Noise in mesoscopic systems

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Basics of scattering theory (Landauer-Büttiker theory)
Fundamentals of shot noise
- Khlus formula (*T*, *V*, and *f* dependence of noise)
- shot noise thermometry
- examples from research
- duality between charge and phase

Shot noise measurement techniques
- basic problems
- present correlation spectrometer

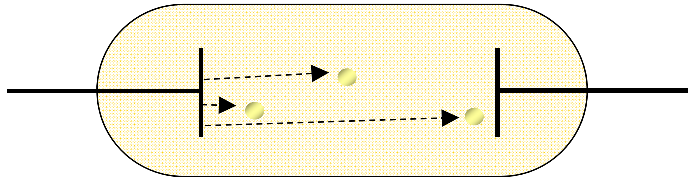
Noise temperature consideration
- amplifiers and matching

Shot noise

Classical shot noise:

W. Schottky, Ann. Phys. (Leipzig) 57, 541 (1918)

$$\langle (\Delta I)^2 \rangle_\nu = 2e\langle I \rangle$$



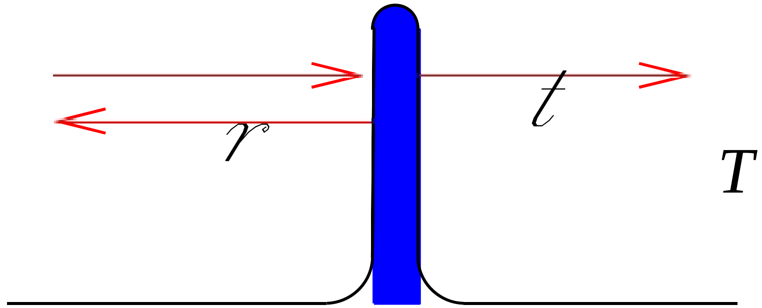
Quantum shot noise:

Khlus (1987), Lesovik (1989), Yurke and Kochanski (1989), Buttiker (1990)

$$|\Psi>_{inc} = e^{ikx}$$

$$|\Psi>_{ref} = re^{-ikx}$$

$$|\Psi>_{tra} = t\, e^{ikx}$$

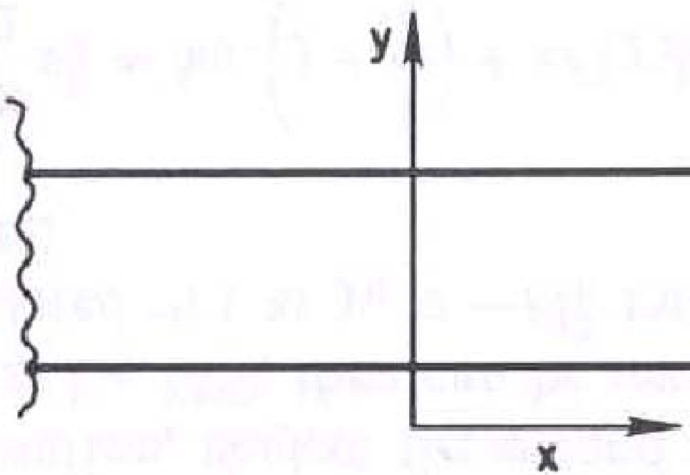


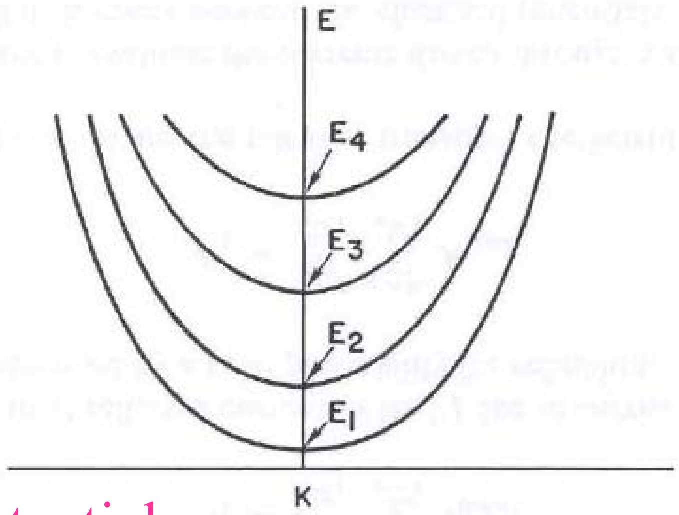
$$T = |t|^2$$

$$\langle (\Delta n_T)^2 \rangle = T(1-T) \quad \Rightarrow \quad \langle (\Delta I)^2 \rangle_\nu = 2e\langle I \rangle (1-T)$$

*Ya.M. Blanter and M. Büttiker, Physics Reports **336**, 166 (2000)*

Scattering channels





asymptotic perfect translation invariant potential

$$V(x,y) = V(y)$$ ⇨

separable wave function

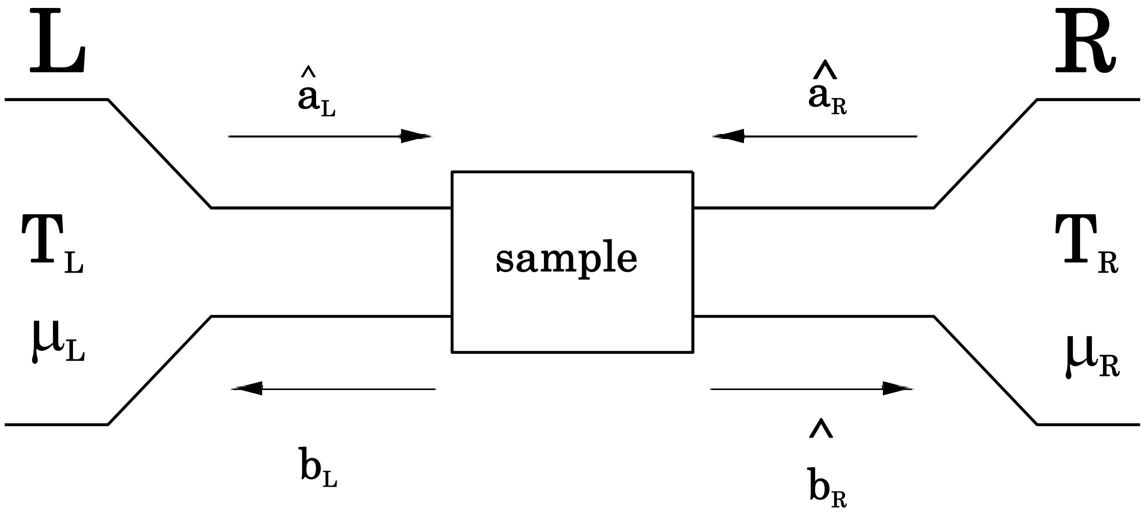
$$\phi_n^{\pm}(\mathbf{r},E) = e^{\pm i k_n(E) x}\,\chi_n(y)$$

energy of transverse motion E_n ***channel threshold***

energy for transverse and longitudinal motion

$$E = E_n + \hbar^2 k^2/2m$$ ***scattering channel***

Scattering matrix and the Fermi field



$$b_{\alpha n} = \sum_{\beta m} s_{\alpha\beta nm} a_{\beta m} \qquad \mathbf{b}_\alpha = \sum_\beta \mathbf{s}_{\alpha\beta}\mathbf{a}_\beta$$

Fermi field in contact α

$$\hat{\Psi}(\mathbf{r},t) = \sum_n \int \frac{dE_{\alpha n}\,\chi_{\alpha n}(y)}{[hv_{\alpha n}]^{1/2}}[e^{ikx}\hat{a}_{\alpha n} + e^{-ikx}\hat{b}_{\alpha n}]e^{-iE_{\alpha n}t/\hbar}$$

Current matrix

$$j(r,t) = -\frac{e\hbar}{2mi}[\Psi^\dagger(r)\nabla\Psi(r) - \Psi(r)\nabla\Psi^\dagger(r)]$$

$$I_\alpha(t) = \int dx_\alpha dy_\alpha j(x_\alpha, y_\alpha, z_\alpha, t)$$

$$I_\alpha(t) = \frac{e}{h}\int dE' dE [a_\alpha^\dagger(E')a_\alpha(E) - b_\alpha^\dagger(E')b_\alpha(E)]e^{i(E'-E)t/\hbar}$$

$$I_\alpha(t) = \frac{e}{h}\int dE' dE \sum_{\beta,\gamma} a_\beta^\dagger(E')A_{\beta\gamma}(\alpha, E', E)a_\gamma(E)e^{i(E'-E)t/\hbar}$$

$$A_{\beta\gamma}(\alpha, E', E) = 1_\alpha\delta_{\alpha\beta}\delta_{\alpha\gamma} - s_{\alpha\beta}^\dagger(E')s_{\alpha\gamma}(E)$$

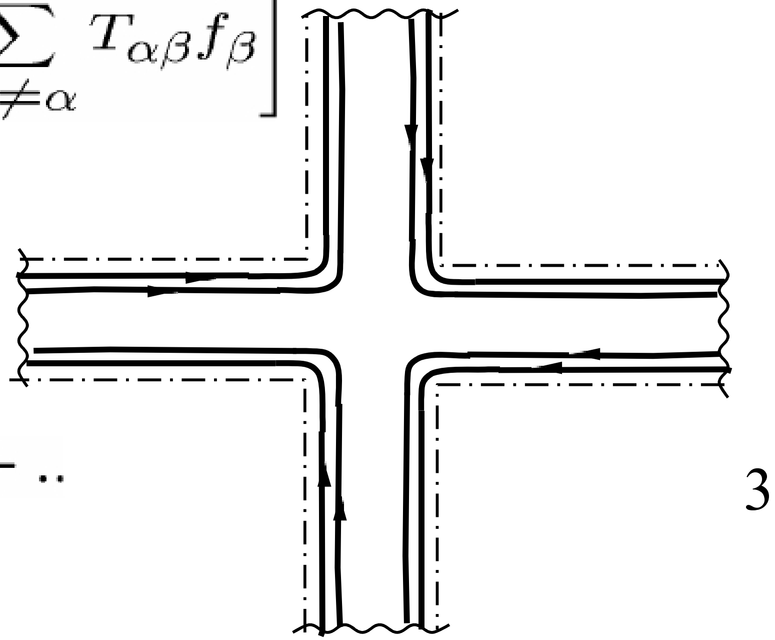
⇨ dc-transport, ac-transport, noise

Conductance matrix

$$I_\alpha = \frac{e}{h}\int dE\left[(N_\alpha - R_{\alpha\alpha})f_\alpha - \sum_{\beta\neq\alpha} T_{\alpha\beta} f_\beta\right]$$

$$R_{\alpha\alpha} = Tr(s^\dagger_{\alpha\alpha} s_{\alpha\alpha})$$

$$T_{\alpha\beta} = Tr(s^\dagger_{\alpha\beta} s_{\alpha\beta})$$



$$f_\alpha(\mu_\alpha) = f(\mu_0) - (df/dE)eV_\alpha + ..$$

⇨ $$I_\alpha = \frac{e}{h}\sum_\beta G_{\alpha\beta} V_\beta$$

$$G_{\alpha\alpha} = \frac{e^2}{h}\int dE(-df/dE)(N_\alpha - R_{\alpha\alpha})$$

$$G_{\alpha\beta} = -\frac{e^2}{h}\int dE(-df/dE)T_{\alpha\beta}$$

$$\sum_\alpha G_{\alpha\beta} = 0$$

$$\sum_\beta G_{\alpha\beta} = 0$$

Shot-noise: two-terminal

$$S_{\alpha\beta} = 2\frac{e^2}{h}\sum_{\gamma\delta}\int dE\,Tr[A_{\gamma\delta}(\alpha)A_{\delta\gamma}(\beta)]f_\gamma(E)(1-f_\delta(E))$$

Consider a two-terminal conductor at $T = 0$, $V > 0$:

$$S = S_{11} = -S_{12} = -S_{21} = S_{22};$$

Quantum partition noise

$$S = 2\frac{e^2}{h}|eV|\,Tr[tt^\dagger rr^\dagger] = 2\frac{e^2}{h}|eV|\sum_n T_n(1-T_n)$$

If all $T_n \ll 1$

$$S = 2e\left(\frac{e^2}{h}\sum_n T_n\right)|V| = 2e\,|I|$$

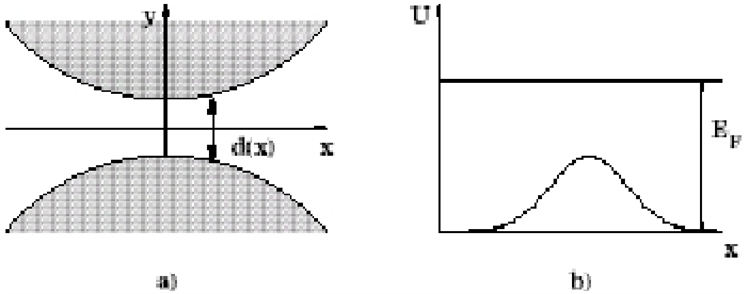
Schottky (Poisson)

Fano factor

$$F = \frac{S}{S_P} = \frac{\sum_n T_n(1-T_n)}{\sum_n T_n}$$

Khlus (1987)
Lesovik (1989)
Buttiker (1990)

