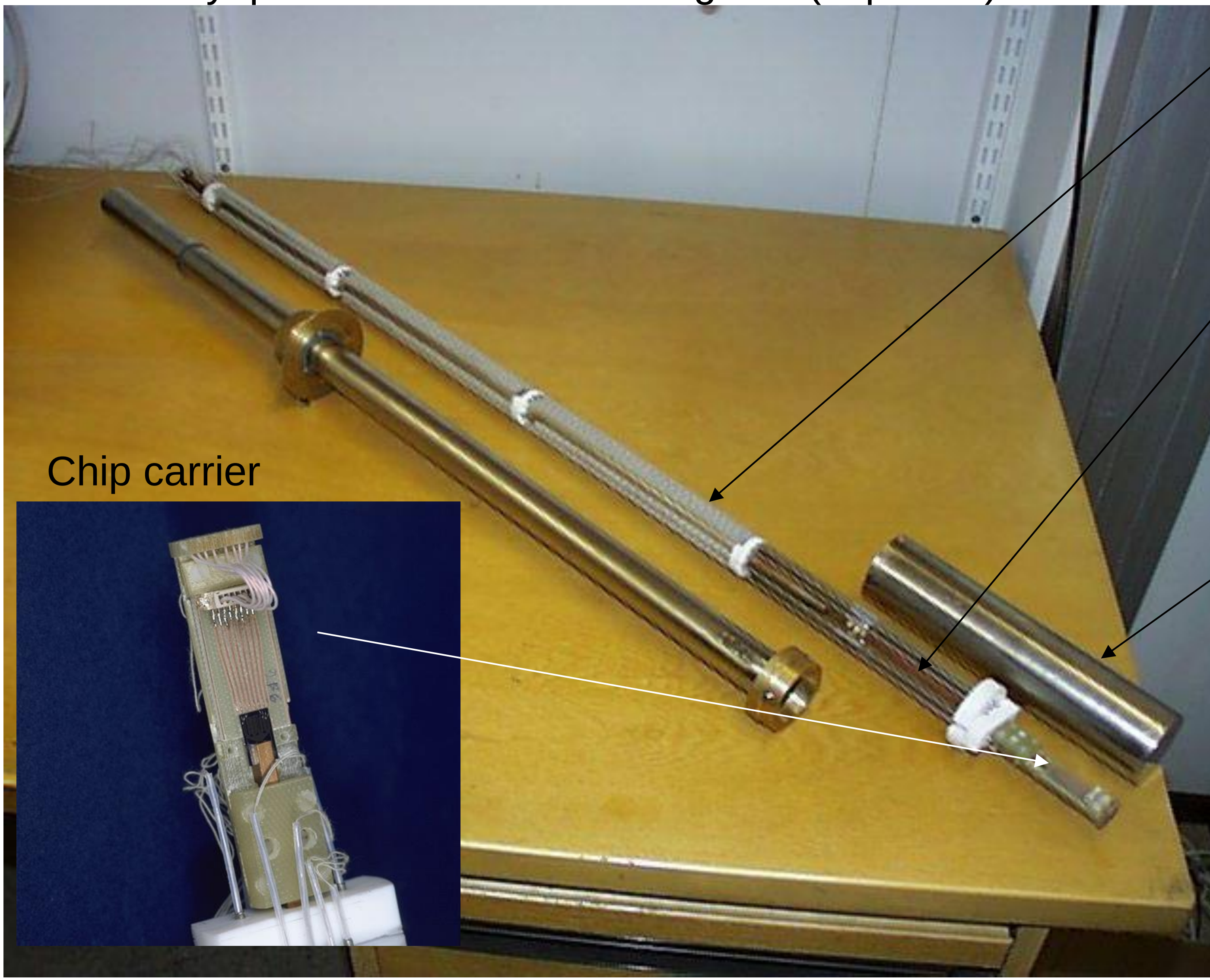


1. Top view (micro photograph)

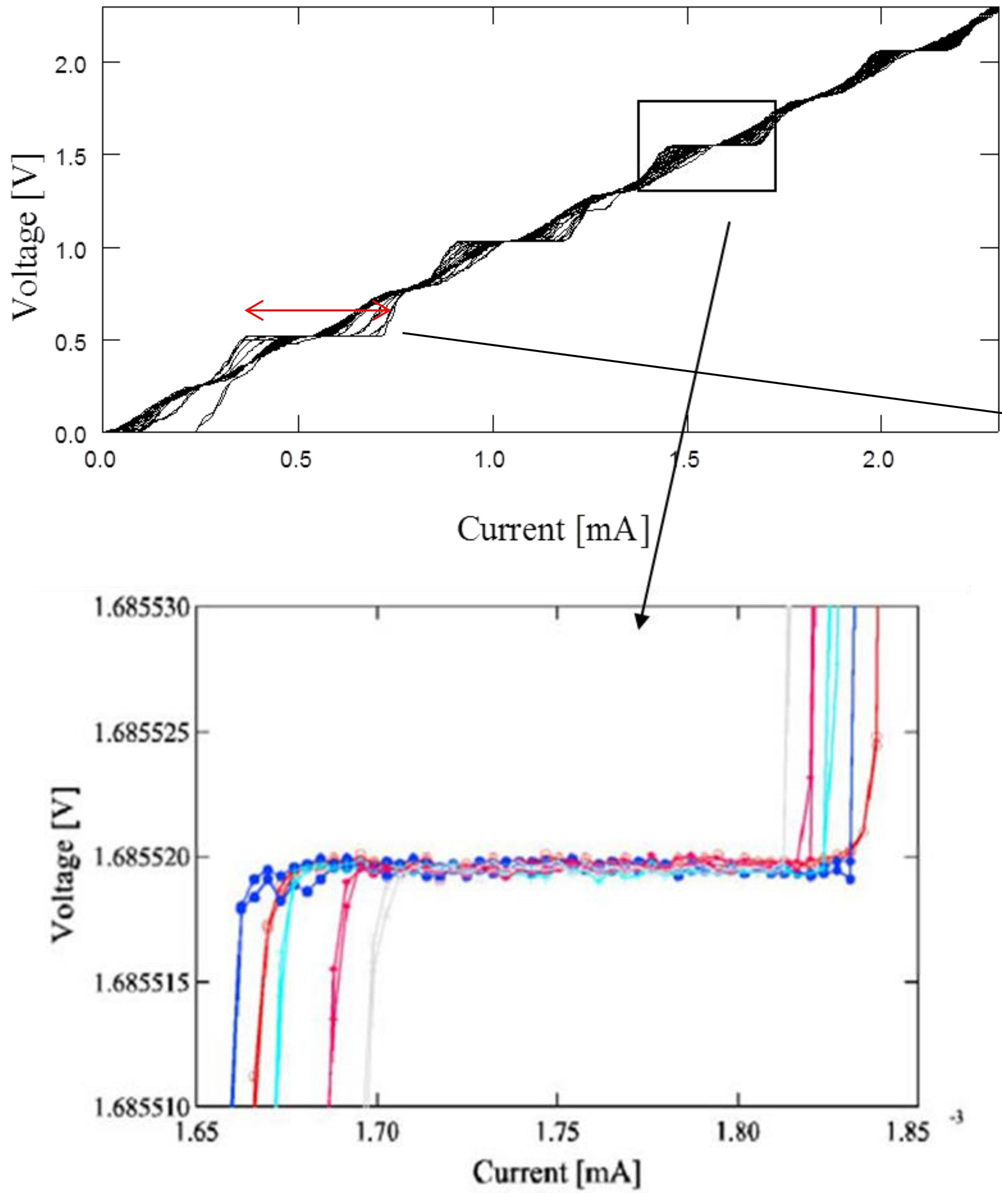


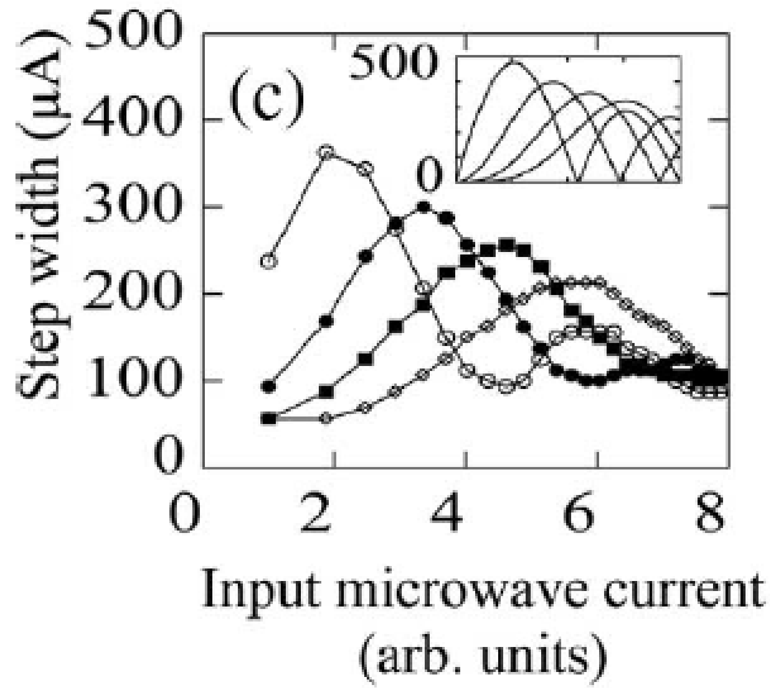
2. Cross section

- Parts of the measurement setup:
  - Cryoprobe with E-band waveguide (in pieces)



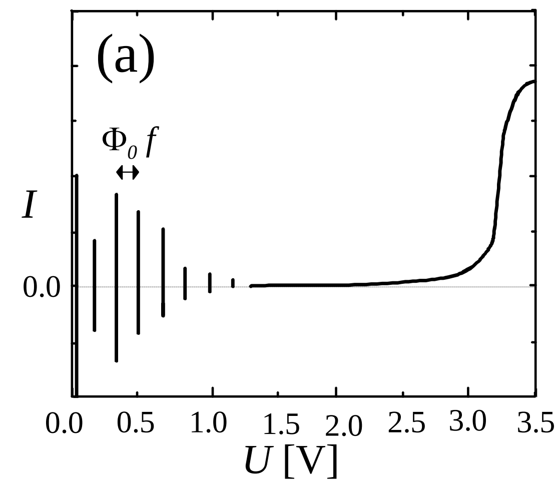
- 3577 junctions irradiated with 75 GHz





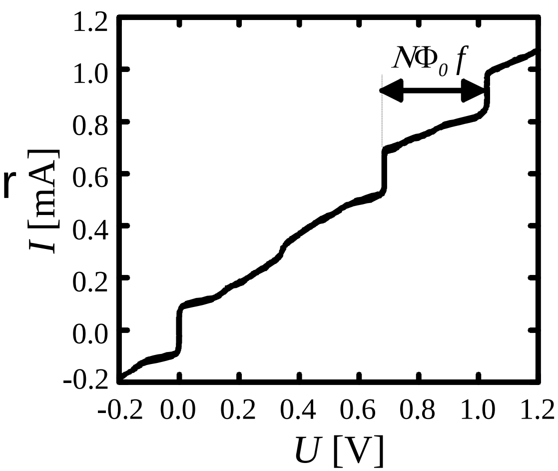
J. Hassel, et al., IEEE Trans. Appl. Supercond. 17, 930, 2007.