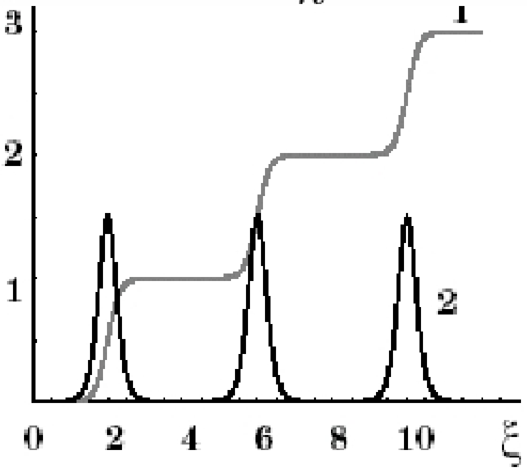
Shot-noise: Quantum point contact

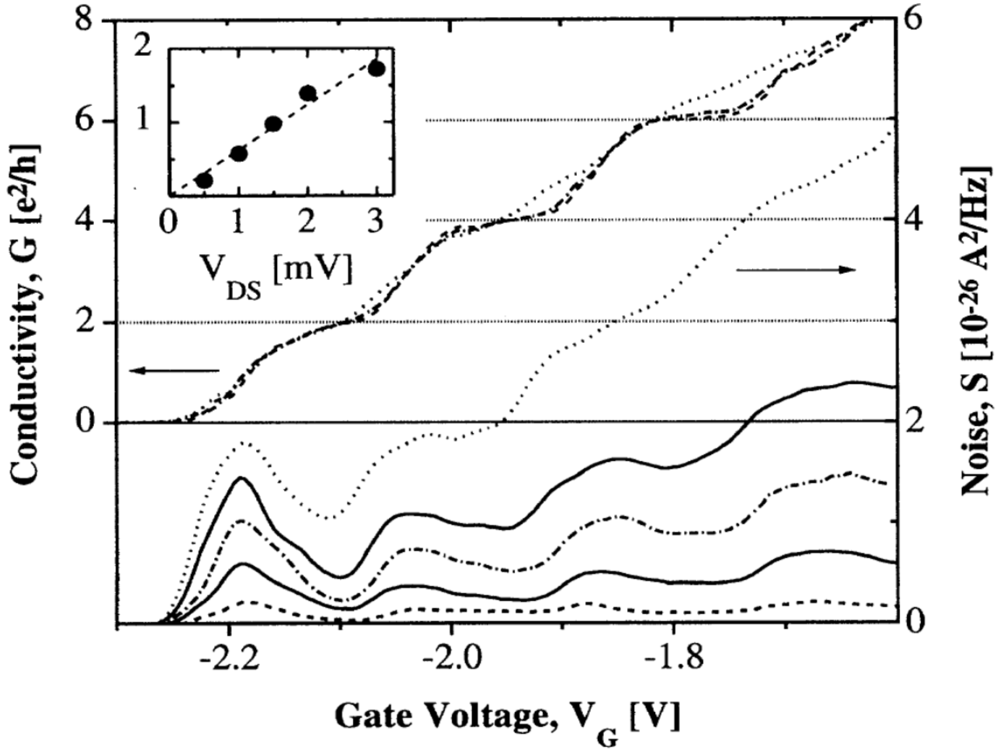


A. Kumar, L. Saminadayar, D. C. Glattli, Y. Jin, B. Etienne, PRL 76, 2778 (1996)

M. I. Reznikov, M. Heiblum, H. Shtrikman, D. Mahalu, PRL 75, 3340 (1996)

$$S = 2\frac{e^2}{h}|eV| \sum_n T_n(1 - T_n)$$





Ideally only one channel contributes

Finite temparatures

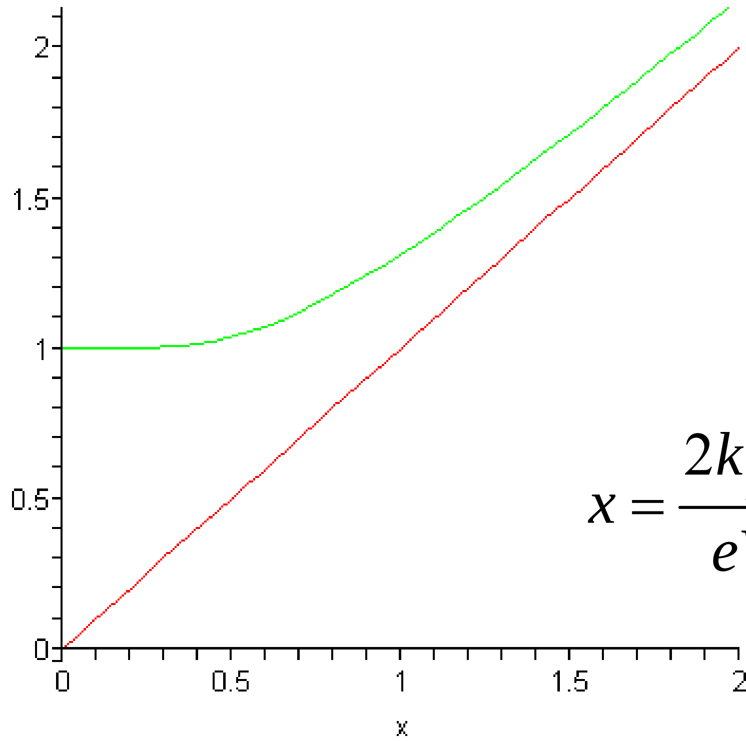
$$S_i(\omega=0)=2\frac{e^2}{h}\int dE\left\{T_i f_L(1-f_L)+T_i f_R(1-f_R)+T_i(1-T_i)(f_L-f_R)^2\right\}$$

$$S_I(\omega=0)=2\frac{e^2}{h}\left\{k_B T\sum_i T_i^2+eV\sum_i T_i(1-T_i)\coth\left(\frac{eV}{2k_B T}\right)\right\}$$

$$S_I=4\frac{e^2}{h}k_B T G, \quad V\to 0$$

Johnson-Nyquist noise

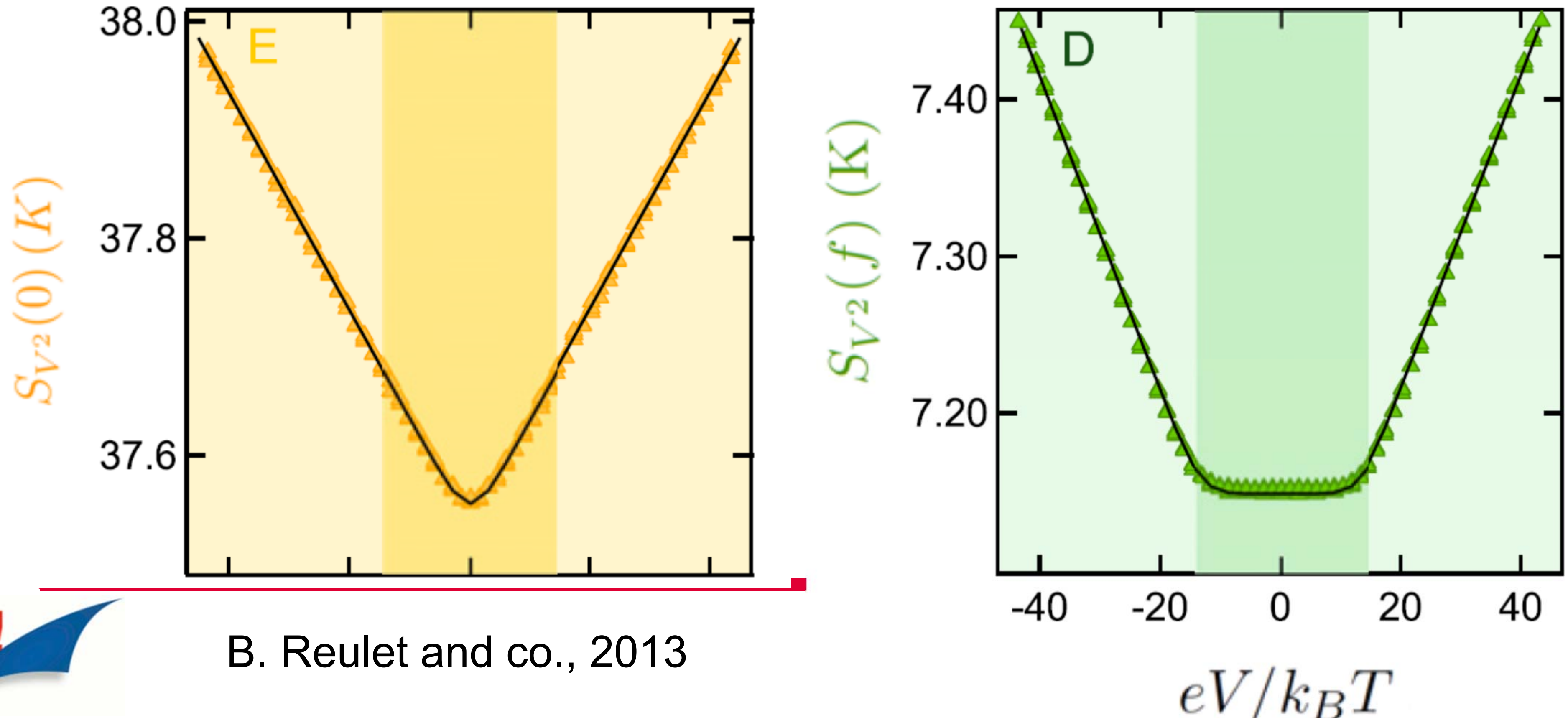
$$G=\frac{e^2}{\pi\hbar}\sum_i T_i$$



$$x=\frac{2k_B T}{eV}$$

Khlus formula (1987)

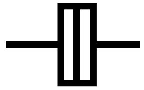
$$S(\omega) = \frac{e^2}{2\pi\hbar}\left\{2\hbar\omega\coth\left(\frac{\hbar\omega}{2k_B T}\right)\sum_n T_n^2 + \left[(\hbar\omega + eV)\coth\left(\frac{\hbar\omega + eV}{2k_B T}\right) + (\hbar\omega - eV)\coth\left(\frac{\hbar\omega - eV}{2k_B T}\right)\right]\sum_n T_n(1 - T_n)\right\}.$$



B. Reulet and co., 2013

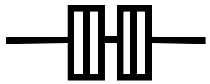
A few examples

- **Tunnel junction (TJ)**



$$F = 1 \quad (T \to 0)$$

- **TJs in series**



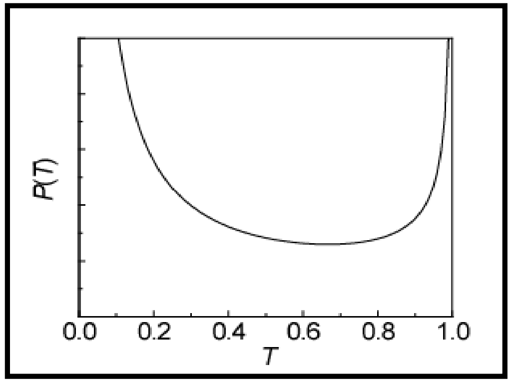
$$\frac{R_1^2 + R_2^2}{(R_1 + R_2)^2}$$

- **1D diffusive conductor**

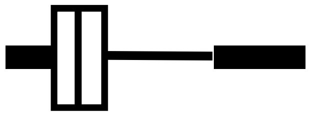
$$F = \frac{1}{3}$$

1D diffusive conductor

$$P(T) = \frac{1}{2L} \frac{1}{T\sqrt{1-T}}$$

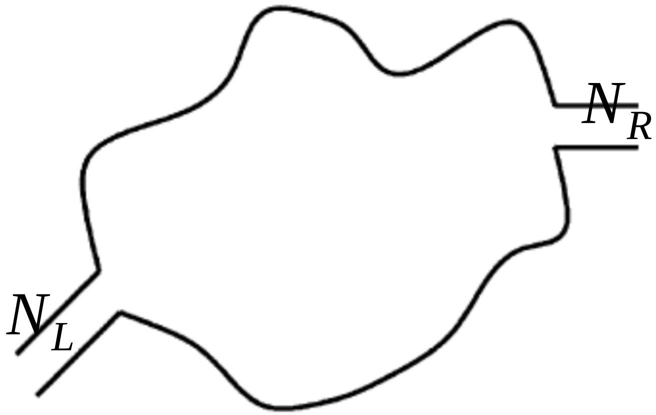


- **TJ plus 1D diffusive conductor**

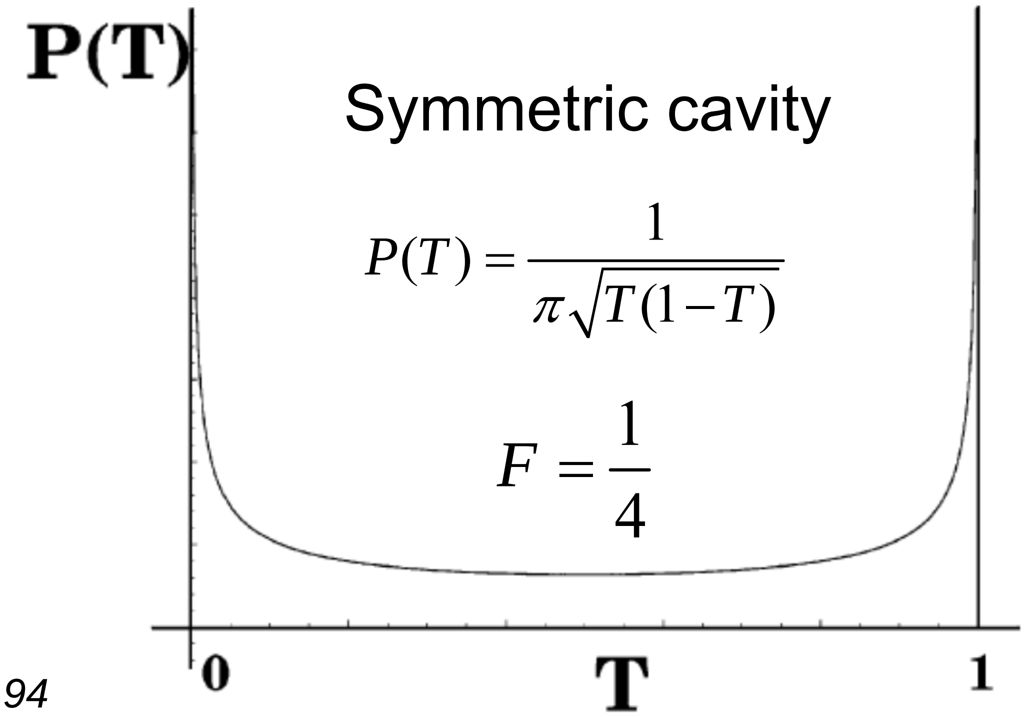


$$F = \frac{1}{3}\left(1 - \frac{3T - 2}{(1 + TL/l_{el})^3}\right)$$

Chaotic cavity



Baranger and Mello 94
Jalabert, Pichard, and Beenakker 94



$$F = \frac{N_L N_R}{(N_L + N_R)^2}$$

→ zero in very asymmetric cavities

$$F = \frac{1}{4}\exp\left(-\frac{t_E}{t_D}\right)$$

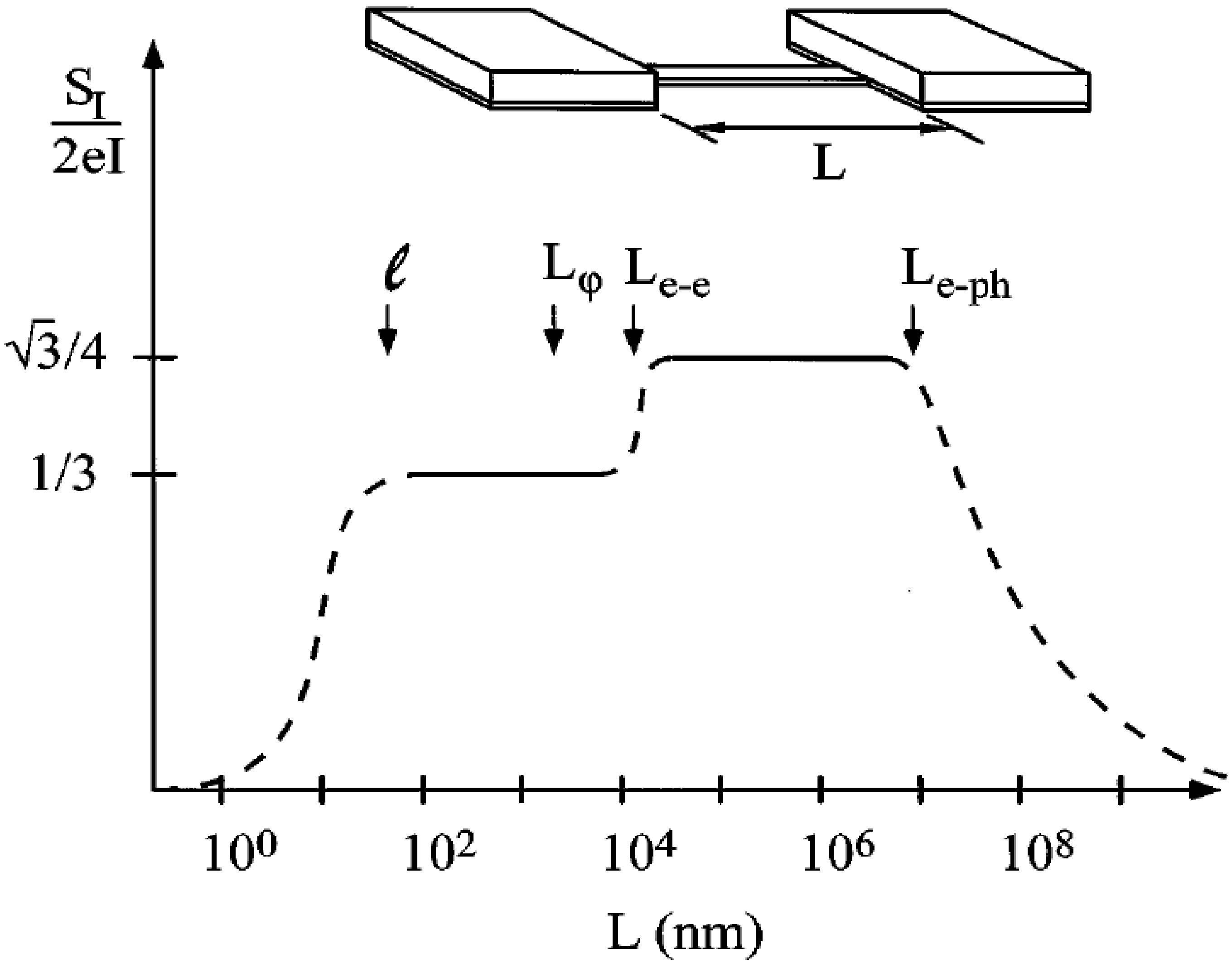
Quantum vs. classical

t_E - Ehrenfest time (el. loses memory about its initial position in phase space

t_D - dwell time

Agam, Aleiner, and Larkin 2000

Diffusive wire noise: length scales



Observation of Hot-Electron Shot Noise in a Metallic Resistor
A. H. Steinbach, J. M. Martinis
M. H. Devoret, PRL **76**, 3806 (1996)

$L_\phi \sim L_{inelastic}$

In graphene:

$L_\phi \sim 1\ \mu\text{m}$ at 1 K

More exotic examples: interactions

Chiral Tomonaga-Luttinger liquid ***F* = *g***

- C.L. Kane and M.P.A. Fischer, *PRL* **72**, 724 (1994)
- Edge channels in Fractional Quantum Hall Effect samples: **fractional charge**

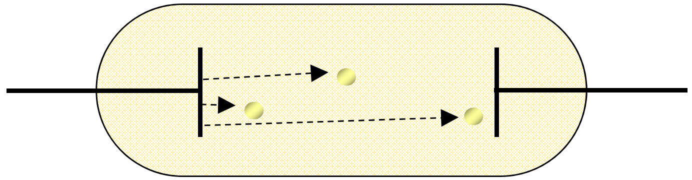
Non-chiral TL-liquid ***F* = 1**

- B. Trauzettel, I. Safi, F. Dolcini, and H. Grabert, *PRL* **92**, 226405 (2004)

Single walled carbon nanotubes **$F = V^{\alpha}$**

- N. Y. Kim, P. Recher, W. Oliver, and Y. Yamamoto, *PRL* **99**, 036802 (2007)
- F. Wu, P. Queipo, … PH, PRL **99**, 156803 (2007) Noise from Fabry-Perot transmission channels

Shot noise with fractional charge



$$\overline{I} = q\,\overline{N}/\tau$$

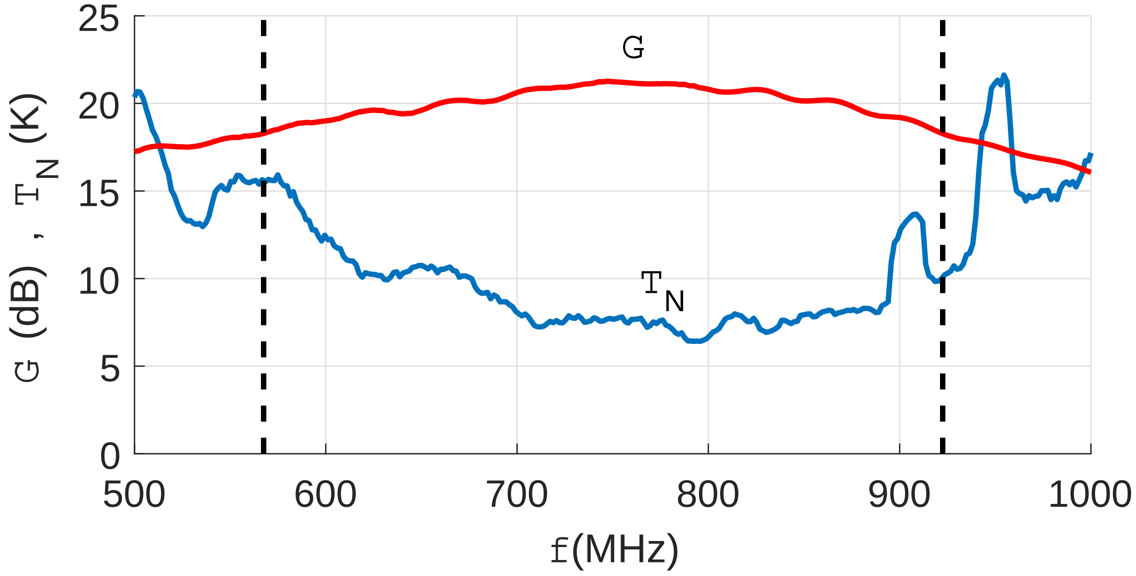
$$\overline{\Delta N^2} = \overline{N}$$

$$\overline{\Delta I^2} = (q/\tau)^2\,\overline{\Delta N^2} = q\,\overline{I}\,(1/\tau)$$

$S_I = 2qI$ where *q* can be a fraction of *e*

$$S_I = 2eI = 2e^2 f$$

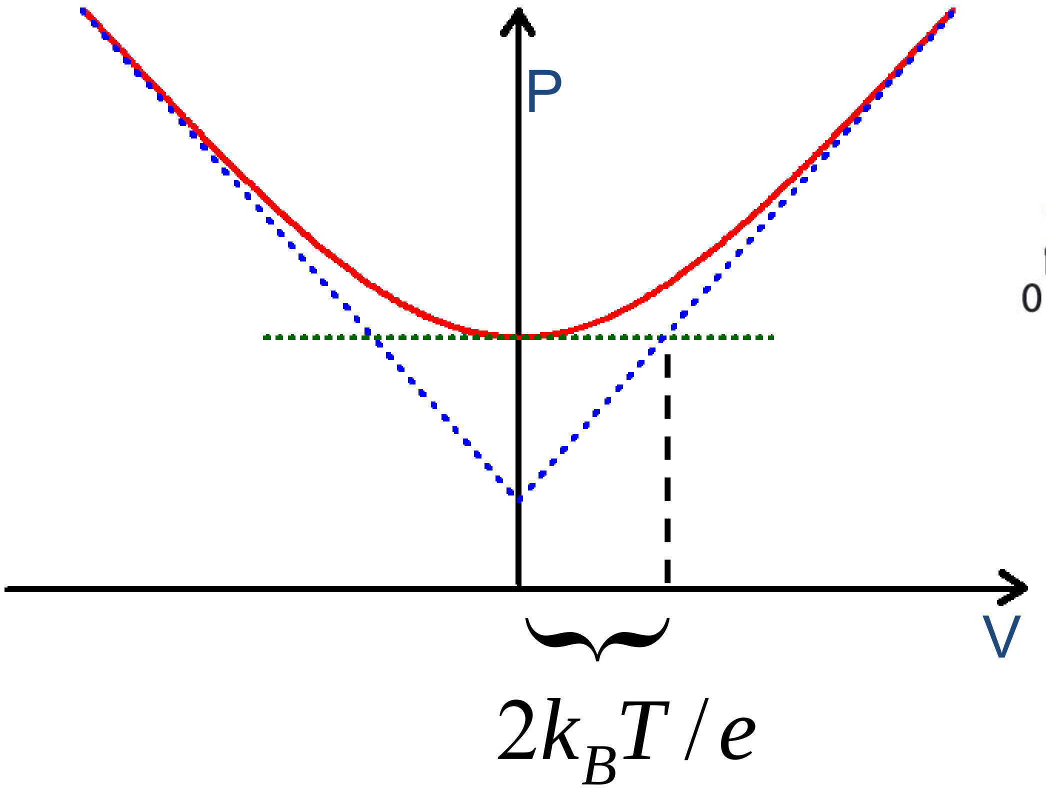
Noise thermometry: why it is hard?



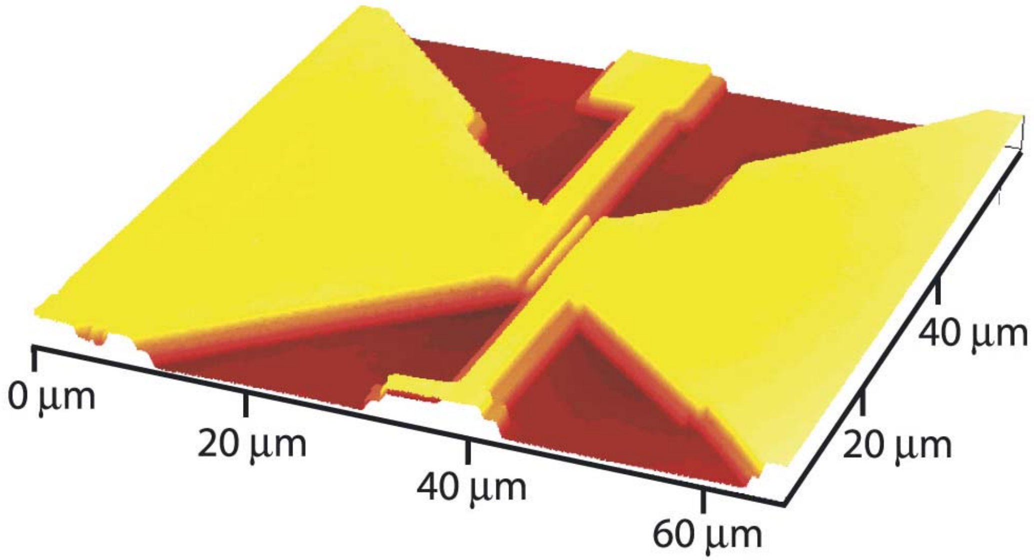
- Must be calibrated accurately to measure temperature accurately

- Cross correlation without any spurious correlations

Shot noise tunnel junction thermometer



- Built-in calibration like in CBT

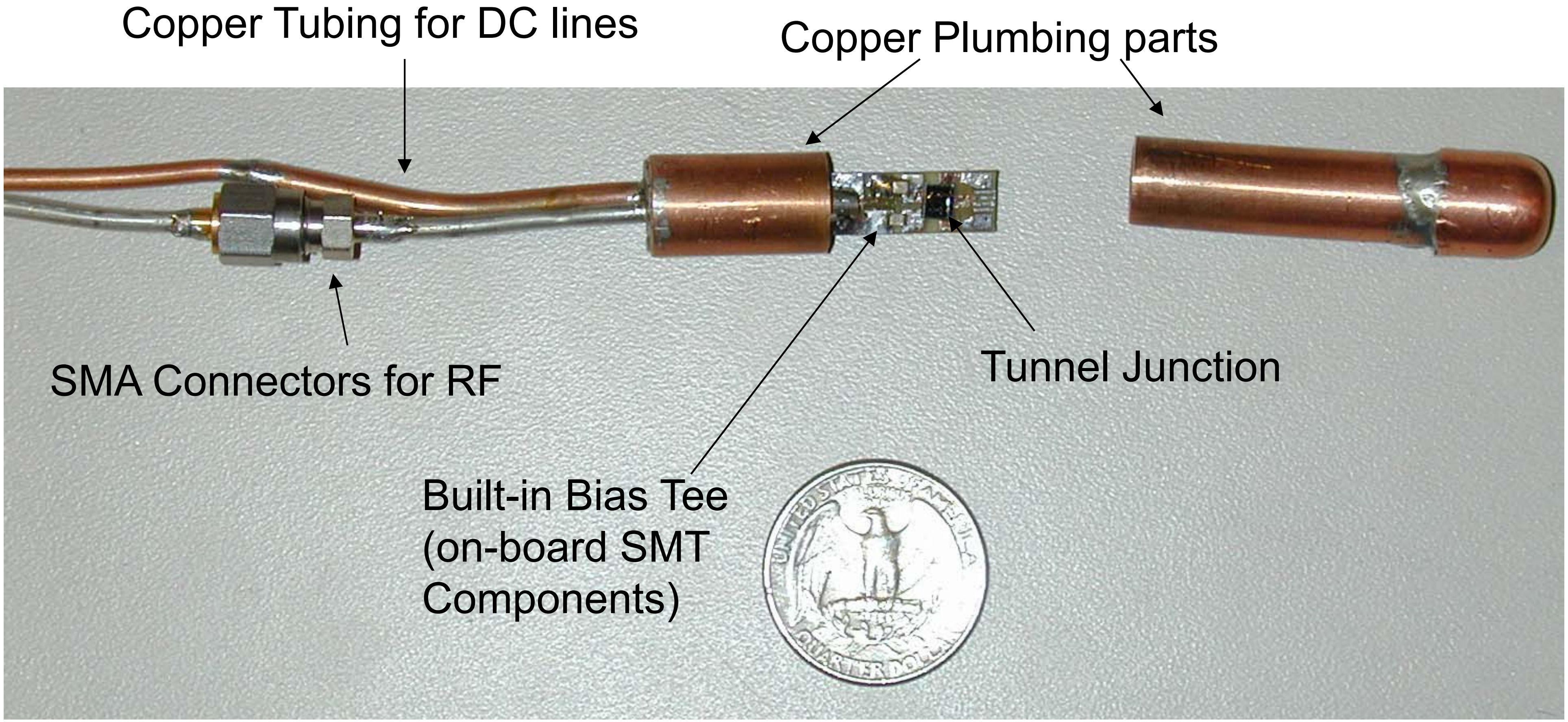


- 50 Ω tunnel junction

- Junction fabrication by shadow evaporation without a suspended bridge
F. Lecocq, et al., Nanotechnology (2011)

L. Spietz, K.W. Lehnert, I. Siddiqi, and R.J. Schoelkopf, Science ***300****, 1929 (2003).*

The Shot Noise Thermometer:



Total cost of package <10$

Lafe Spietz, Yale

Hot electron shot noise thermometry

1) Hot electrons

$$\frac{\pi^2}{6}\frac{dT_e^2}{dx^2} = -\left(\frac{eE}{k_B}\right)^2 + \Gamma\left(T_e^5 - T_0^5\right)$$

$$E = \rho(T_e)I$$

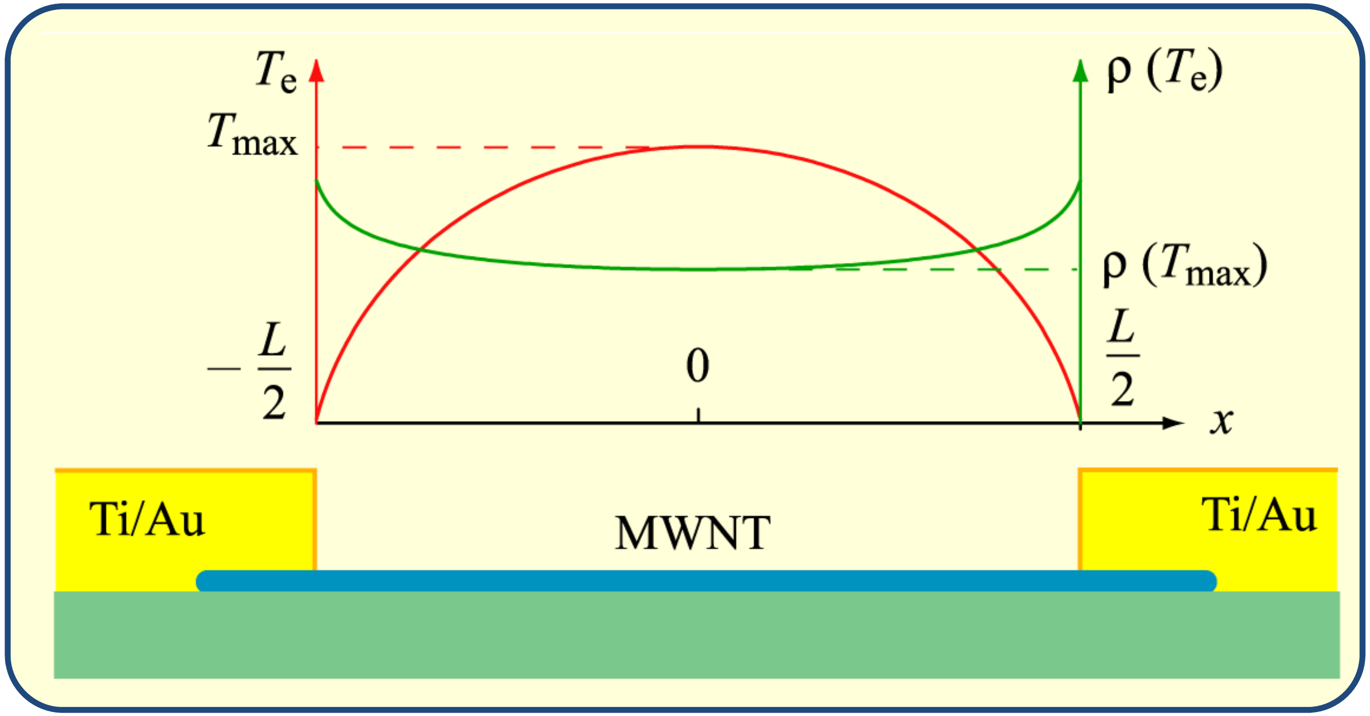
Wiedemann-Franz law

$$\frac{\kappa}{\sigma} = LT \qquad L = \frac{\pi^2}{3}\left(\frac{k_B}{e}\right)^2$$

$$T(x) = \sqrt{T_0^2 + \frac{3}{4\pi^2}\frac{e^2 I^2 R_N^2}{k_B^2}\left(1 - \frac{4x^2}{L^2}\right)}$$

F = constant

F = $3^{1/2}/4$



Hot electron shot noise thermometry

Aalto University

2) Regime where electron temperature is governed by electron-phonon coupling

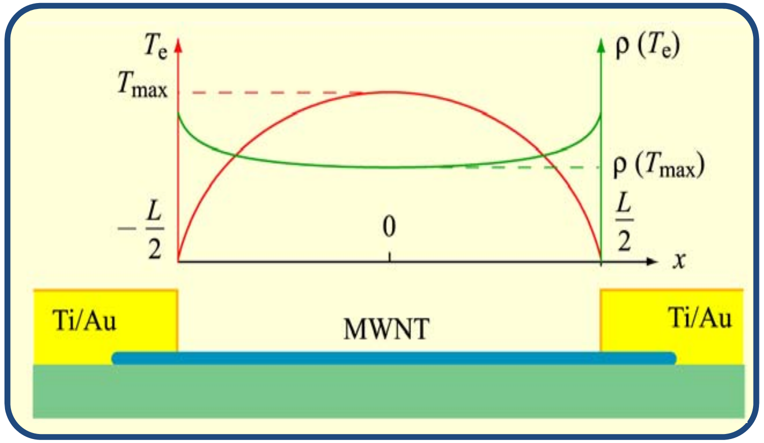
$$\frac{\pi^2}{6}\frac{dT_e^2}{dx^2} = -\left(\frac{eE}{k_B}\right)^2 + \Gamma\left(T_e^5 - T_0^5\right)$$

$$E = \rho(T_e)I$$

- Homogeneous overheating of the main part of the wire except close vicinity of the leads

$$S = \frac{4k_B T_{\max}}{R_N} = \frac{4k_B}{R_N}\left(T_0^5 + \frac{I^2}{\sigma S^2 \Gamma}\right)^{1/5}$$

$F \propto I^{-3/5}$



Electron-phonon coupling in graphene

Joule heating

$$P_{Joule} = UI$$

Shot noise thermometry

Electron diffusion

$$P_{diff} = \Sigma_{diff}(T_e^2 - T_{ph}^2)$$

Electron system, T_e

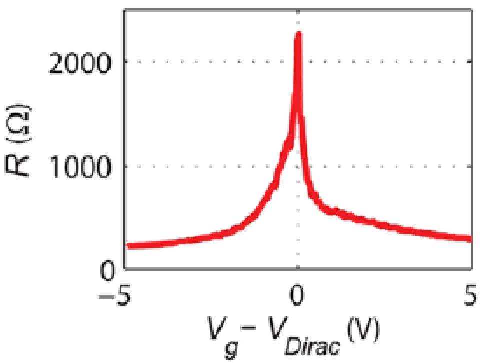
$$P = \Sigma(T_e^{\delta} - T_{ph}^{\delta})$$

Thermalization to the substrate

$$P_{ph,ex} = (T_{ph} - T_{sub})G_{ph-sub} = \frac{T_{ph} - T_{sub}}{R_K}$$

Phonon system, T_{ph}

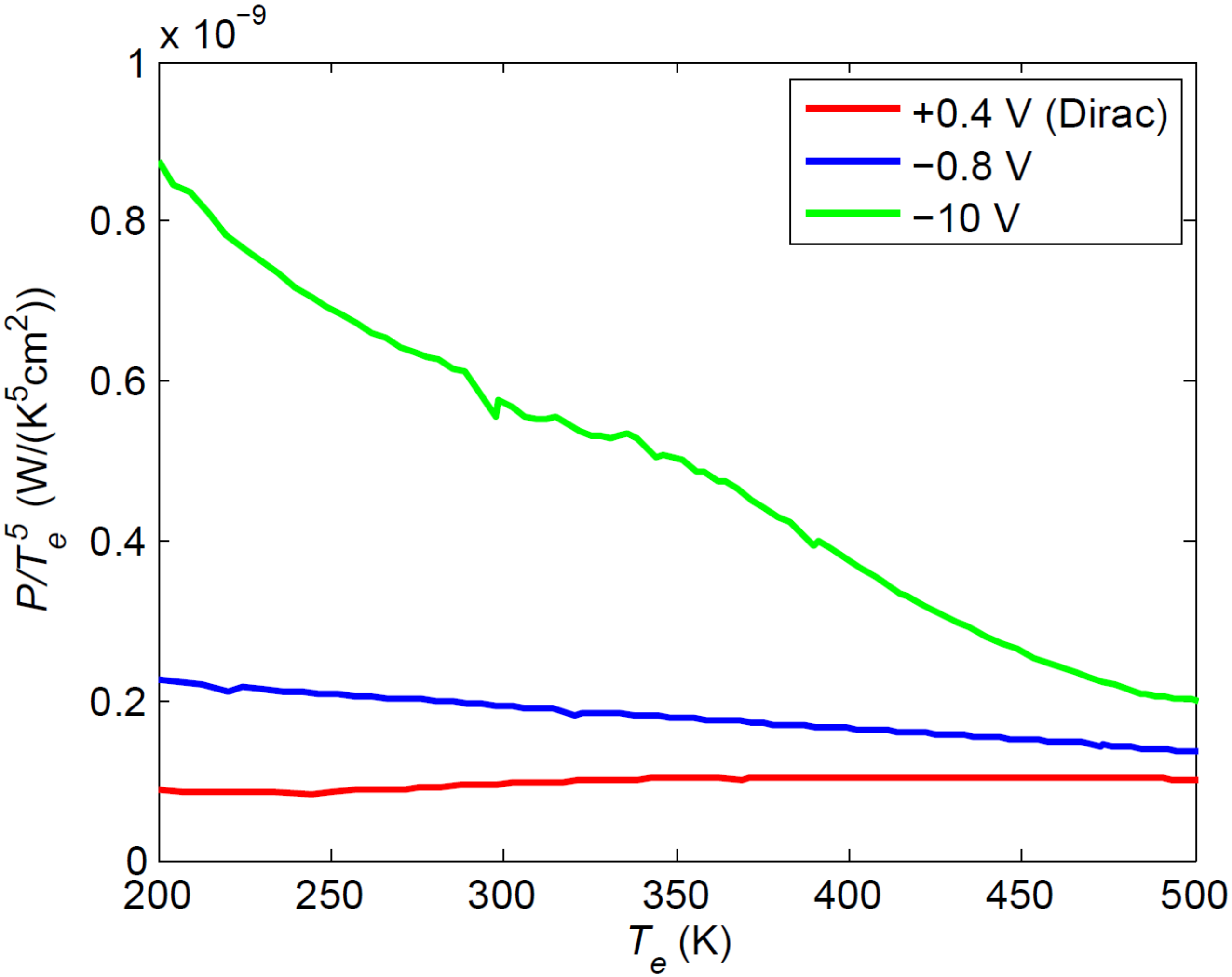
Substrate, T_{sub}

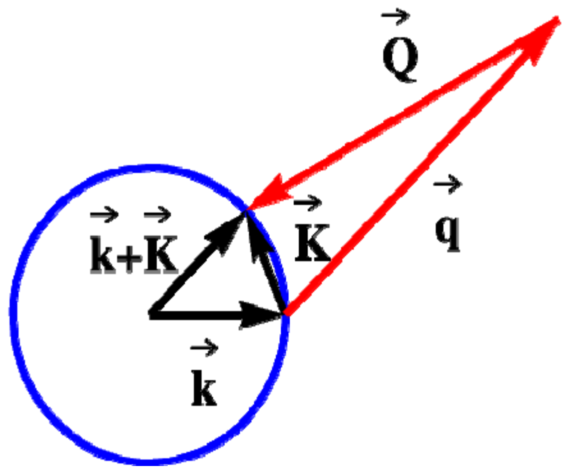


A. Laitinen, …, PH, Nano Lett. ***14****, 3009 (2014)*

Fifth power at Dirac point

- Supercollisions with dynamic ripples





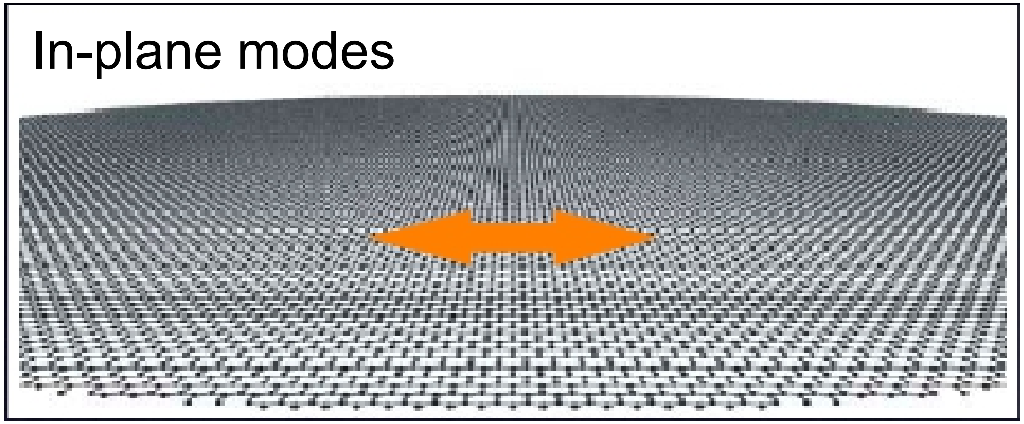
$$P = \Sigma (T_e^{\delta} - T_{ph}^{\delta})$$

$$\delta = 3, \text{ when } \mu >> T$$

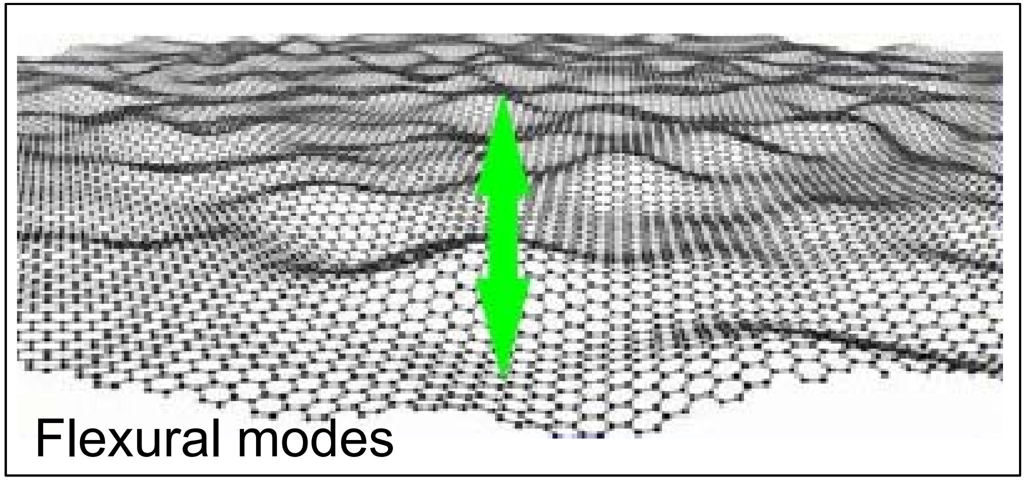
$$\delta = 5, \text{ when } T >> \mu$$

A. Laitinen et al., Nano Lett. ***14****, 3009 (2014)*

Vibration modes in graphene



- Hard to excite
- Good coupling to electrons

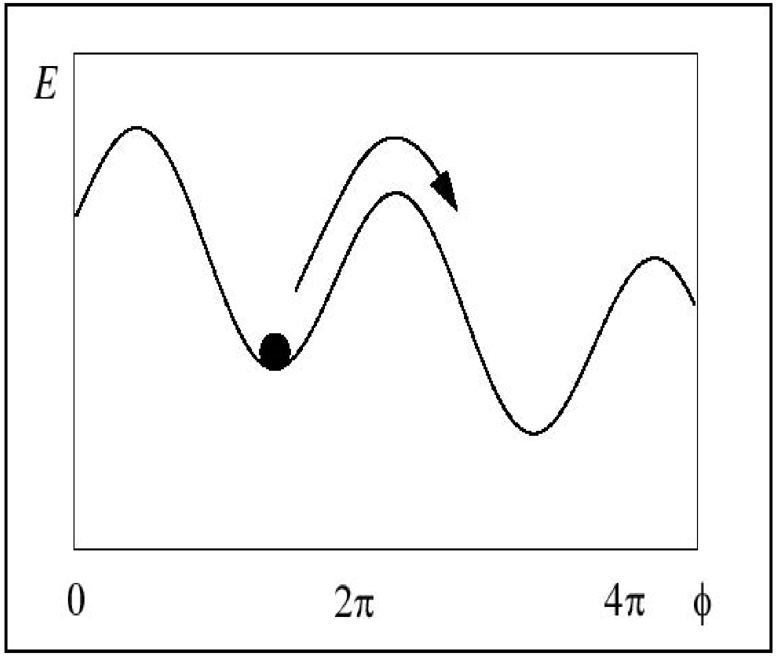


- Easy to excite
- Weak coupling to electrons

Coupling strength to electrons?