- Types of Josephson voltage arrays
  - "Conventional" arrays based on unshunted Superconductor - Insulator - Superconductor junctions
  - Extremely large $\beta_c \Rightarrow$ slow, hysteretic IV curves
  - Used as primary DC references
    - Not useful in AC metrology

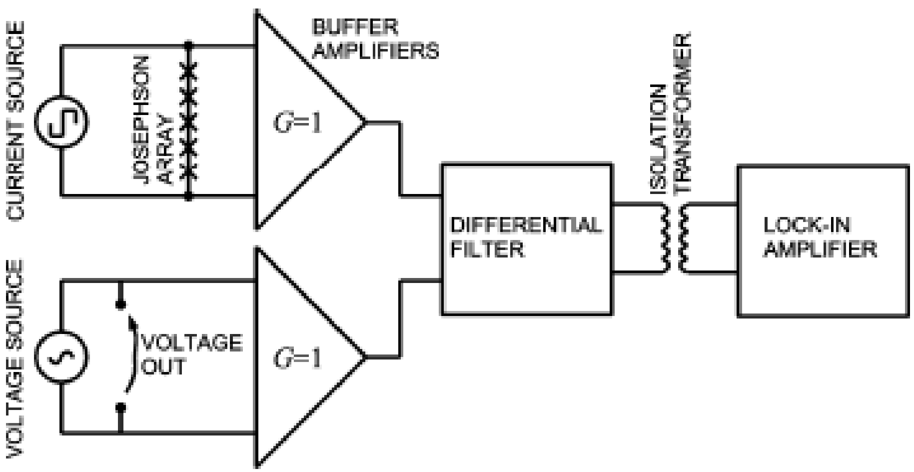


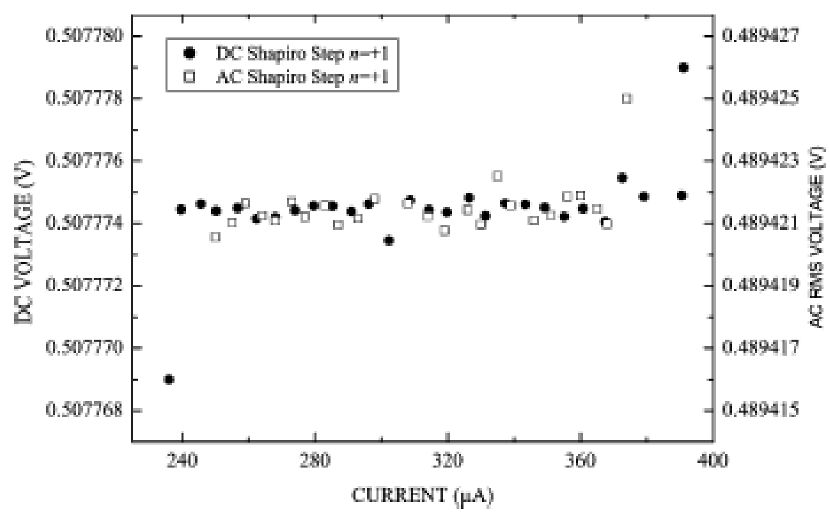
  - "Programmable" arrays (e.g. those presented earlier)
    - Dissipation added to junctions
      - Either by external shunts (as in previous slides)
      - Or by making a intrinsically dissipative tunnel barrier
        - e.g. Superconductor - Normal conductor - Superconductor junctions

      $\Rightarrow$Small $\beta_c \Rightarrow$ Fast, nonhysteretic



- AC generation by programmable arrays
  - Method developed by VTT and MIKES:
    - switch JVS between steps +n and -n
    - extract the fundamental Fourier component of the square wave
    - compare to the source being calibrated

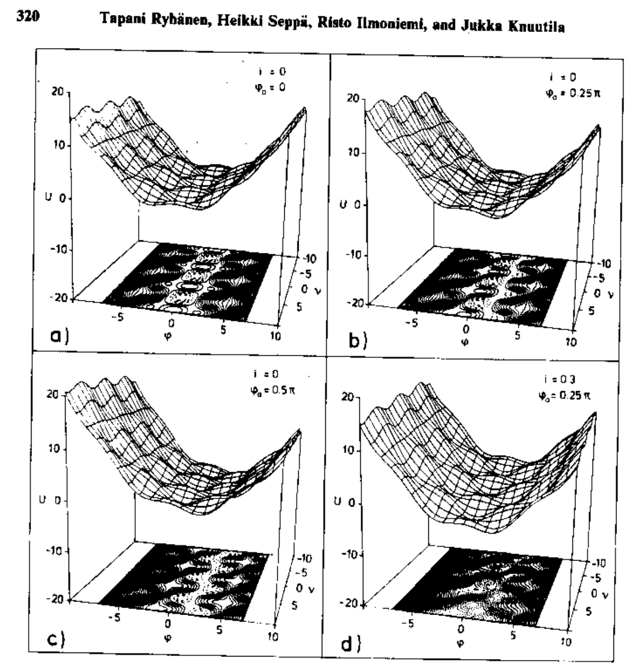




J. Nissilä, et al., IEEE Trans. Instrum. Meas. 54, 636, 2005.

2B SQUIDs in (bio)magnetometry

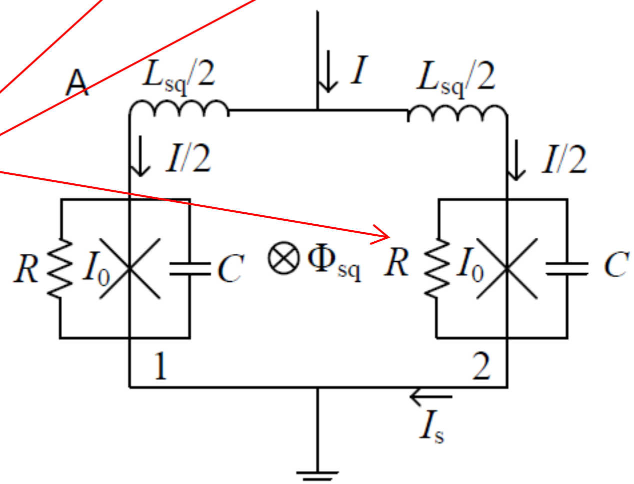
- Autonomous SQUID as magnetometer?
  - Stability/noise optimization:
    - Nonhysteretic JJ behavior β_c <~ 1
    - Single-valued flux behavior β_L ~ 1

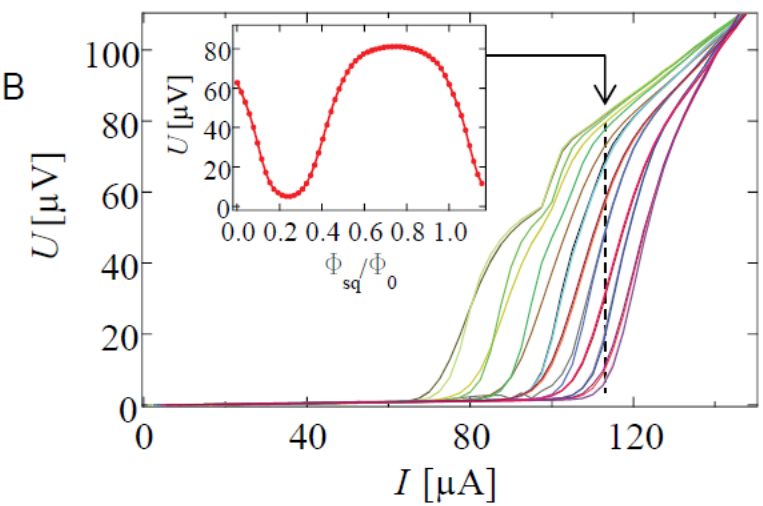


$$\beta_c \frac{d^2\nu}{dt^{*2}} + \frac{d\nu}{dt^*} + \sin\nu\cos\varphi = i + i_{n,\nu}(t^*)$$

$$\beta_c \frac{d^2\varphi}{dt^{*2}} + \frac{d\varphi}{dt^*} + \cos\nu\sin\varphi + \frac{2}{\beta_L}(\varphi - \varphi_a) = i_{n,\varphi}(t^*)$$

- Sensitivity fundamentally limited by the noise from the shunt resistors.





- Noise from resistive source
  - Energy resolution: minimum detectable magnetic field energy within a SQUID loop.

Thermally-limited energy resolution (from simulations) ε

$$\varepsilon = 12 k_B T \sqrt{L_{SQ} C_j}$$

Quantum-limited $\varepsilon \geq \hbar$

Flux noise

$$\Phi_n = \sqrt{2 L_{SQ} \varepsilon}$$

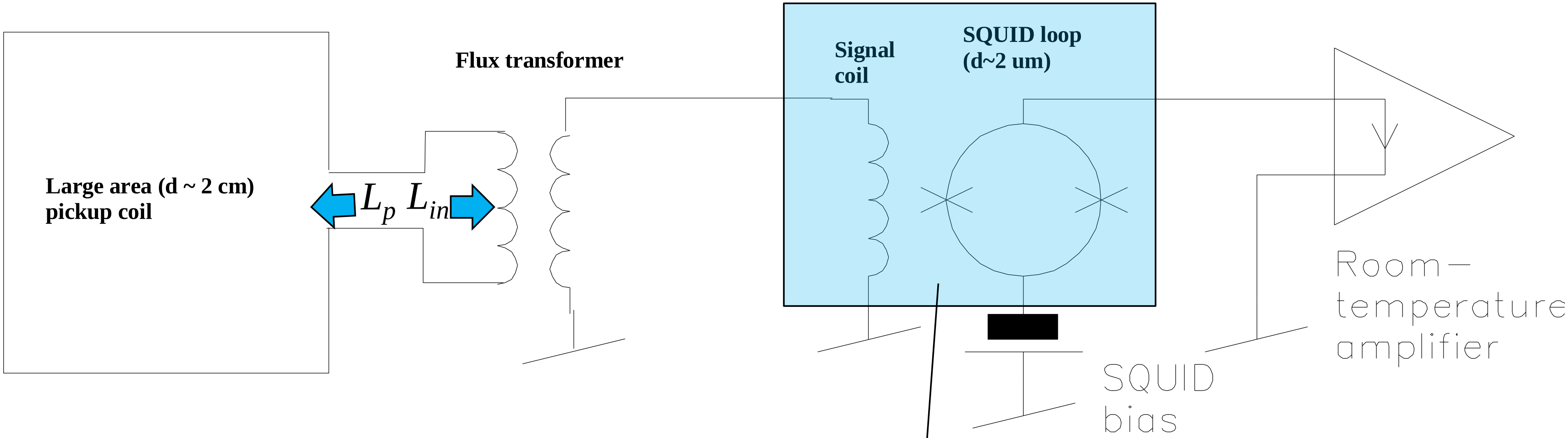
- Typical parameters limit detection area
  - Junction technology => I_c ~ 50 µA, C ~ 2 pF
  - β_L ~ 1 => L_{SQ} ~ 7 Ph
  - L_{SQ} ~ $\mu_0 d$ => SQUID loop dimension ~ 5 µm

⇒Flux noise $\Phi_n \sim 10^{-7}\, \Phi_0/\mathrm{Hz}^{1/2}$

⇒Magnetic field noise $S_B^{1/2} = \Phi_n/d^2 \sim 8$ pT/Hz$^{1/2}$

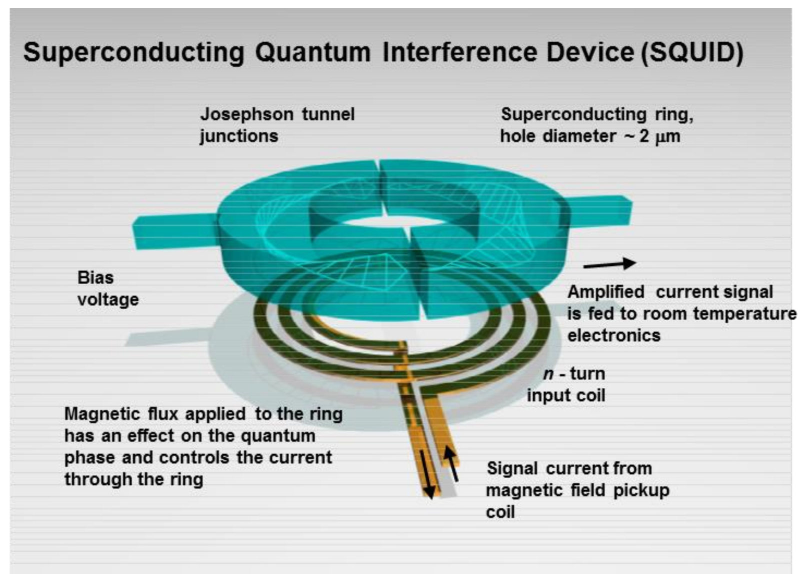
⇒Not good enough

- Typical magnetometer configuration
  - Magnetic field from e.g. brain converted to a flux on a SQUID loop
  - For optimal field-to-flux conversion a matching circuit used