A. Fasolino, J. Los, and M. Katsnelson, Nat Mater. **6**, 857, (2007)

RCSJ-MODEL: noise by phase slips



$$\frac{\partial^2\varphi}{\partial t^2}+\frac{1}{\tau}\frac{\partial\varphi}{\partial t}+\omega_p^2\sin\varphi=\omega_p^2\frac{I}{I_c}$$

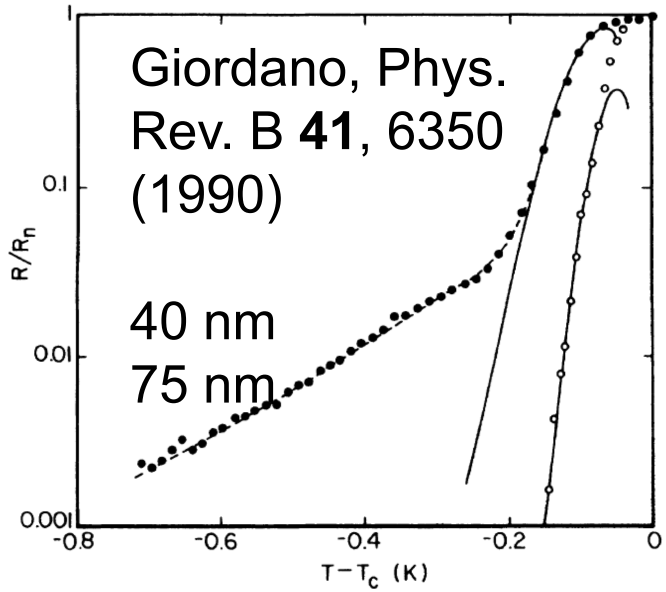
$$\omega_p^2=\frac{2eI_c}{\hbar C}=\left(\frac{2e}{\hbar}\right)^2\frac{E_J}{C}$$

$$Q=\omega_p\tau=\sqrt{\beta_c}\qquad \tau=RC$$

1) Fixed phase

2) Running phase solutions

noise by voltage pulses



Shot noise due to phase slips

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Current shot noise *due to charge pulses:* $Q = \int I dt$

$$S_I = 2eI = 2e^2 f$$ *random pulses – white noise*

Switch to flux quantum: $e \Leftrightarrow \varphi_0$

Voltage shot noise *due to phase slips:* $\Phi = \int V dt$

$$S_V = 2\varphi_0{}^2 f = 2\varphi_0 V$$ *random pulses – white noise*

$$F_V = \frac{S_V}{2\varphi_0 V}$$

$\varphi_0 = \frac{h}{2e}$ ***is the flux quantum***

Thermally activated phase slips

Aalto University

D.S. Golubev and A.D. Zaikin
Phys. Rev. B **78**, 144502 (2008)

$$V = \varphi_0 \Gamma \sinh\left(\frac{\varphi_0 I}{2k_B T}\right) \qquad S_V = 2\varphi_0^2 \Gamma \cosh\left(\frac{\varphi_0 I}{2k_B T}\right)$$

$$F_V = \frac{S_V}{2\varphi_0 V} = \coth\left(\frac{\varphi_0 I}{2k_B T}\right)$$

$$F_V = \frac{S_V - S_V(0)}{2\varphi_0 V} = \coth\left(\frac{\varphi_0 I}{2k_B T}\right) - \frac{2k_B T}{\varphi_0 I}$$

Fano factor due to phase slips in RCSJ

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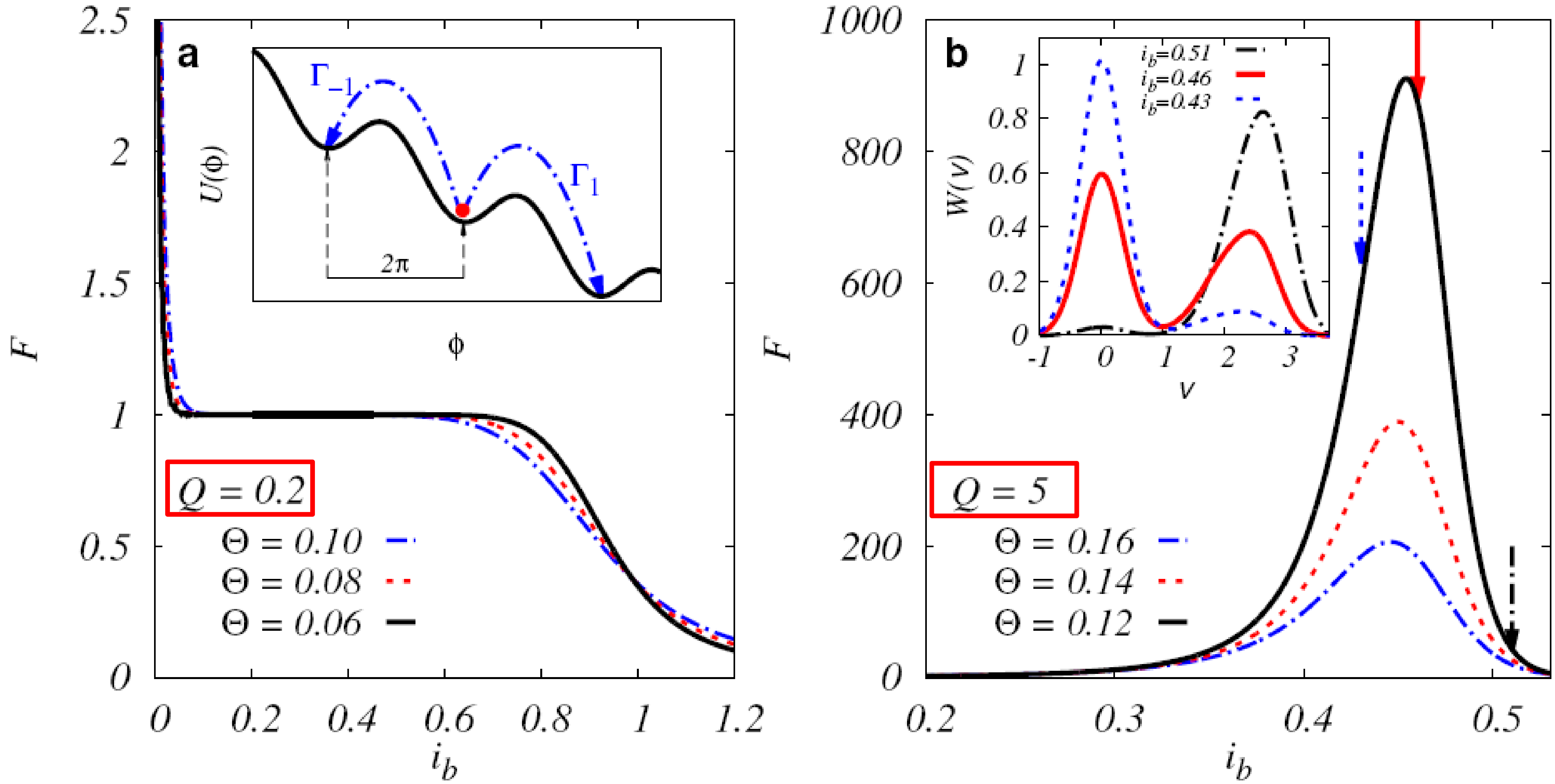
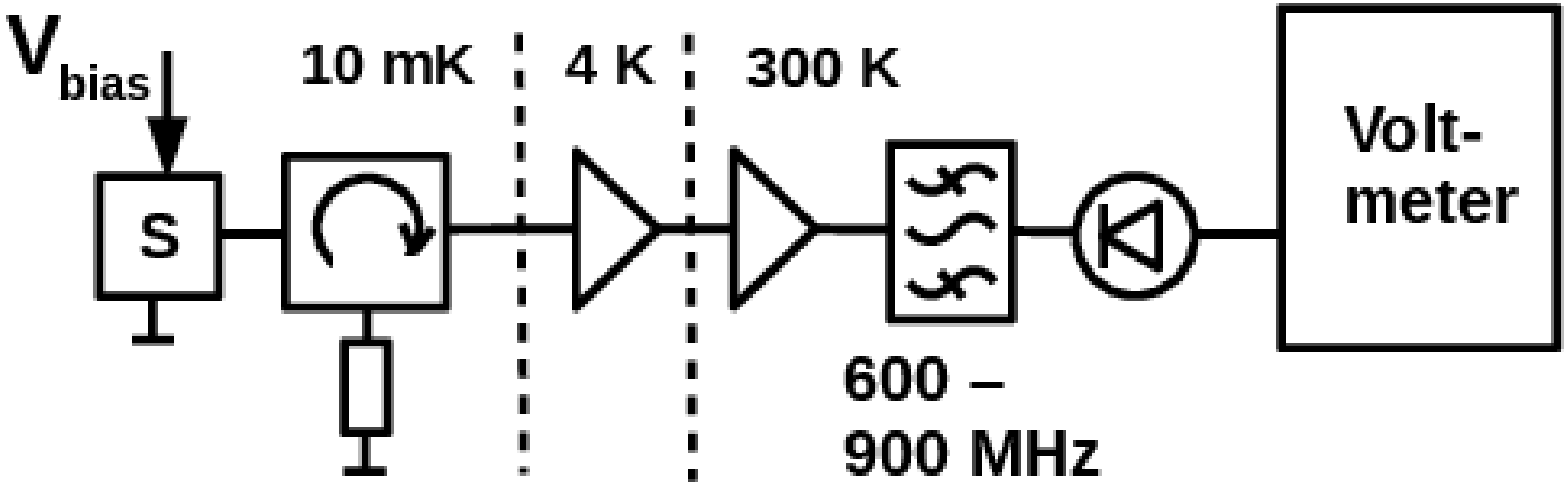


Martin Žonda, Wolfgang Belzig, Tomáš Novotný
Phys. Rev. B **91**, 134305 (2015).

Noise measurements

- Shot noise measurement in practice
- Overview of our present scheme
- Mixers
- Data acquisition
- Calculating correlations
- GPU architecture
  - Benefits of data processing on GPU
  - Structure of data processing program
- Sensitivity of our setup

Basic diode measurement



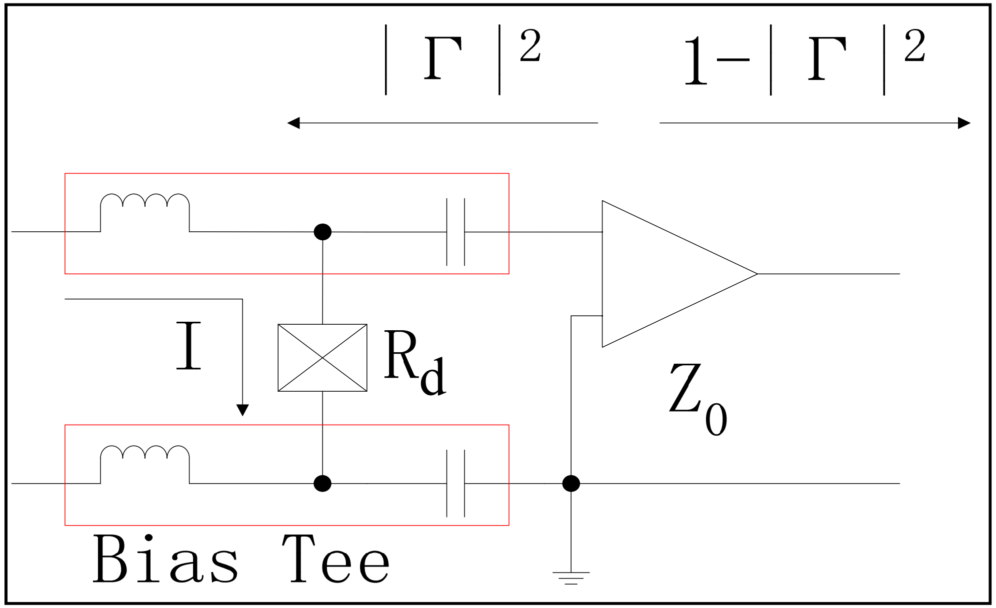
- Basic noise measurement with diode detector
- Amplifier noise dominates (<1 % comes from sample with high impedance)
- Measure DC voltage (about -10 mV)
- Drift and gain fluctuations cause problems (AC helps)
- Keep an eye on non-linearities

Noise power transmission

$$P_{noise} = S_I \cdot R = 4kT(1-F) + F\,2eV \coth\frac{eV}{2kT}$$

Thermal and shot noise mixed

$$\begin{cases} = 4kT & (eV << kT) \\ = 4kT(1-F) + F\,2eV \approx F\,2eI \cdot R_{TOT} & (eV >> kT) \end{cases}$$

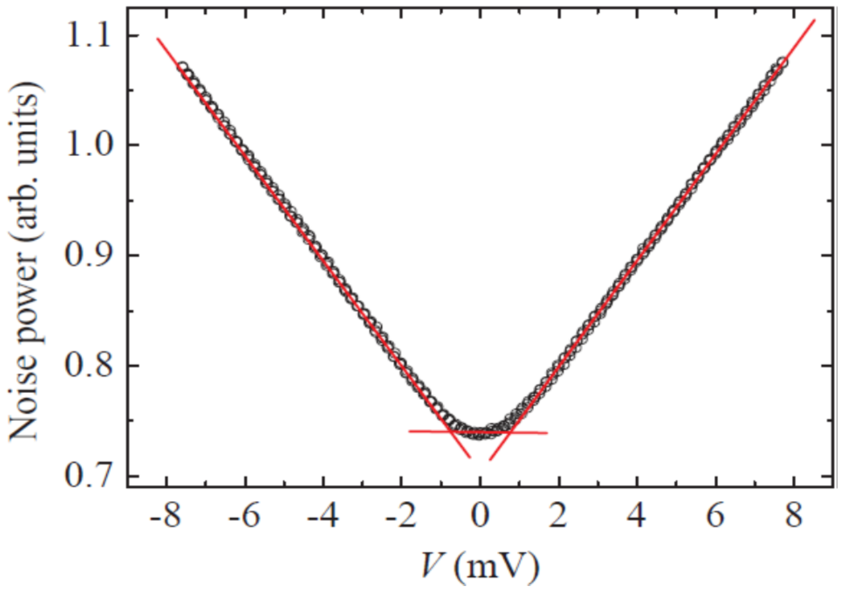


$$\Gamma = \frac{|R_d - Z_0|}{|R_d + Z_0|}$$

$$1 - |\Gamma|^2 \approx \frac{4R_d Z_0}{(R_d + Z_0)^2}$$

$$P_{measured} \approx F\,2eI \cdot 4Z_0 \frac{R_{TOT}}{R_d}$$

Measurement of F (without cross correlation)



$$S(V,T) - S(0,T) = \frac{4k_BT}{R}\left(F\,\frac{eV}{2k_BT}\coth\left(\frac{eV}{2k_BT}\right) - 1\right)$$