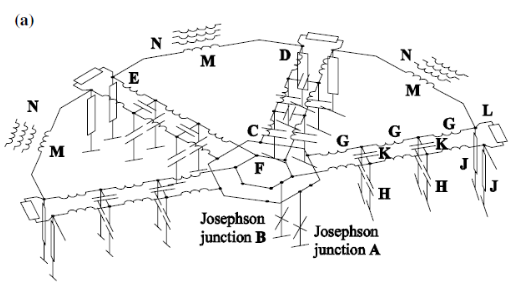


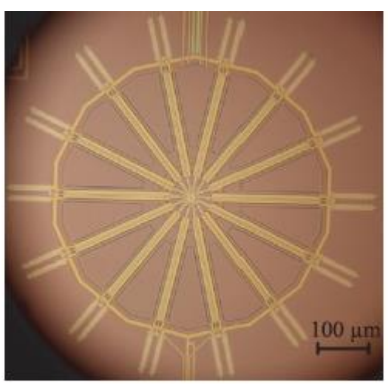
- For a SQUID with given energy resolution the magnetic field resolution improves:

$$S_B^{1/2} = \frac{L_\mathrm{p} + L_\mathrm{in}^\mathrm{eff}}{kA}\sqrt{\frac{2\epsilon_\mathrm{sq}}{L_\mathrm{in}}}.$$

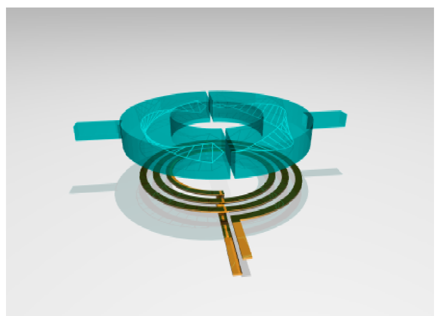


- Ways of doing the inductance transformation
  - Washer
  - Intermediate transformer
  - Multiloop
    - Several loops in parallel to drop the inductance as seen by the junctions
  - Array SQUID





Multiloop



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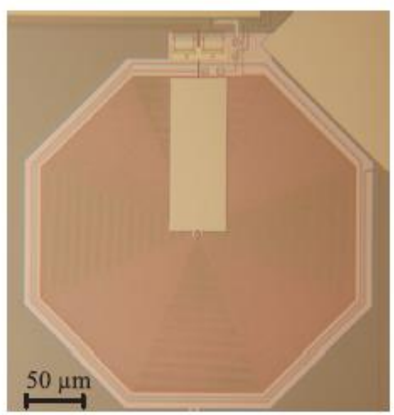
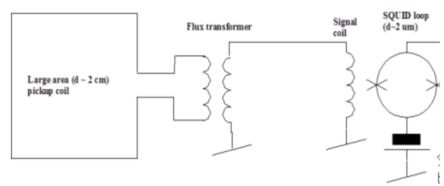
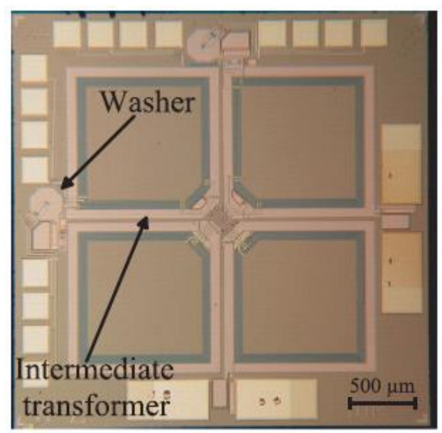
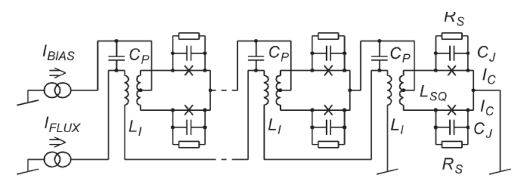


Washer





Washer + intermediate transformer



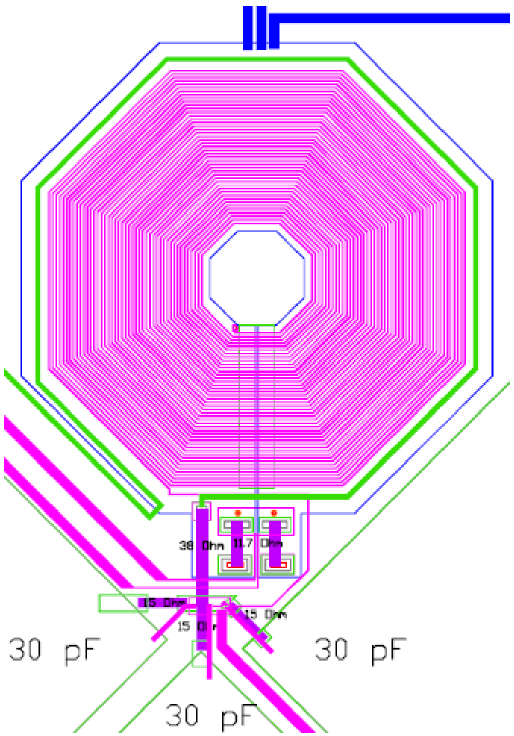
SQUID array

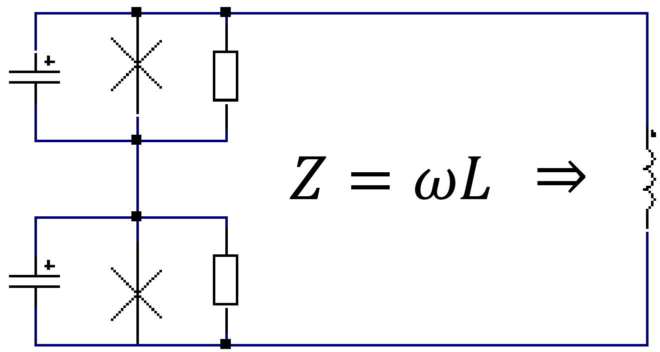
- Considerations about magnetometer SQUID design
  - Input transformers tend to generate parasitic resonances in the circuitry:

  $\Rightarrow$ Autonomous SQUID model no longer strictly valid.

  $\Rightarrow$ Handling (damping) the resonances needed in practical SQUID designs.

Autonomous
SQUID
Ideal mpedance
environment



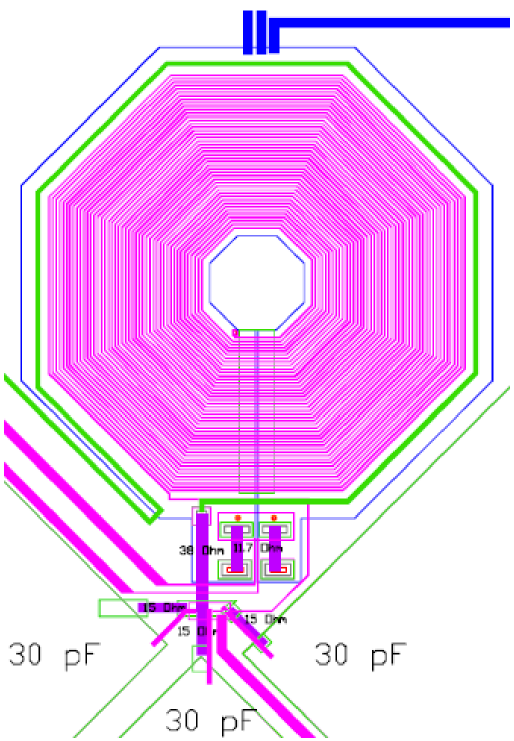


$$Z = \omega L \;\Rightarrow$$

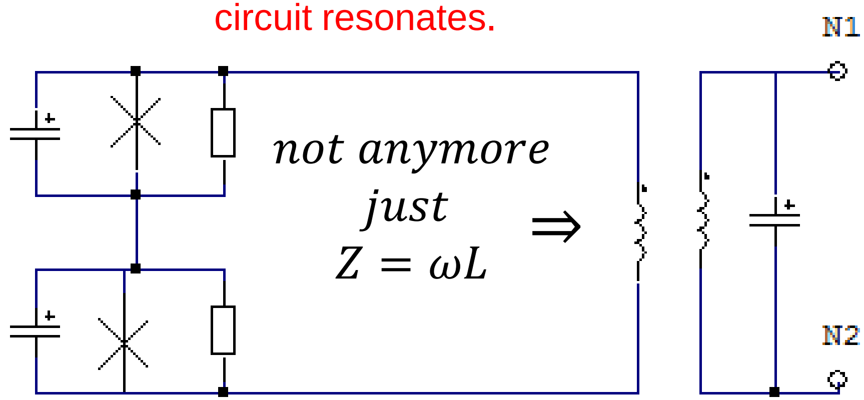
- Considerations about magnetometer SQUID design
    - Input transformers tend to generate parasitic resonances in the circuitry:

    $\Rightarrow$ Autonomous SQUID model no longer strictly valid.

    $\Rightarrow$ Handling (damping) the resonances needed in practical SQUID designs.



First order
problem: input
circuit resonates.



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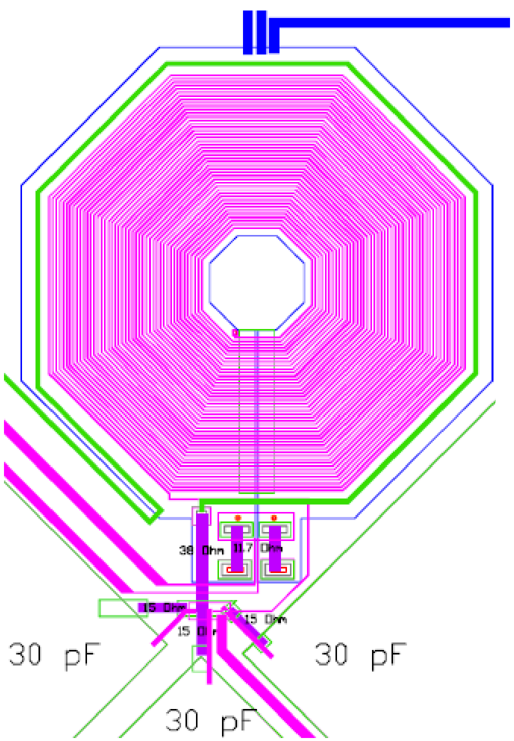
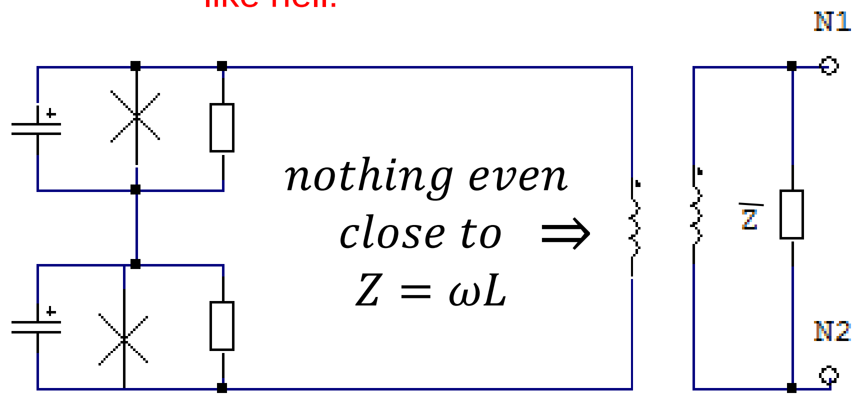
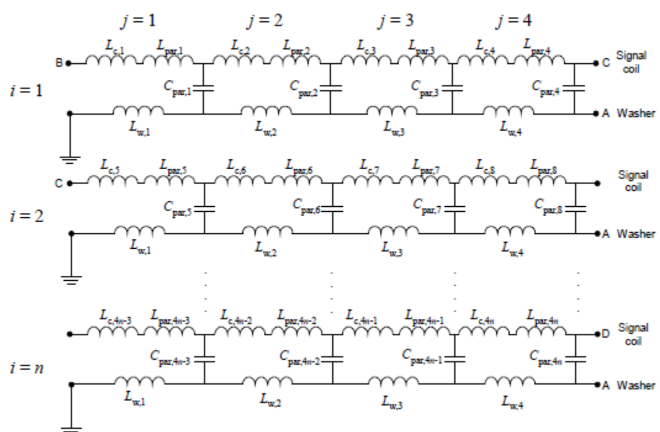
  $\Rightarrow$ Handling (damping) the resonances needed in practical SQUID designs.

Real-life situation:
input resonates
like hell!







Model for Z (between A and ground),

- Considerations about magnetometer SQUID design
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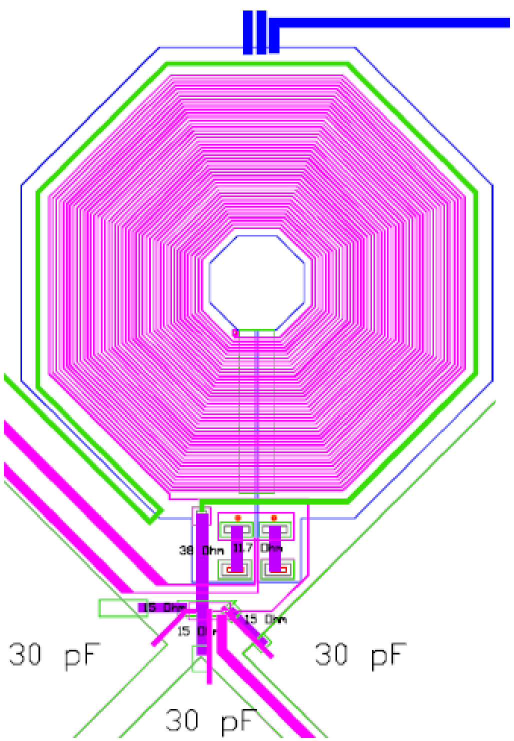
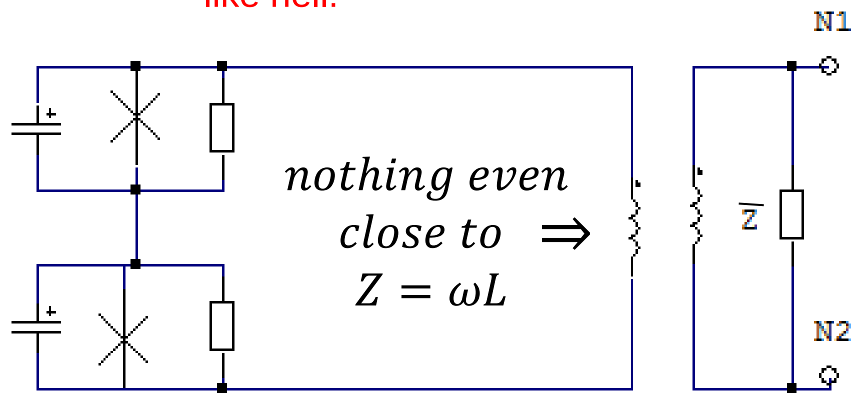
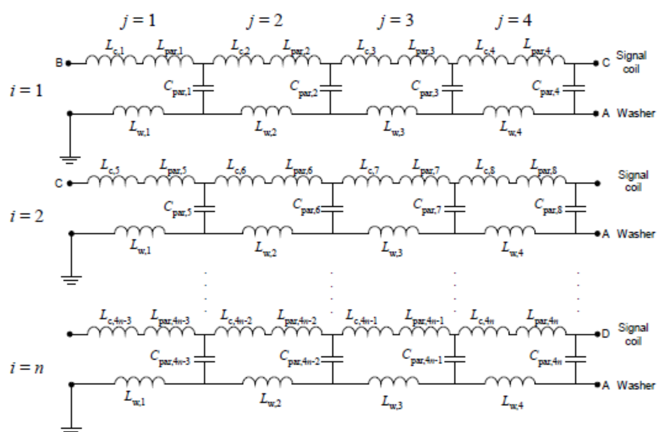
like hell!

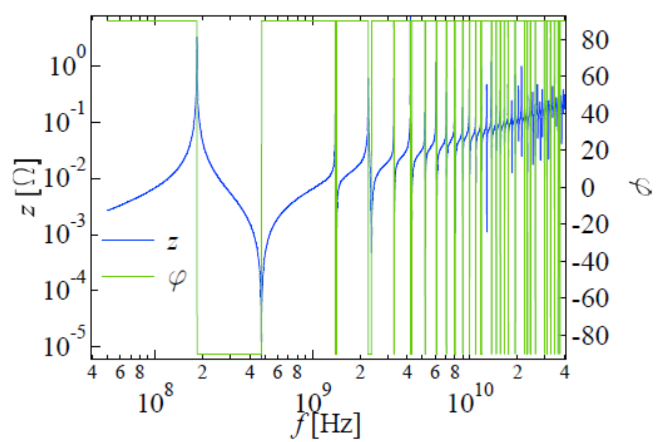
J. Luomahaara, Masters Thesis, Aalto Univ. 2009.







Model for Z (between A and ground),



Simulated Z.

- Considerations about magnetometer SQUID design
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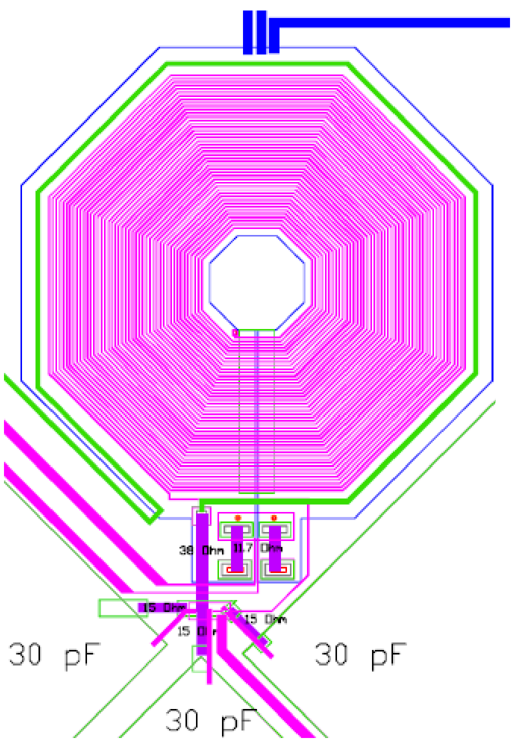
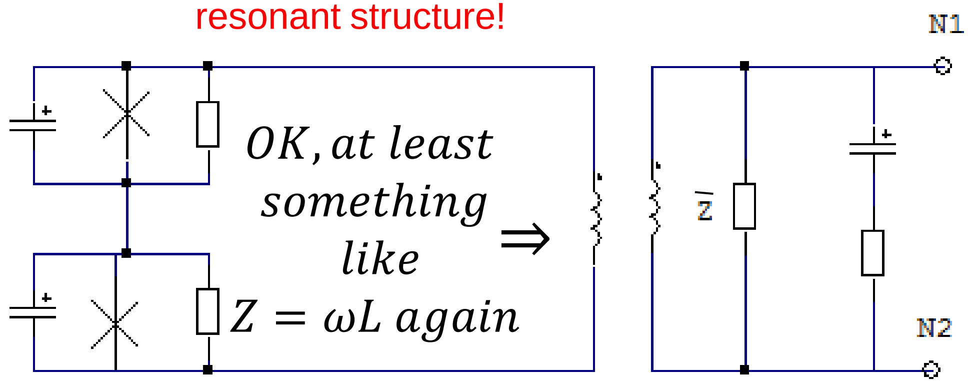
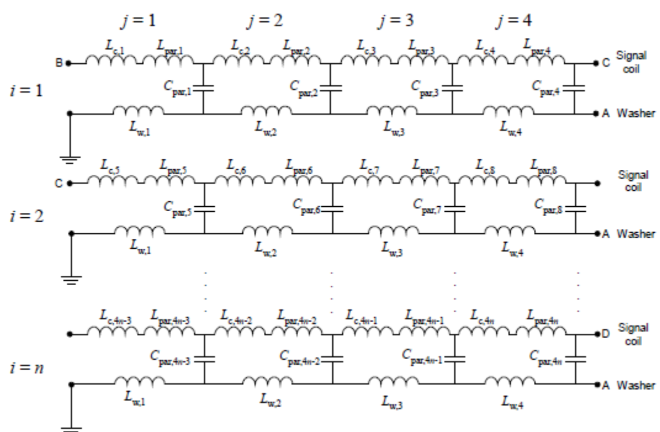
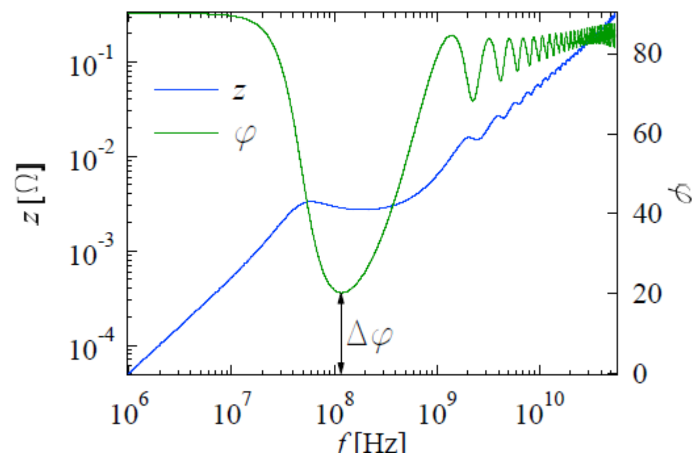


Try to damp the
resonant structure!