$$F_d \equiv \frac{1}{2e}\frac{dS}{dI} = \frac{1}{2}\frac{dS/dV}{dI/dV}$$

1) "differential F"
taken by lock-in technique

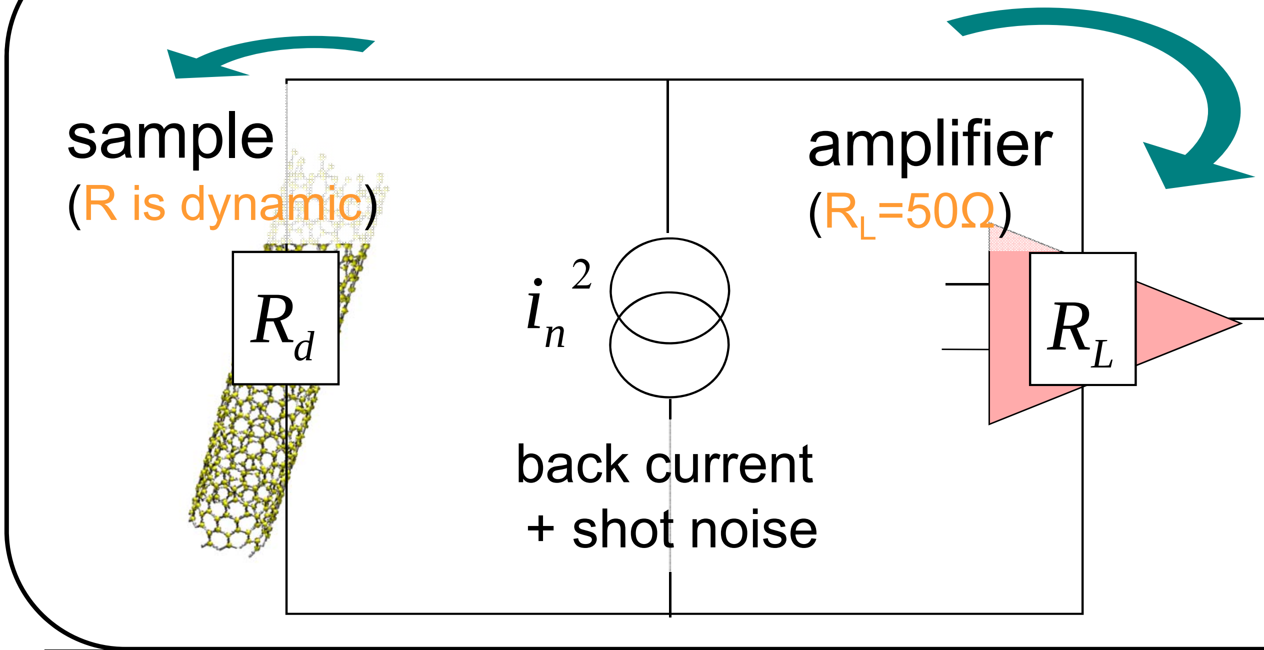
$$S(I) - S(0) = \int_0^I 2eF_d\,dI$$

2) integration
to get the exact curve

$$\widetilde{F} = \frac{1}{I}\int_0^I F_d\,dI$$

Correction of non-linearity in measurement of F_d

- equivalent circuit



power to the amp

$$\left[\frac{1}{R_L} P_{meas} = \left(\frac{R_d}{R_d + R_L}\right)^2 i_n^{\,2}\right]$$

power measured by lock-in

$$\frac{1}{R_L}\frac{dP_{meas}}{dI} = 2eF_d\left(1 - \frac{2R_L}{R_d}\right) - 2i_n^{\,2} R_d R_L \frac{\partial^2 I}{\partial V^2}$$

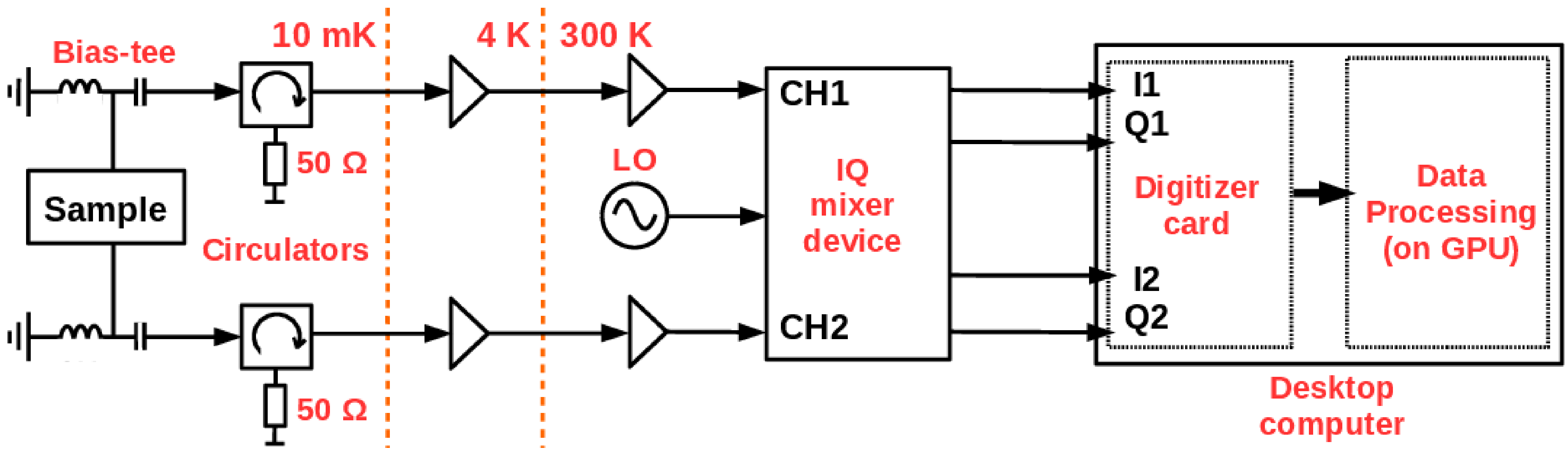
what we want

Correlation measurement

- Two RF channels used
- Noise of two amplifiers is not correlated
  - Correlation measurement removes the uncorrelated part
  - Only sample noise is observed
- Analog setups
  - Wide bandwidth
  - Hard to modify
- Digital setups
  - Narrower bandwidth
  - Calculate other quantities
  - Previous setups limited to few MHz
  - High data rates set requirements for data processing (FPGA etc.)

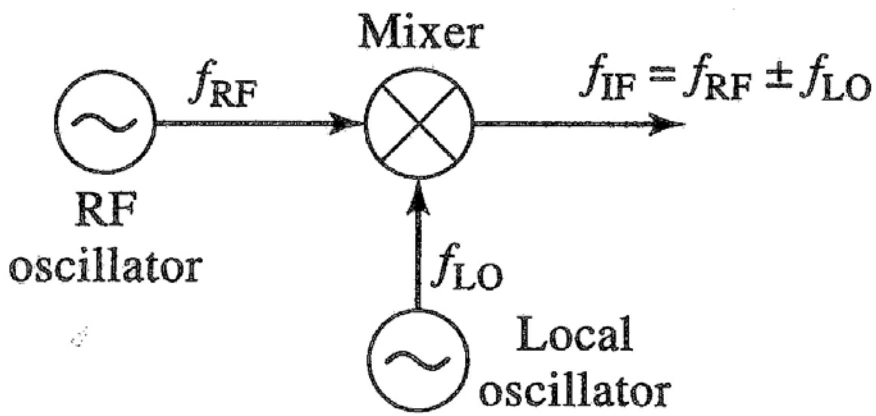
Our solution

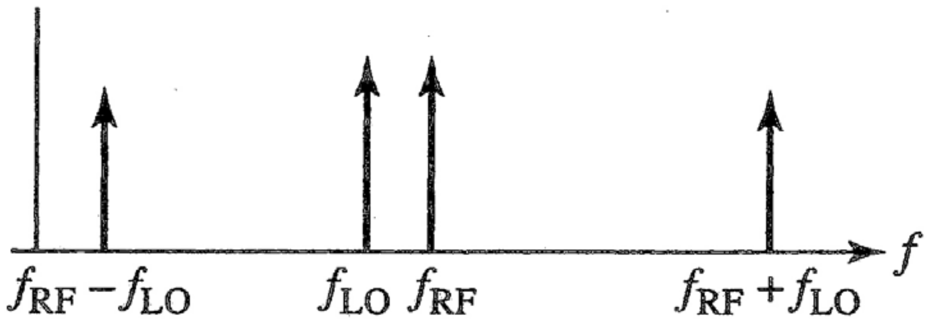
- Digital spectrometer based on 4-channel digitizer card
- Quadrature detection of two channels
- Data processing in real time



RF downmixing

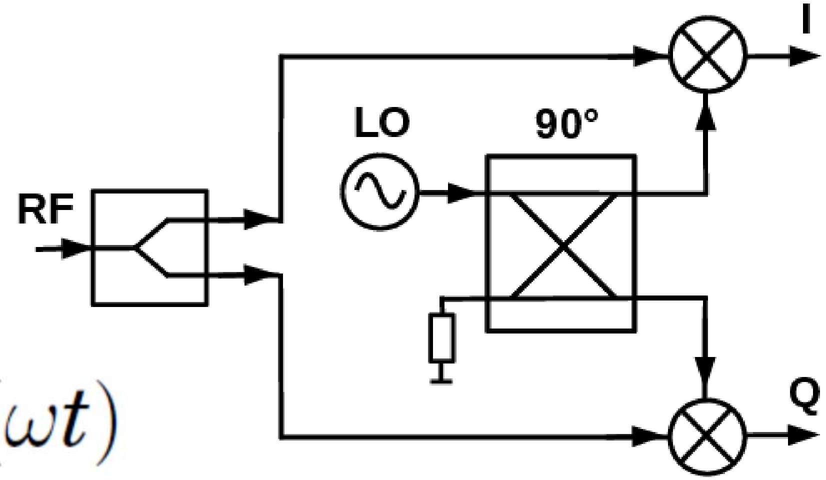
- 800 MHz is too high for direct digitizing
- Shift the frequency band to lower frequencies
- Example: 800 – 850 MHz 0 – 50 MHz
- Mixer multiplies RF signal and local oscillator (LO)





IQ mixer

- Extracts both quadratures of RF signal
- Representation of RF signal:



$$x(t) = \mathrm{A_I} \cos(\omega t) + \mathrm{A_Q} \sin(\omega t)$$

- IF signal after IQ mixer and low-pass filter:

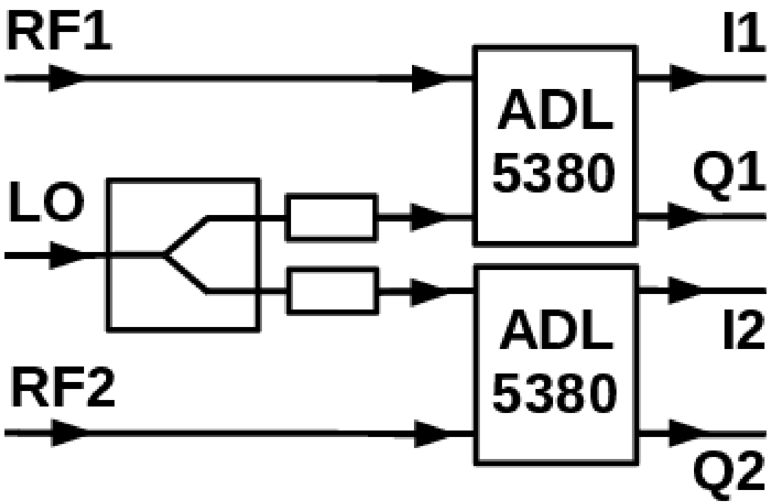
$$\mathrm{I_{LP}}(t) = \frac{\mathrm{A_{LO}}}{2} \left[\mathrm{A_I} \cos\left((\omega - \omega_0)t\right) + \mathrm{A_Q} \sin\left((\omega - \omega_0)t\right)\right]$$

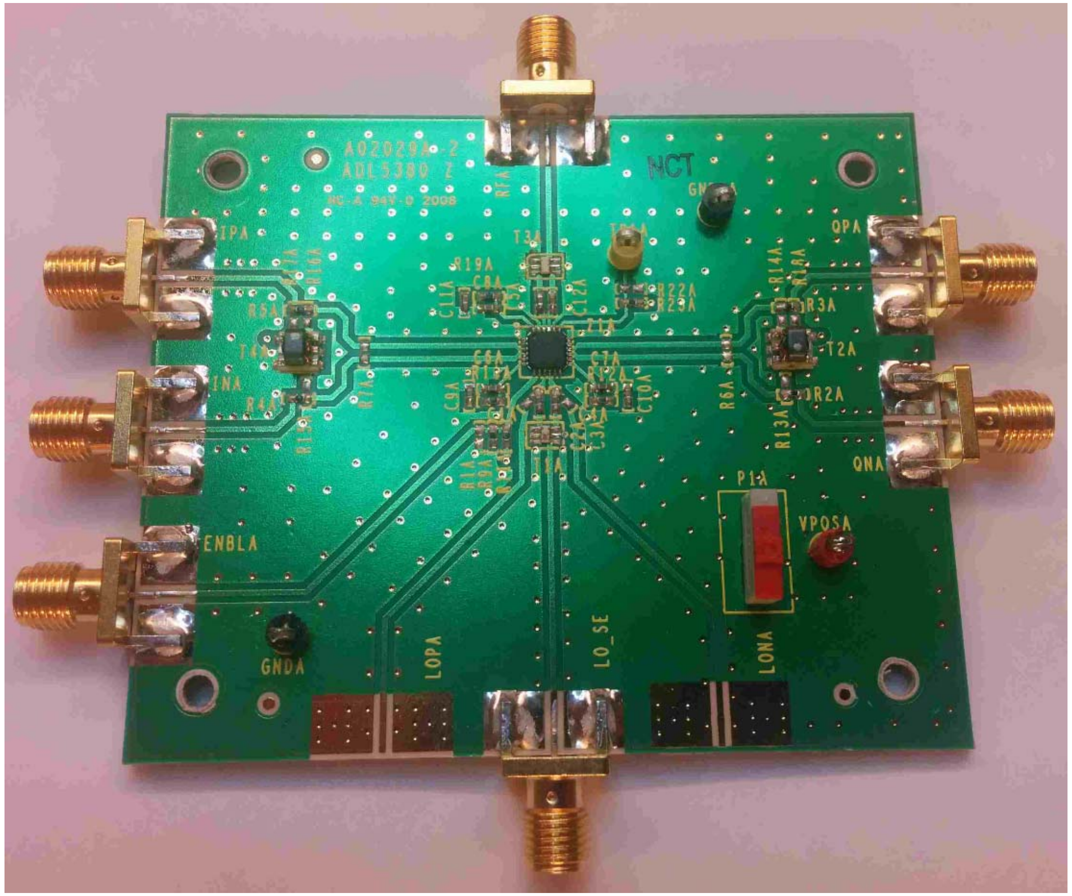
$$\mathrm{Q_{LP}}(t) = \frac{\mathrm{A_{LO}}}{2} \left[-\mathrm{A_I} \sin\left((\omega - \omega_0)t\right) + \mathrm{A_Q} \cos\left((\omega - \omega_0)t\right)\right]$$

Orthogonality is maintained (90 phase difference)

IQ mixer device

- Two-channel IQ mixer device
- Two Analog Devices ADL5380 quadrature demodulator boards
- Split LO signal

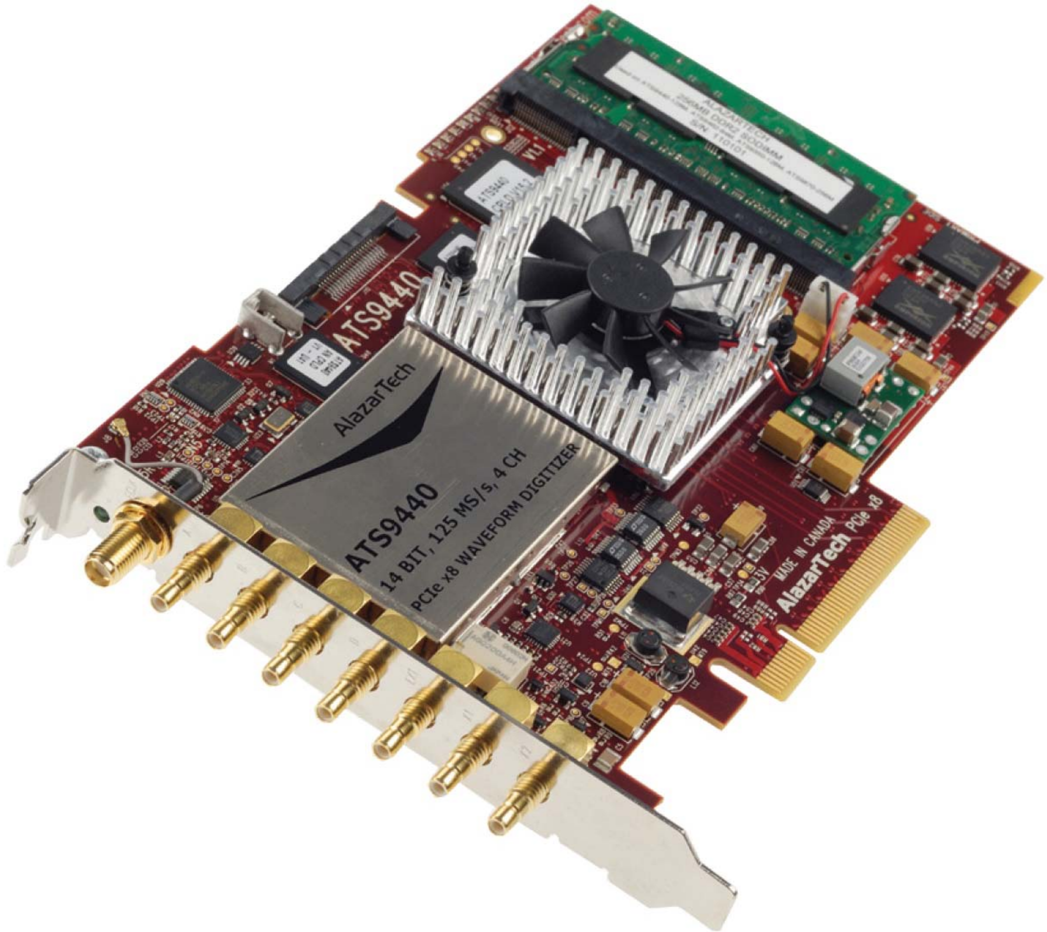




- 2 RF inputs, 4 IF outputs

Data acquisition

- AlazarTech ATS9440 capture card with PCI Express interface
- Four channels, 125 MS/s, 14-bit resolution
- Outputs data to computer RAM
- Data output rate 1 GB/s (2 bytes 4 ch. 125 MS/s)



Data acquisition

- **Cross-correlation in time domain:**

$$z_{cor}[t] = \sum_{\tau=-\infty}^{\infty} \overline{x[\tau]}\, y[t+\tau]$$

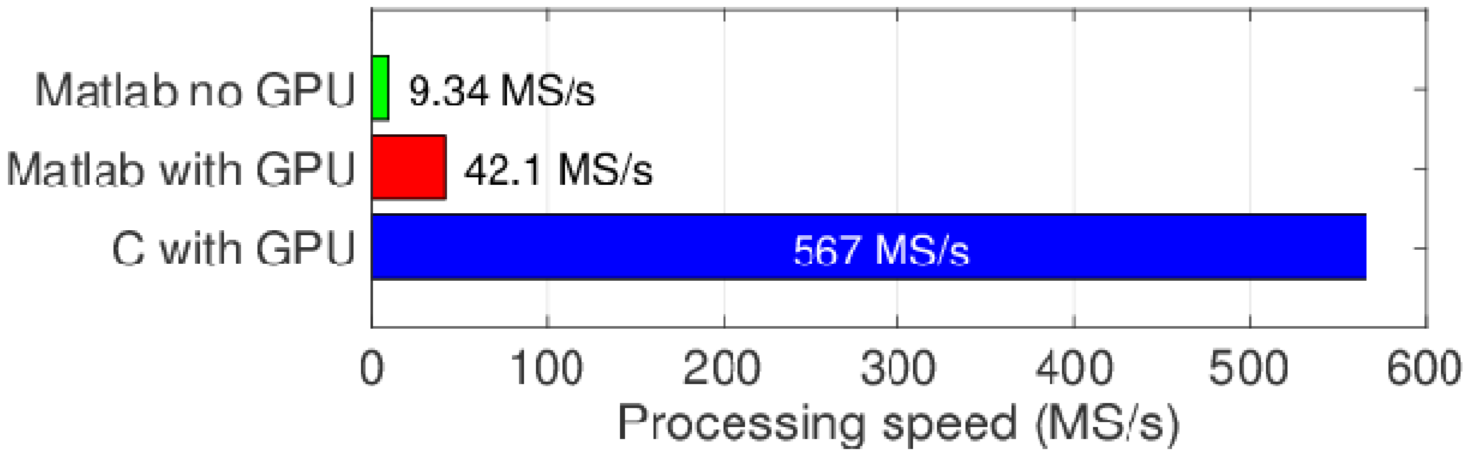
- **Using Fourier transform and average:**

$$z_{cor}[t] = \frac{1}{M}\sum_{k=1}^{M} \mathcal{F}^{-1}\left[\overline{X_k[f]} \cdot Y_k[f]\right] \equiv \frac{1}{M}\mathcal{F}^{-1}\left[\sum_{k=1}^{M} \overline{X_k[f]} \cdot Y_k[f]\right]$$

- **Linearity of Fourier transform (and FFT)**
- **Autocorrelation: vector dot product**

CUDA C instead of MATLAB

- Performance of Matlab was insufficient
- 60-fold speedup to Matlab without GPU, 13-fold speedup to Matlab with GPU
- Achieving 125 MS/s with Matlab would require a very powerful computer
- Speedup justifies the programming effort with C
- Very simple compared with FPGA programming



Theoretical sensitivity

- **Analog detection:**

$$\Delta T = \frac{T_s}{\sqrt{B\tau}}$$

Dicke radiometer formula

- **Digital with both quadratures:**

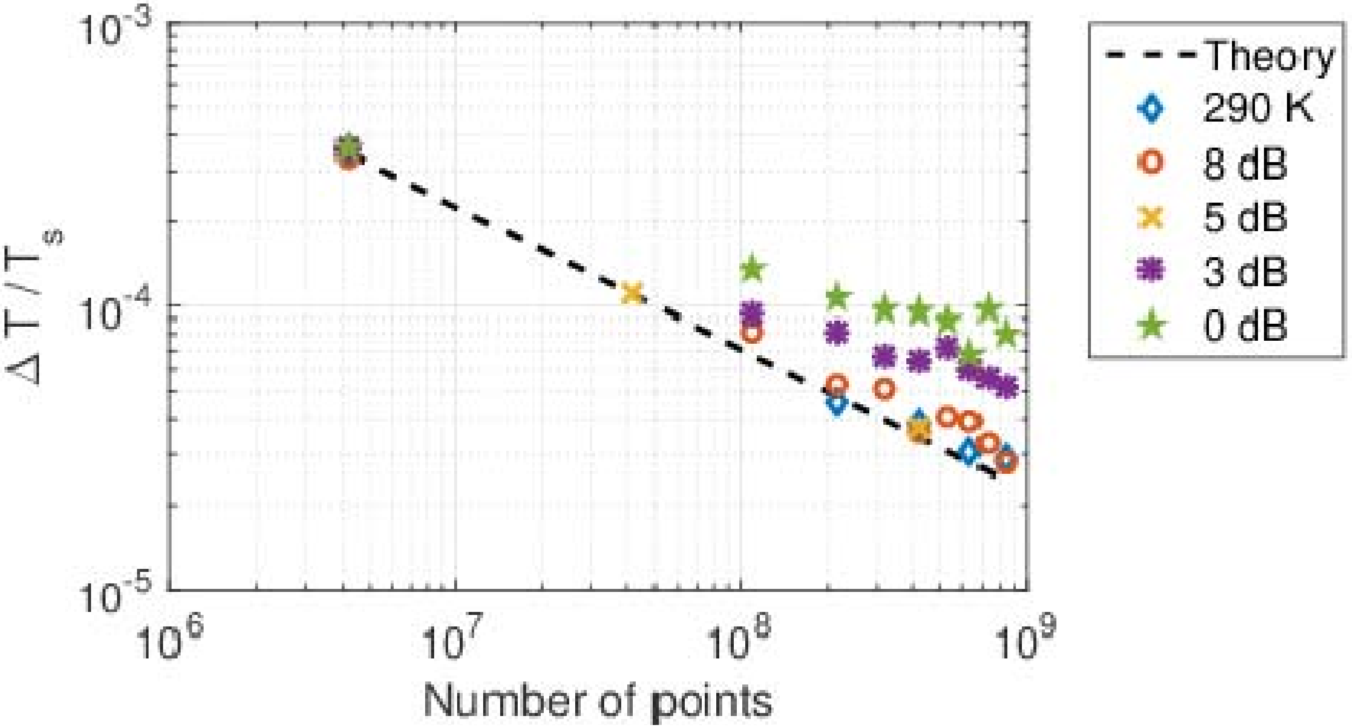
$$\Delta T = \frac{T_s}{\sqrt{2N}}$$

- **Relative sensitivity:**

$$\frac{\Delta T}{T_s} = \frac{1}{\sqrt{2N}}$$

Measured sensitivity

➢ **Relative sensitivity: 2.8e-5 with 8.43 s averaging time**



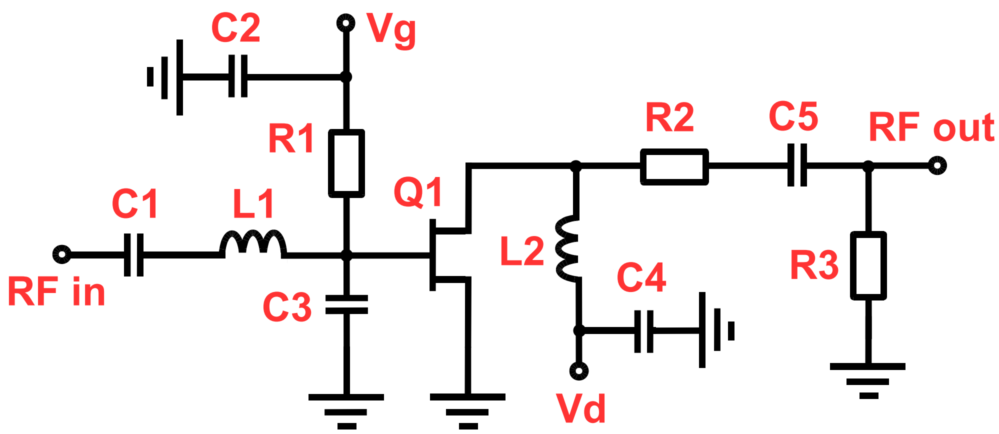
- Corresponds to $810\,\mu\mathrm{K}/\sqrt{\mathrm{Hz}}$

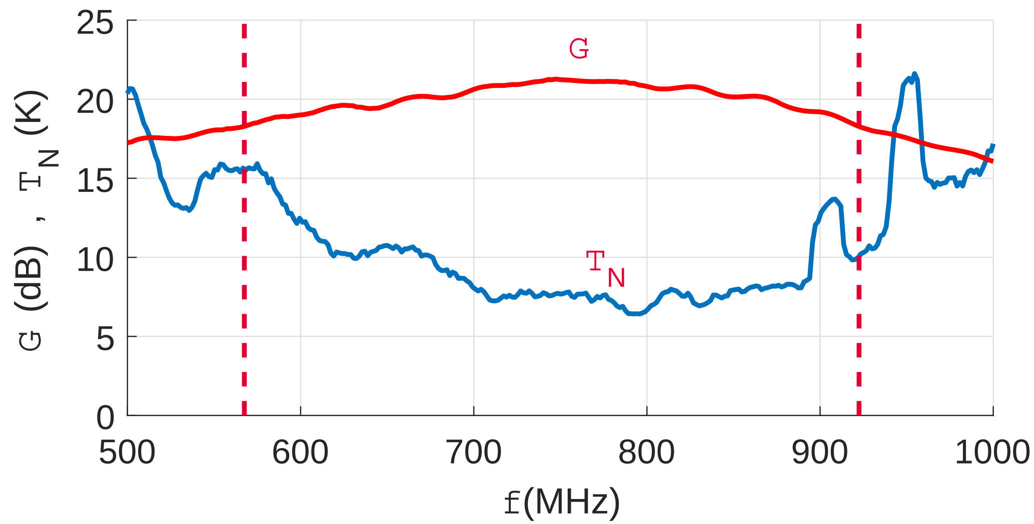
- Best reported sensitivity $710\,\mu\mathrm{K}/\sqrt{\mathrm{Hz}}$ (analog setup)

F.D. Parmentier, et al., Rev. Sci. Instrum. ***82****, 013904 (2011)*

Cryogenic low-noise amplifier

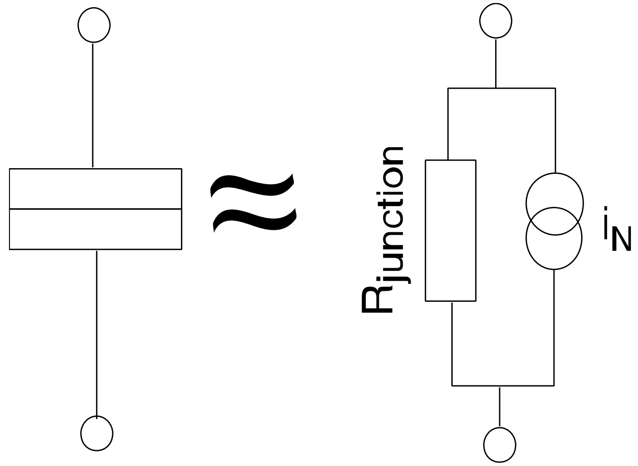
- **Covers 600-900 MHz band**
- **Single HEMT (Avago ATF-36077)**
- **Lumped element design:**
  - ✓ **Simple L-section impedance matching (L1,C3)**
  - ✓ **Stabilizing resistors (R2,R3)**
  - ✓ **DC-blocking capacitors (C1,C5)**
  - ✓ **Bias voltage feed (R1,C2,L2,C4)**
  - ✓ **Gain 21 dB**
  - ✓ **Noise temperature 7 K**
  - ✓ **Power consumption 7.5 mW**



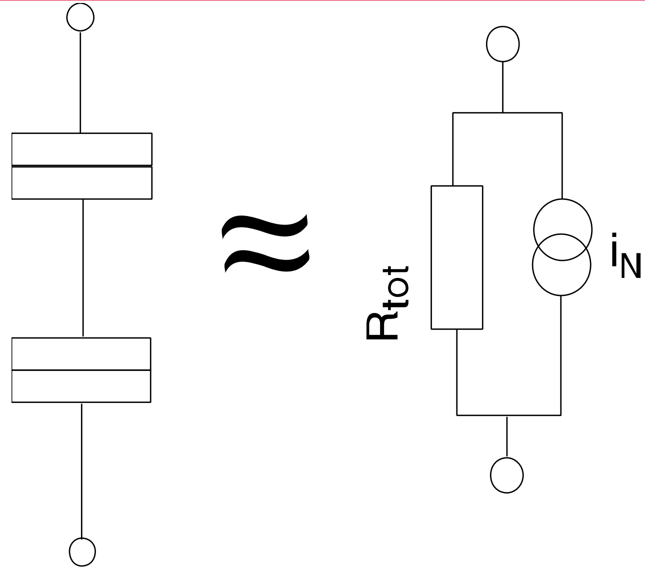


T. Elo, P. Lähteenmäki, Z. Tan, D. Cox, and P. Hakonen, RSI to be published.

Calibration of noise spectrometers



$$F = 1$$



$$F = \frac{1}{2}$$

$$S_I = \frac{2eV}{R_{junction}} \coth \frac{eV}{2k_B T}$$

$$S_I = \frac{eV}{R_{tot}} \coth \frac{eV}{4k_B T}$$

available power: $S_I R / 4$
(when source and
amplifier matched)