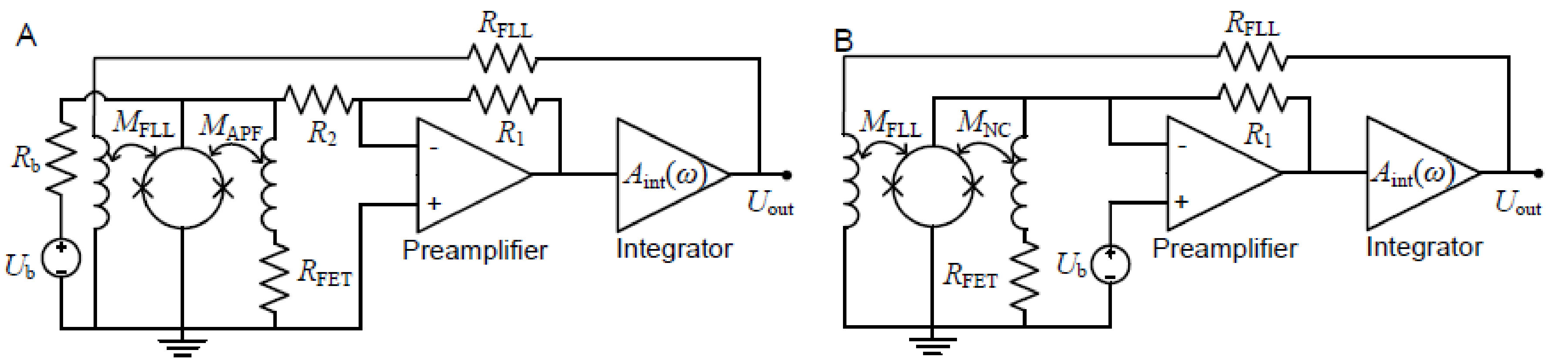


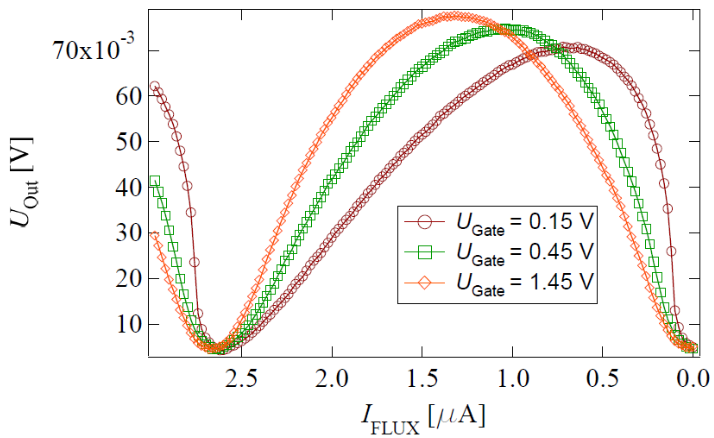
Model for $\bar{Z}$ (between A and ground),



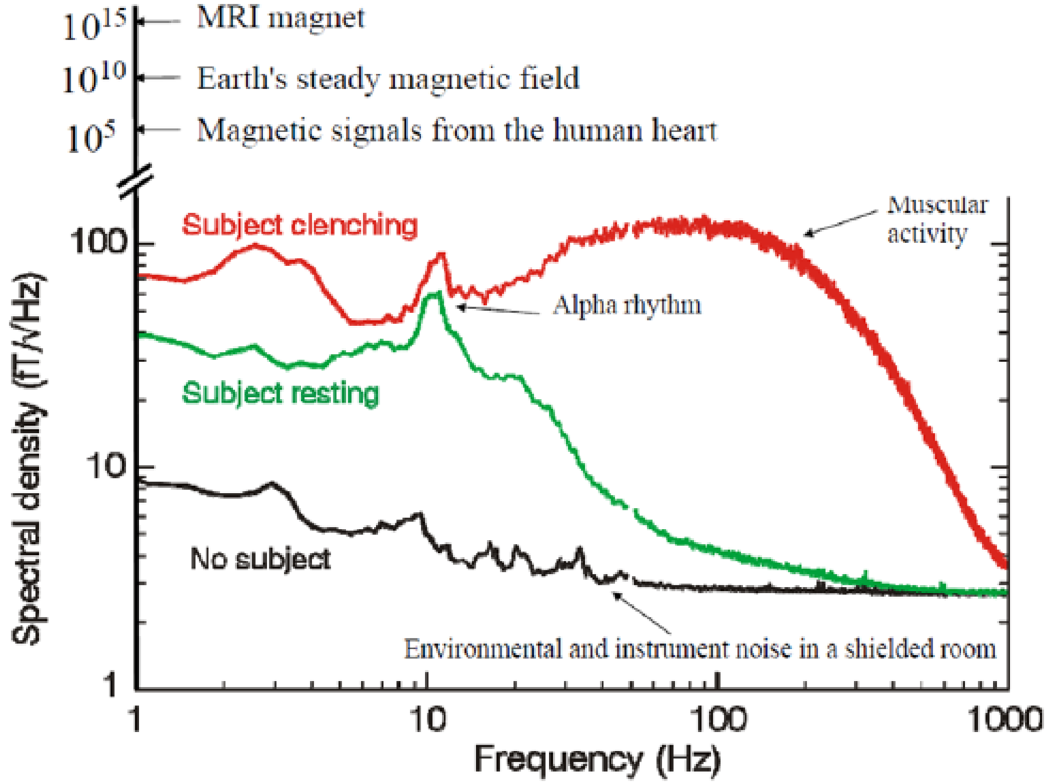
Simulated Z.

- Electronics for reading out the SQUIDs?
  - Problem 1: The linearity of the SQUID is limited
    - Solution: Flux-locked loop to linearize the response
  - Problem 2: sensitivity of commercial room temperature amplifiers is not sufficient for reading out the
    - Solutions: local feedback based readout methods (see below).



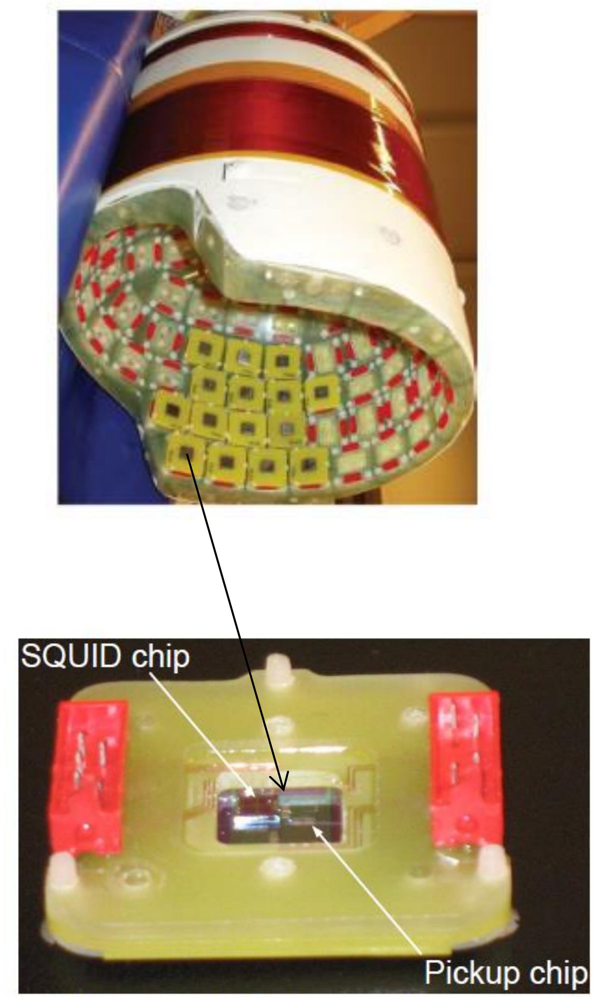


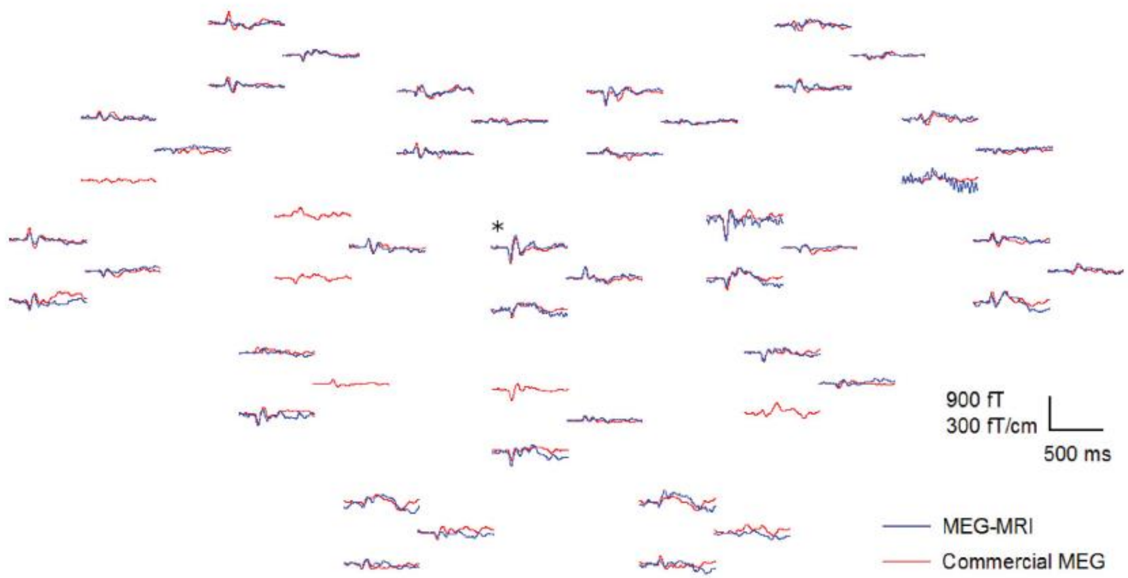
- Magnetoencephalography
  - Direct recording of magnetic signature of neuronal activity of brain
  - Applications:
    - Presurgical mapping (epilepsy, brain tumours).
    - Fundamental studies of cognition.
  - A few hundred sensor channels.
  - Sensitivity requirement ~fT/Hz$^{1/2}$

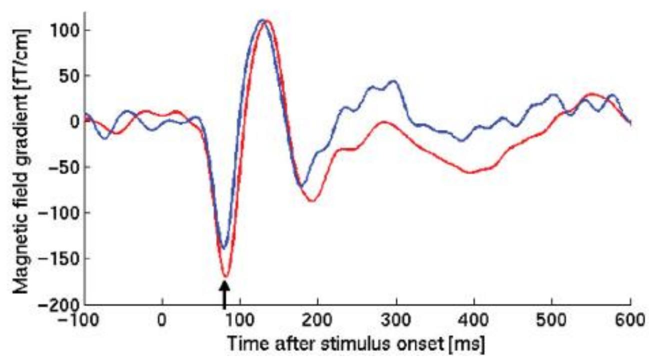


Noise/signal levels in MEG system
[Courtesy of prof. Lauri Parkkonen, Aalto Univ].

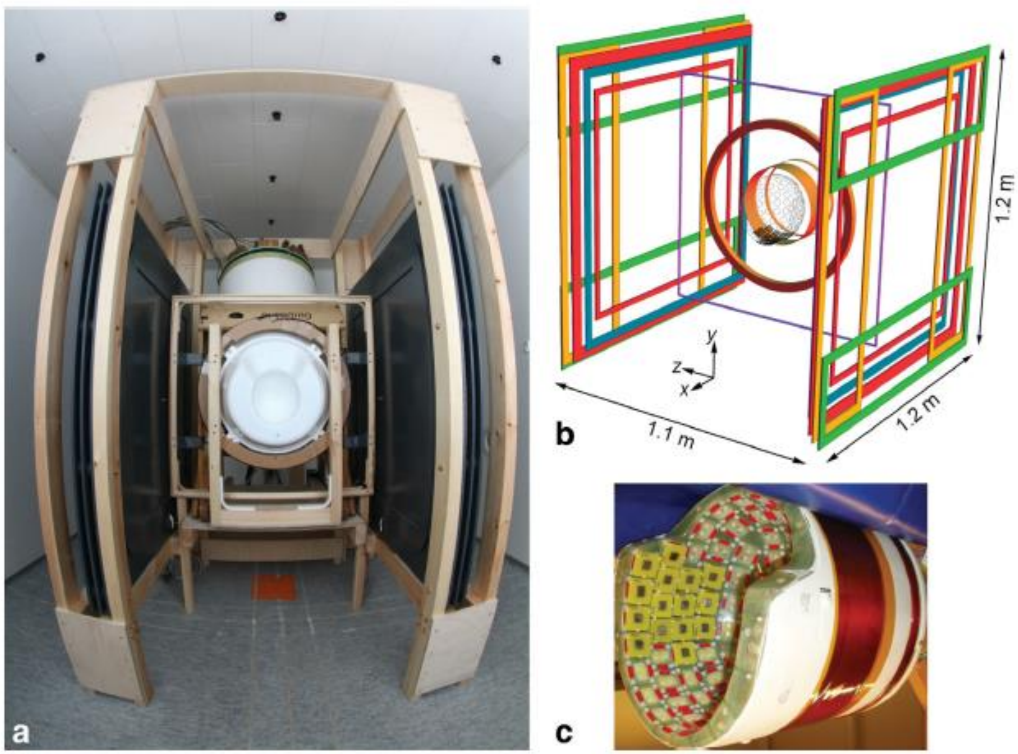
- Sensor data from MEG systems
    - Magnetic field after visual stimulus at $t = 0$ for a human subject

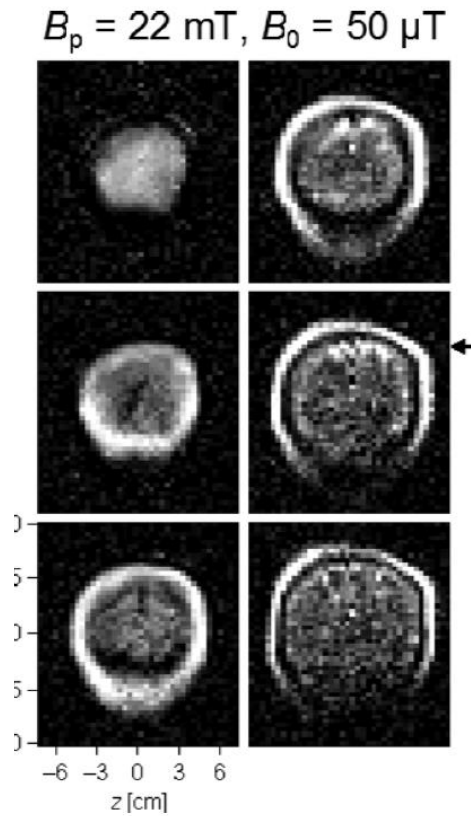


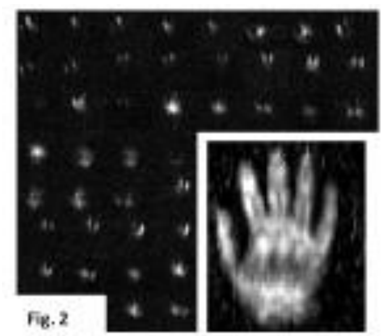




- MRI:
  - Detecting Larmor preseccion
- Ultra-low-field magnetic resonance imaging (ULF-MRI)
  - Conventional MRI at 1.5 – 3 T fields (~100 MHz Larmor frequency)
  - SQUID-based systems demonstrated in a few locations
  - European MEGMRI collaboration demonstrated integration of commercial MEG system and ULF-MRI modality.







P. Vesanen et al., Magn. Reson. Med. 69, 1795, 2013.

2C SQUIDs as low(esh) frequency amplifiers

- Low/intermediate frequency applications
  - General about amplifiers:
    - The performance of amplifier is chacaterised by:
      A) Its optimal noise temperature
      B) Its noise matching resistance
      - And the frequency dependencies of these.
    - A measure of 'goodness' of an amplifier is its impedance dependent noise temperature.

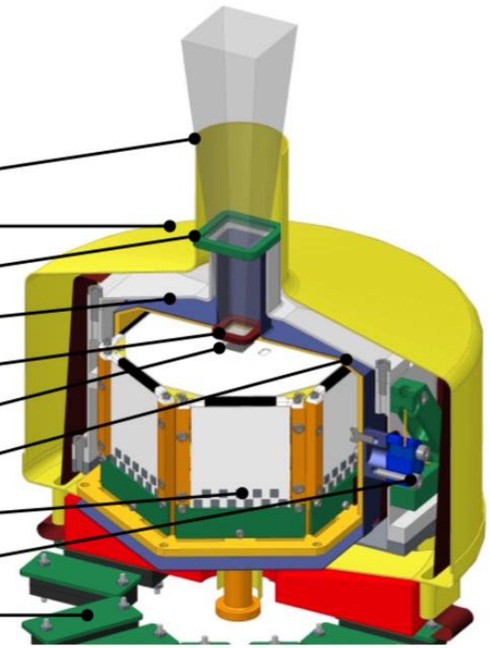
$$T_n = \frac{T_{n,opt}}{2}\left(\frac{R_d}{R_{opt}} + \frac{R_{opt}}{R_d}\right) \approx \frac{R_d i_n^2}{4k_b}$$

- The last approximation valid for SQUIDs at low *f* (very low optimum noise matching resistance R_{opt})

⇒SQUID is intrinsically good for reading out very low impedance sources

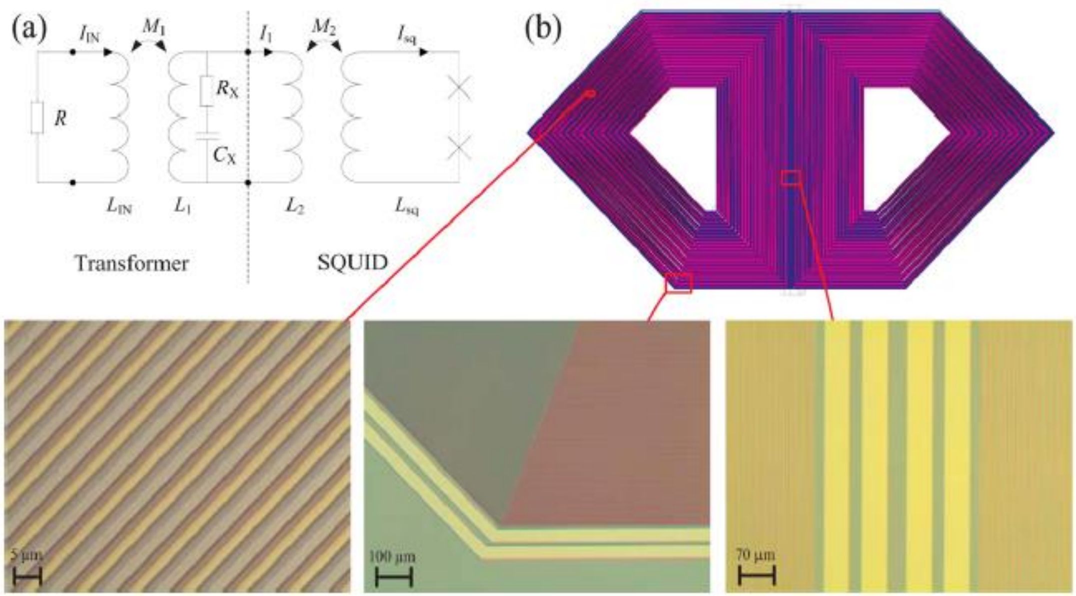
- Example: superconducting transition edge sensors (TESs)
  - Superconducting films biased to transition => $R_d \sim 1\ \mathrm{m}\Omega$
  - Applications in astronomical imaging (X-ray, THz).

Advanced Telescope for High-Energy Astrophysics (ATHENA)



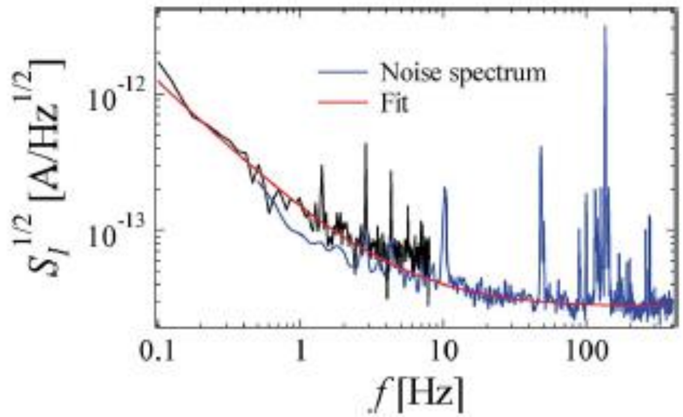
X-ray Integral Field Unit (X-IFU)

- For larger impedeance applications:
  - Intrinsic current sensitivity (and thus the noise temperature) can (in principle indefinitely) improved by adding turns to input transofrmer.
  - ⇒Optimal for reading out low-impedance sources.
  - ⇒Large impedance sources require large input transformers (in analogy to magnetometer inductance matching)



1284:2 transformer configuration aimed for metrology applications. This one optimises to about ~ 100 kΩ at 1 K operating temperature, but needs ~100 m of thin film line!

J. Luomahaara, et al., Supercond. Sci. Technol. 25, 035006, 2012.

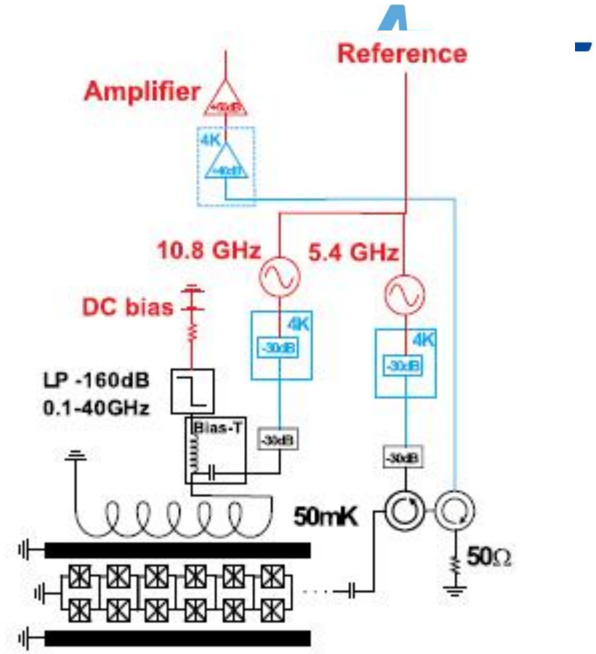


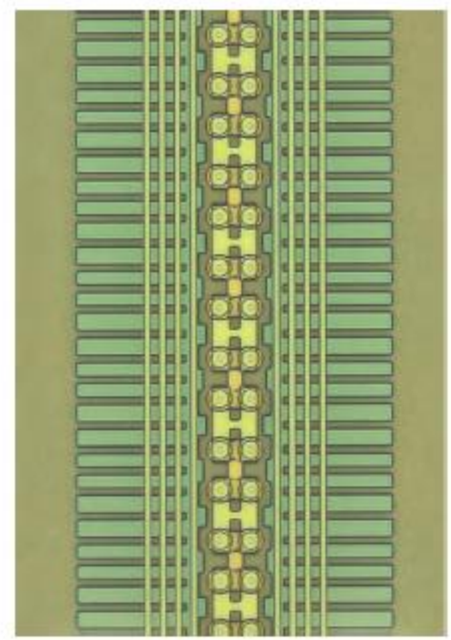
Input referred current noise measured with the transformer/SQUID entity.

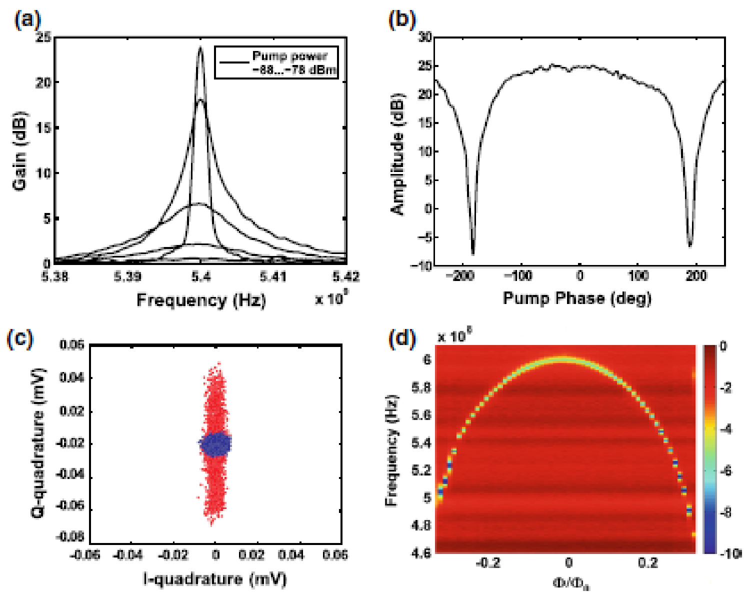
J. Luomahaara, et al., IEEE Trans. Appl. Supercond 25, 1601705, 2013.

2D SQUIDs as microwave amplifier

- Josephson junction is a current or flux tunable inductor
  - => Parametric amplification (believe Sorin covered a lot of this on Friday)

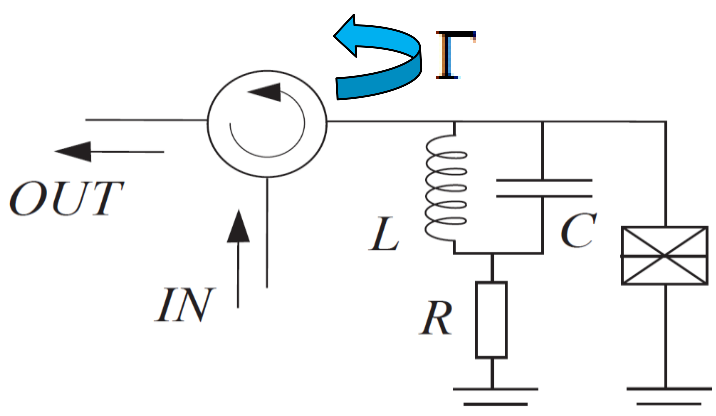




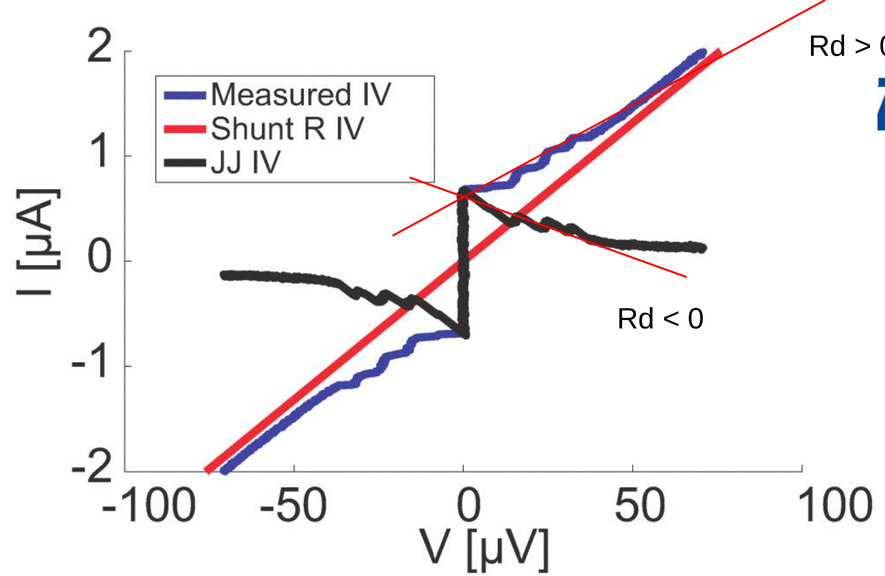


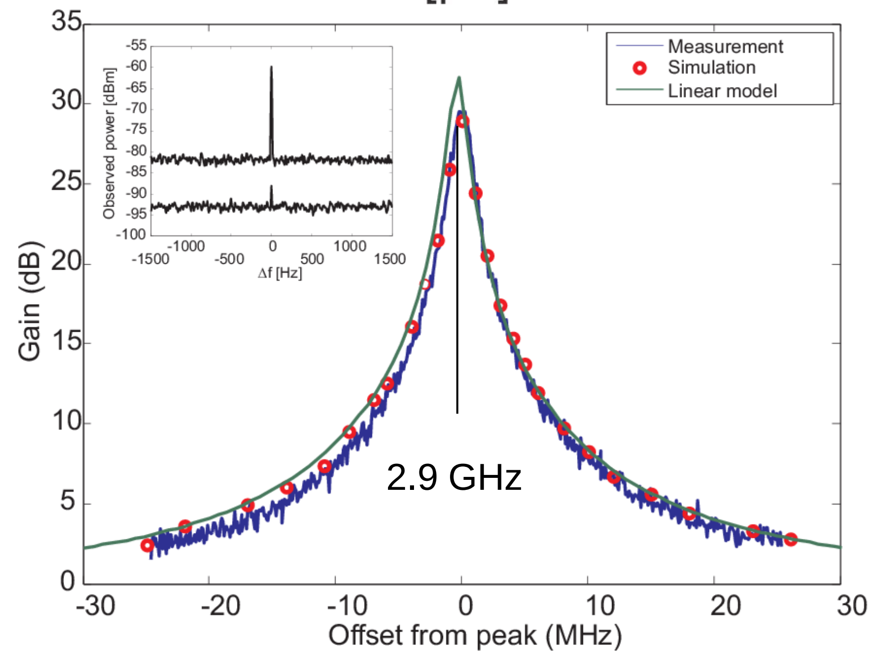
P. Lähteenmäki, et al., J. Low Temp. Phys. 175, 868, 2014.

- Intrinsic dynamic resistance R_d of JJ negative
    - Leads to hysteretic characteristics
- Overdamped JJ -> positive dynamic impedance, stable system.
- Remove damping in *LC* centered band:
    - Instability ('negative Q' osillations).
- Couple to external circuitry as below
    - Stability restored.
    - In signal band stability restoration leads to the gain in the *LC* centered band!



$$G_{max} = |\Gamma|^2_{max} = \left|\frac{R_d - Z_0}{R_d + Z_0}\right|^2$$



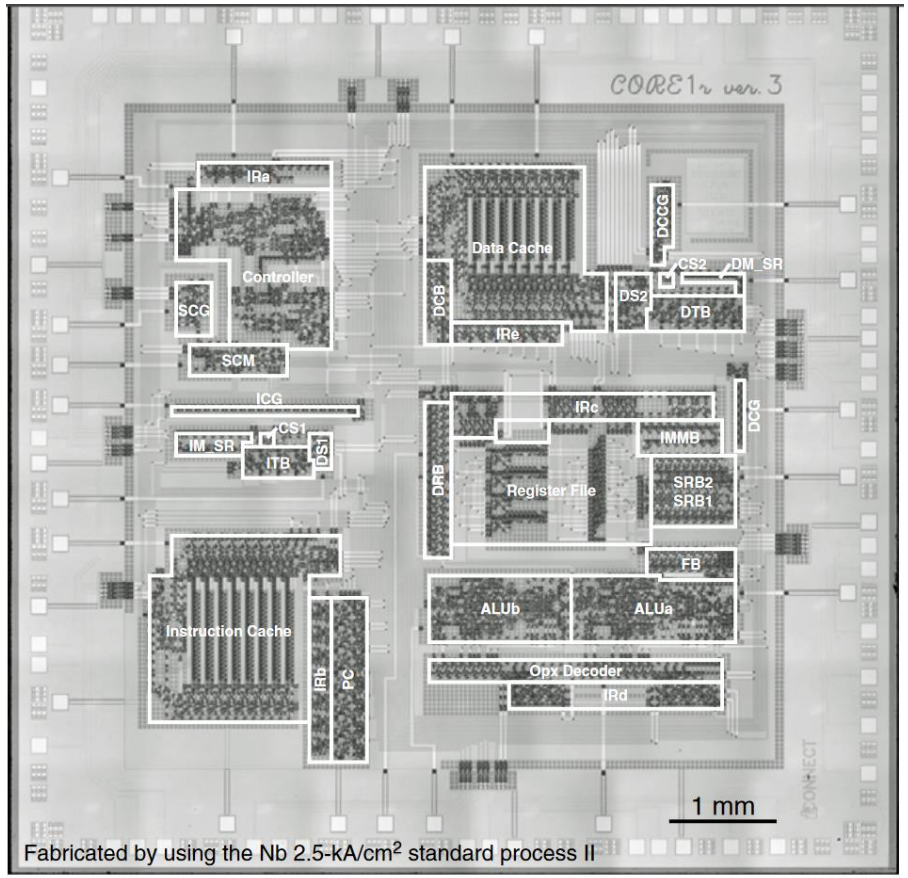


Noise and gain calibration: Experimental T_n = (220 ± 70) mK = (1.6 ± 0.5)*hf*.

V. Vesterinen, et al., Sci. Rep. 2, 276, 2012.

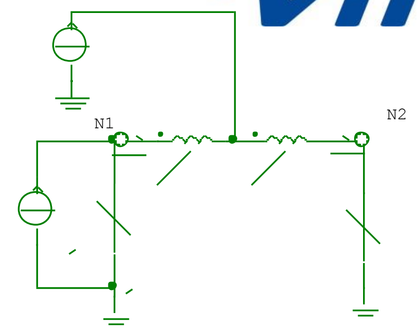
2E Rapid single flux quantum logic

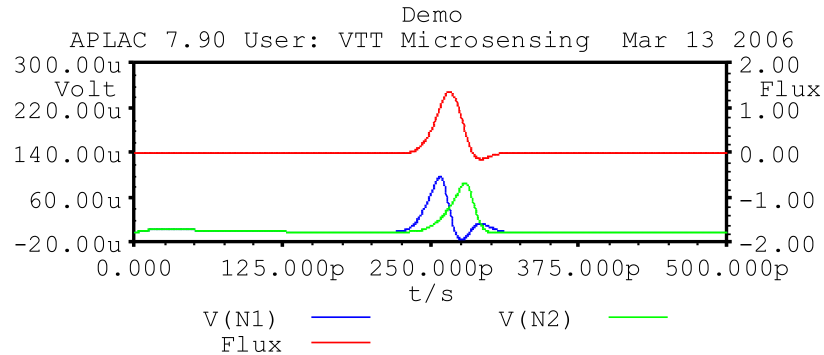
- Previous applications rely on SQUID dynamics at finite voltage dynamics
    - Flux quanta ’flowing’ continuously
- RSFQ technology is based on single 2π phase rotations to produce logic operations
- Aim here understanding of basic Josephson dynamics used in RSFQ



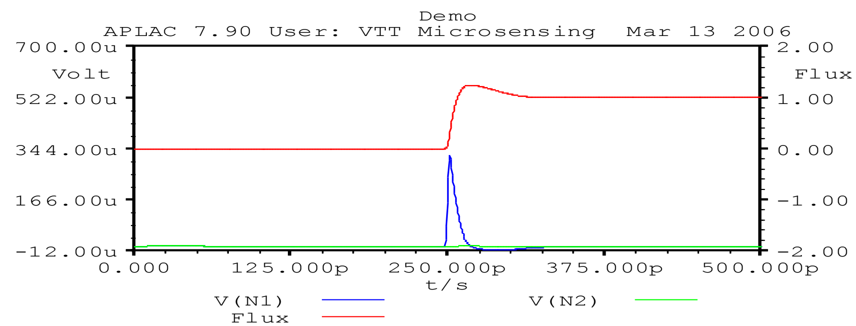
RSFQ microprosessor with ~22 000 JJs [M. Tanka, et. al, Supercond. Sci. Technol. 20, 2007].

- Introduction of phase rotation dynamics: a simple DC SQUID ring biased to a current below I_c
  - Force left junction switch
  - Case 1: $\beta_L < 1$
    - Second junction switches too
    - No flux stored in the ring
      - Remains in "False" state

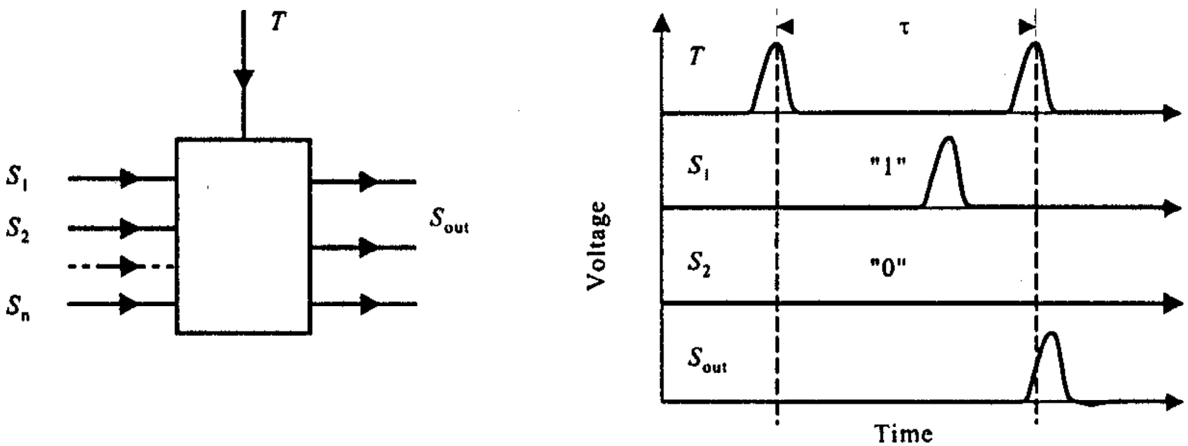




  - Case 2: $\beta_L > 1$
    - Second junction does not switch
    - Flux quantum stored in the ring
      - Switches to "True" state



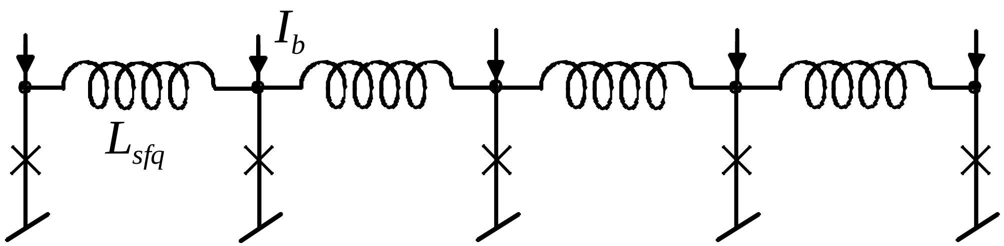
- RSFQ technology uses this type of JJ/SQUID switching dynamics to construct logical operations
- A general RSFQ cell consists of a clock input, signal inputs and outputs
- If a flux quantum approaches to signal input between clock pulses it means, that the input is set to logical "True", if not it is set to "False"
- The ending clock pulse sends or does not send an output pulse depending on the result of the logical function the cell realises

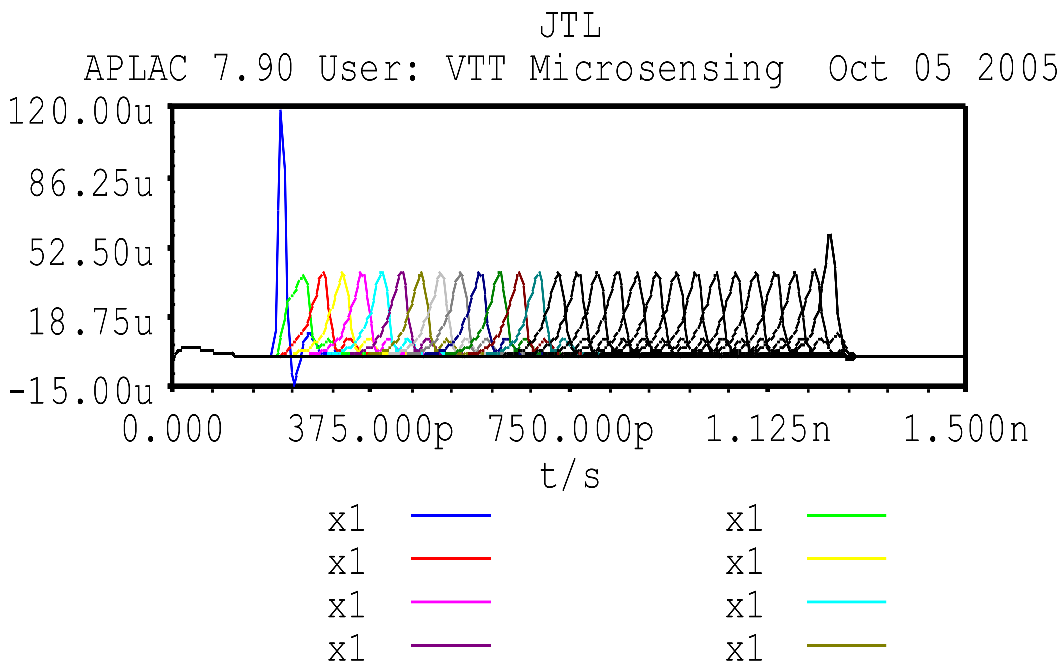


T. van Duzer, Principles of Superconducting Devices and Circuits .

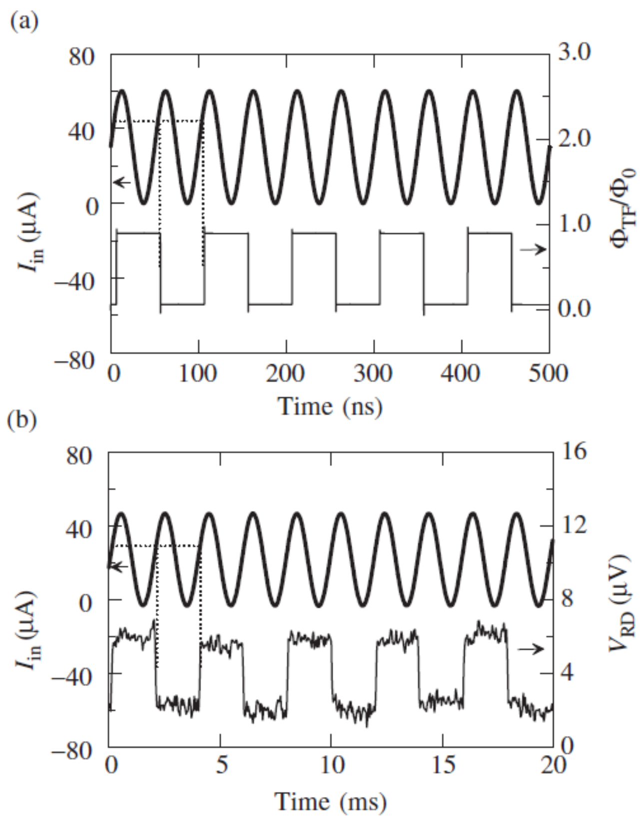
Figure 5.16f Block diagram of a general RSFQ cell and the timing diagram showing incoming pulses on S_1 and S_2 arriving between two clock pulses. The output pulse is produced by the clock pulse ending the period. (Footnote 105.)

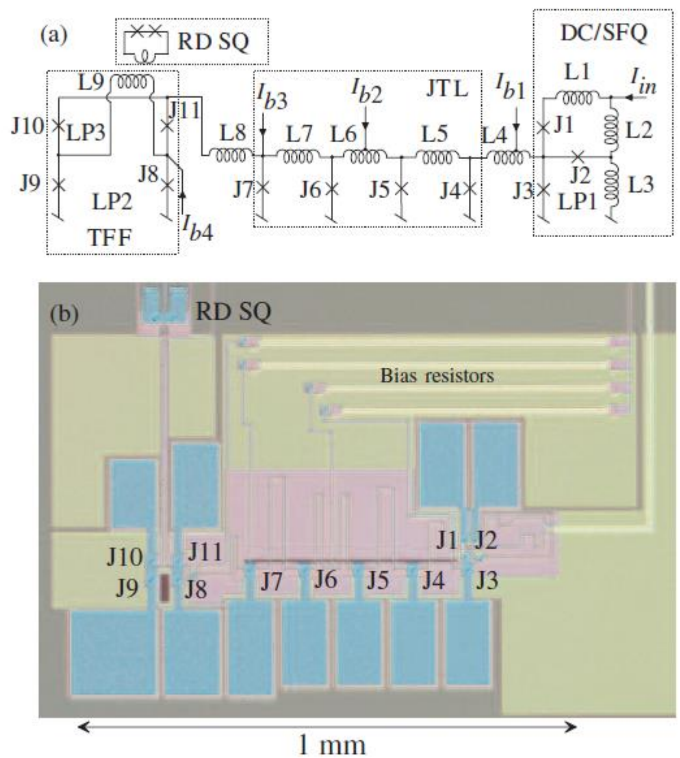
- Basic functionalities of RSFQ (1):
  - 'Propagation'
    - Pulses are propagated in Josephson transmission lines with soliton dynamics:





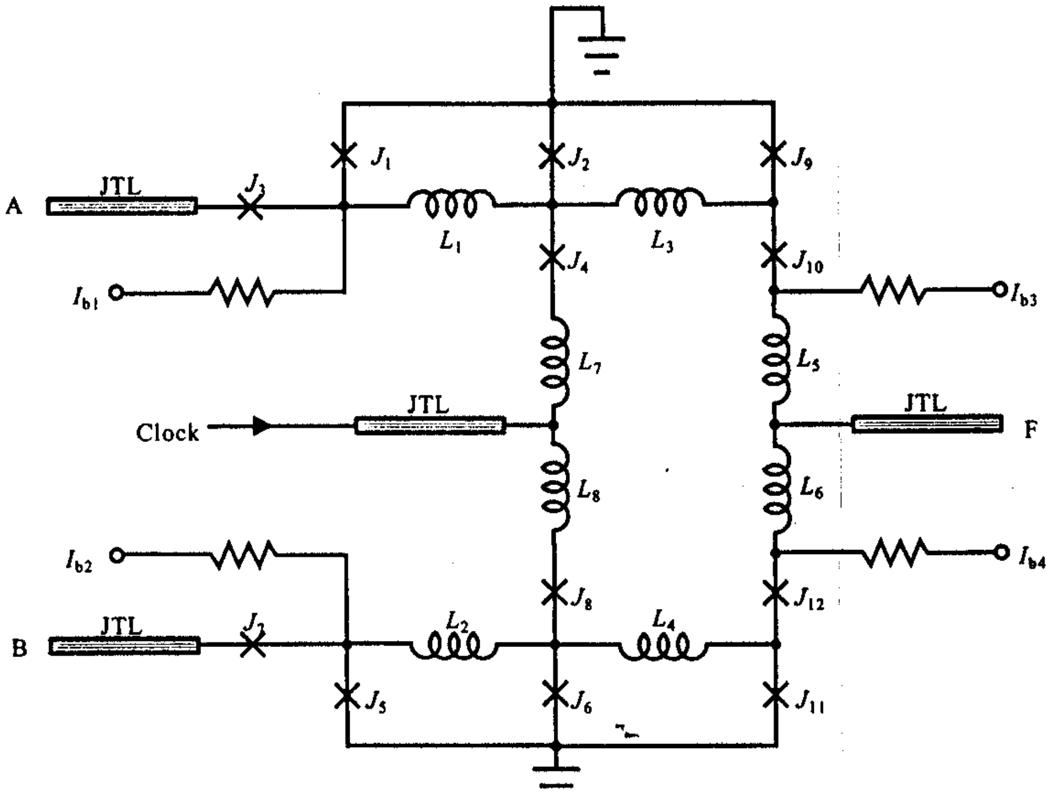
- Basic functionalities of RSFQ (2):
  - ’Storage’
    - The flux quanta are temporarily stored as persistent current in certain SQUID loops
  - ’Selection’
    - Switching conditional to whether a given SQUID loop has flux quantum stored (persistent current) or not.





J. Hassel, et. al., New J. Phys. 9, 158, 2007.

- An example cell:
  - OR gate



T. van Duzer, Principles of Superconducting Devices and Circuits .

Summary

- Josephson dynamics simple but very versatile
  - Phase locked Josephson oscillation => voltage metrology
  - Flux/field controlled phase propagation
    - Magnetometry, current amplifier
  - Tunable/nonlinear inductance, negative resistance
    - Parametric amplification
  - Controlled single phase rotations
    - Ultrafast logic