Also hot electron regime

$$S_I = \frac{\sqrt{3}}{4} 2eI \qquad F = \frac{\sqrt{3}}{4}$$

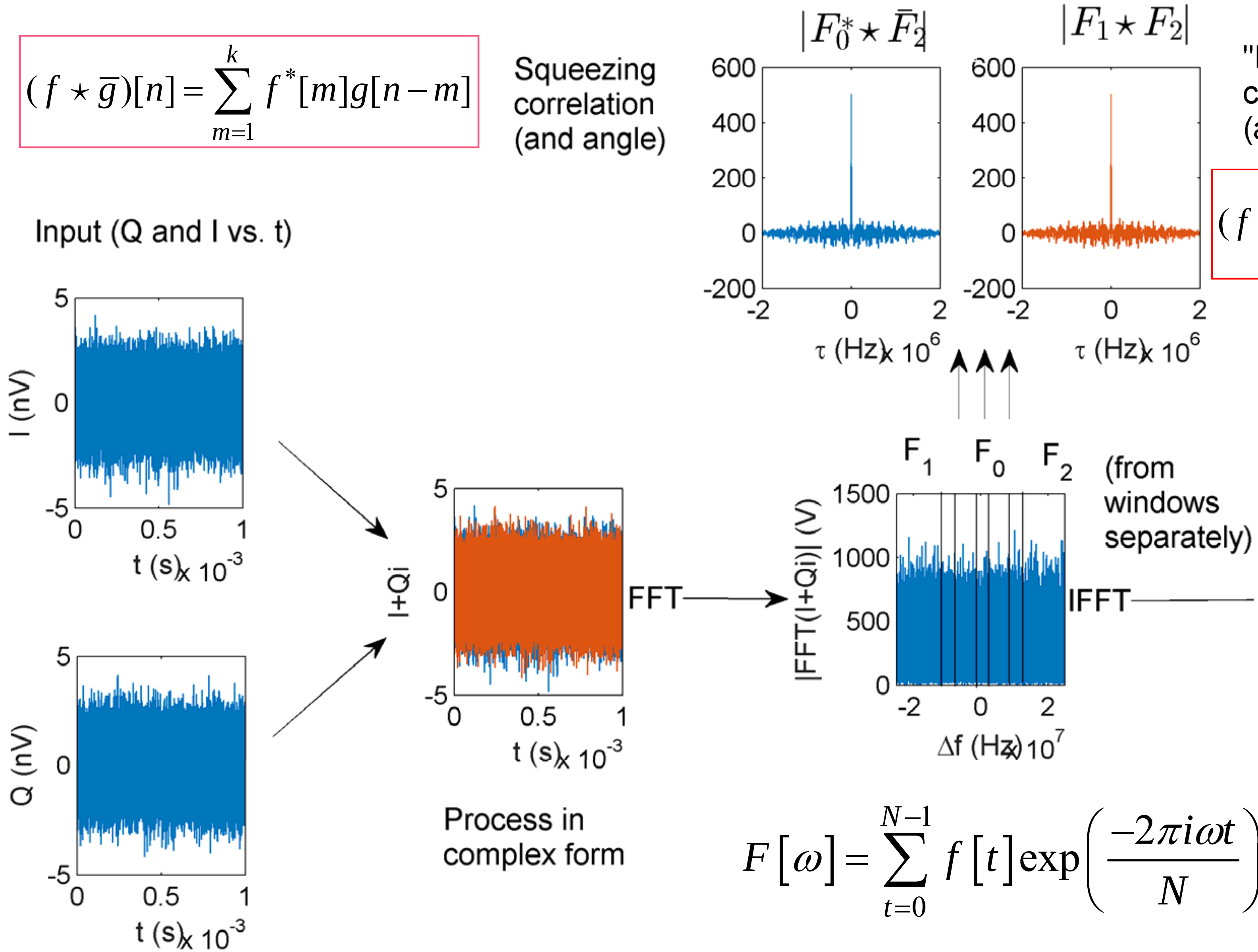
Multifrequency correlations I

$$(f \star \bar{g})[n] = \sum_{m=1}^{k} f^*[m]g[n-m]$$

Squeezing correlation (and angle)

"Beamsplitter correlation" (and angle)

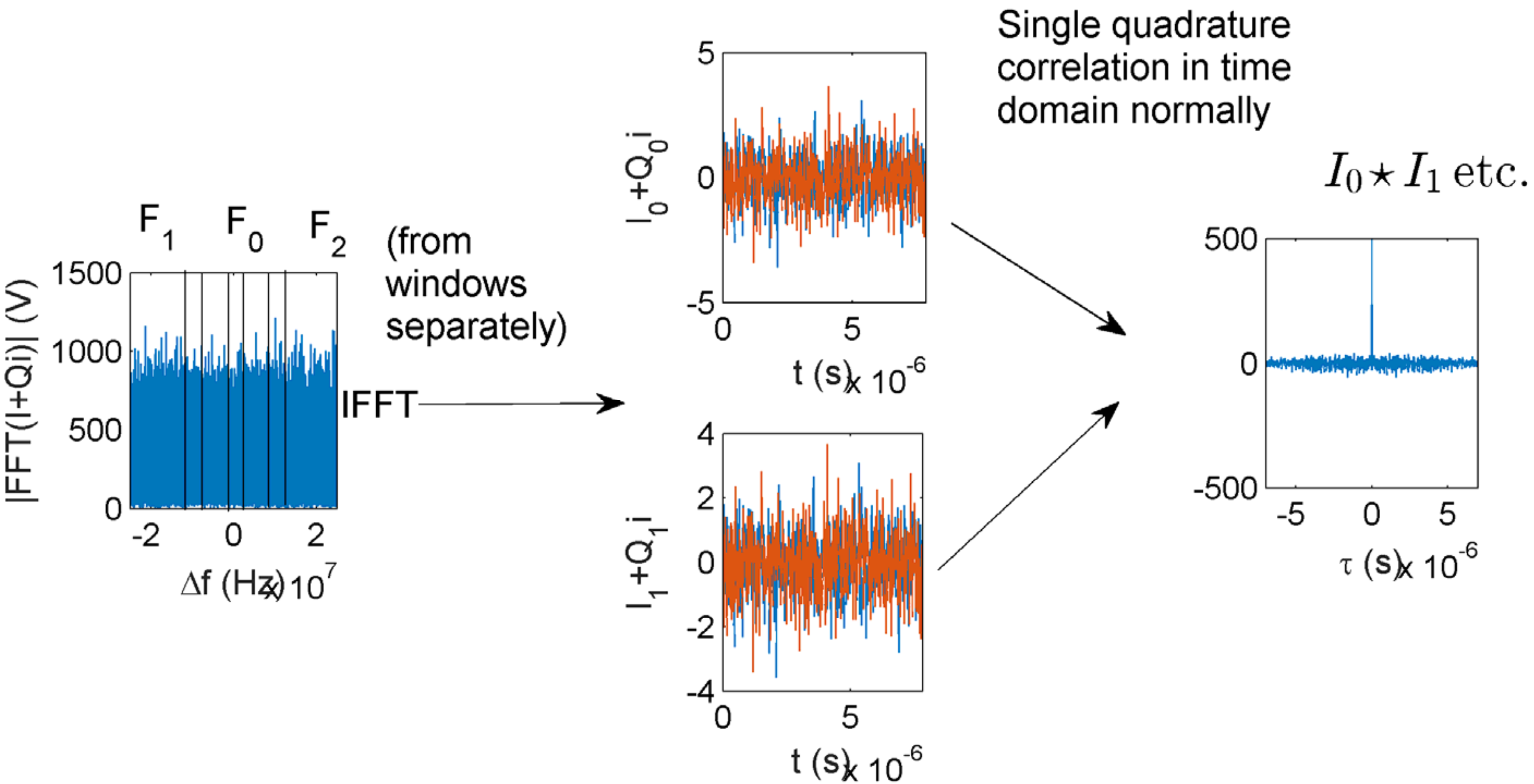
$$(f \star g)[n] = \sum_{m=1}^{k} f^*[m]g[m+n]$$



$$F[\omega] = \sum_{t=0}^{N-1} f[t]\exp\left(\frac{-2\pi i\omega t}{N}\right),$$

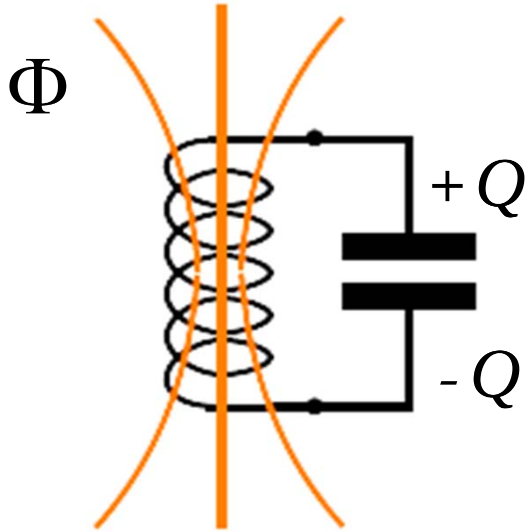
P. Lähteenmäki, G. S. Paraoanu, J. Hassel, P. Hakonen, Nature Comm. ***7****, 12548 (2016)*

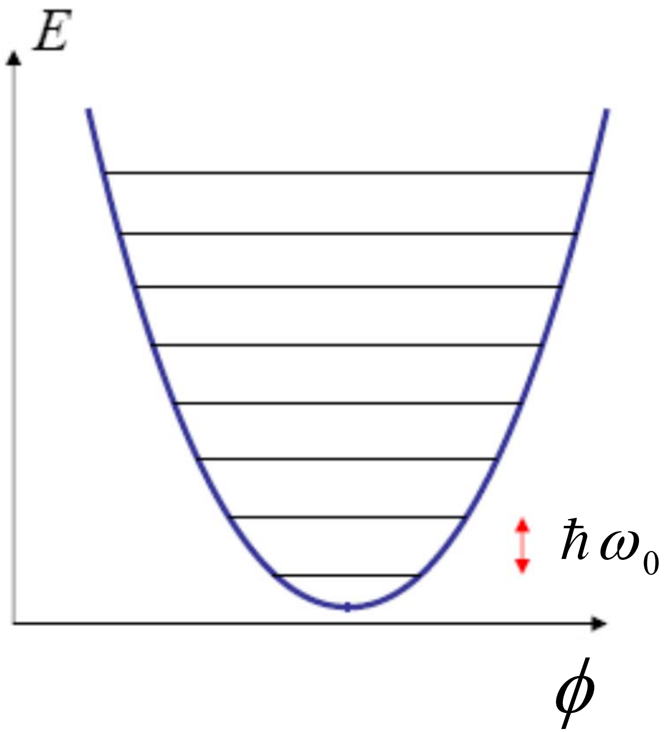
Multifrequency correlations II



P. Lähteenmäki, G. S. Paraoanu, J. Hassel, P. Hakonen, Nature Comm. ***7****, 12548 (2016)*

LC circuit as quantum harmonic oscillator





$$H = \hbar\omega_0\left(a^\dagger a + 1/2\right)$$

$$a = \frac{\widehat{\Phi}}{\Phi_r} + i\frac{\widehat{Q}}{Q_r} = \hat{\phi} + i\hat{q}$$

annihilation and creation operators

$$a^\dagger = \frac{\widehat{\Phi}}{\Phi_r} - i\frac{\widehat{Q}}{Q_r} = \hat{\phi} - i\hat{q}$$

$$\Phi_r = \sqrt{2\hbar\omega_0 L}; \quad Q_r = \sqrt{2\hbar\omega_0 C}$$

$$\left\langle \delta\Phi^2 \right\rangle = \frac{\hbar Z_0}{2}; \quad \left\langle \delta Q^2 \right\rangle = \frac{\hbar}{2Z_0}$$

SQL

Standard quantum limit

$$\left\langle \delta\Phi^2 \right\rangle = \frac{\hbar Z_0}{2}; \ \left\langle \delta Q^2 \right\rangle = \frac{\hbar}{2Z_0}$$

$$\omega_0 = 1/\sqrt{LC}$$

$$S_\Phi = \frac{\hbar Z_0}{2\omega}; \ S_Q = \frac{\hbar}{2Z_0\omega}$$

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$S_V = \frac{\hbar Z_0}{2}\omega; \ S_I = \frac{\hbar}{2Z_0}\omega$$

$$S_V S_I = \left(\frac{\hbar\omega}{2}\right)^2$$

$Z_0 = 50\ \Omega$ at 1 GHz

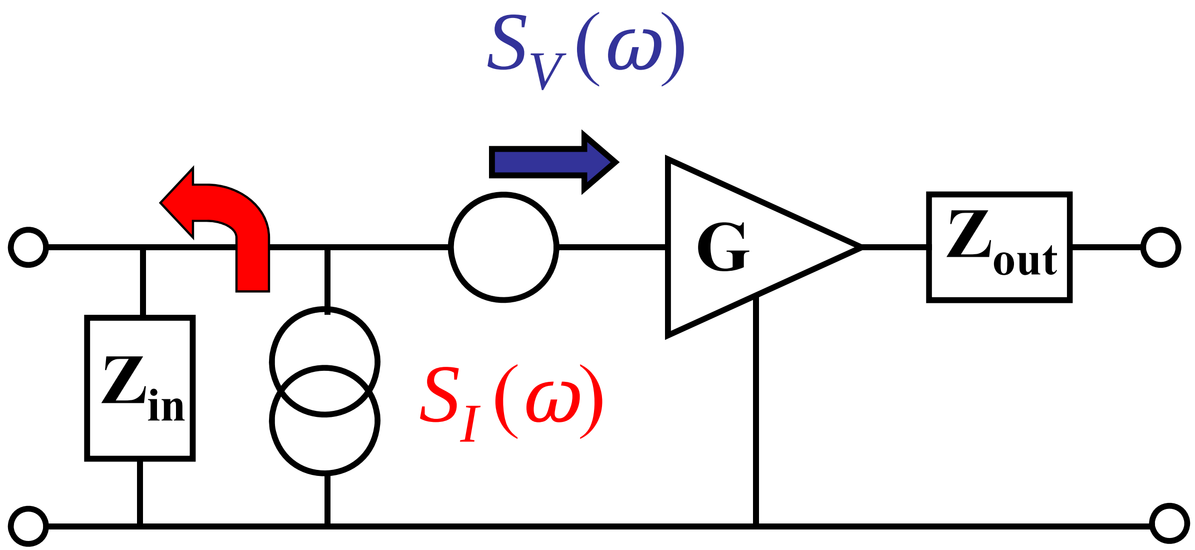
$$\sqrt{\left\langle \delta v^2 \right\rangle} \sim 4\,\mathrm{pV}/\sqrt{\mathrm{Hz}} \qquad \sqrt{\left\langle \delta i^2 \right\rangle} \sim 80\,\mathrm{fA}/\sqrt{\mathrm{Hz}}$$

$Z_0 = 5000\ \Omega$ at 1 GHz

$$\sqrt{\left\langle \delta v^2 \right\rangle} \sim 40\,\mathrm{pV}/\sqrt{\mathrm{Hz}} \qquad \sqrt{\left\langle \delta i^2 \right\rangle} \sim 8\,\mathrm{fA}/\sqrt{\mathrm{Hz}}$$

Equivalent circuit of an amplifier

H. Rothe and W. Dahlke, Proc. IRE **44**, 811 (1956).



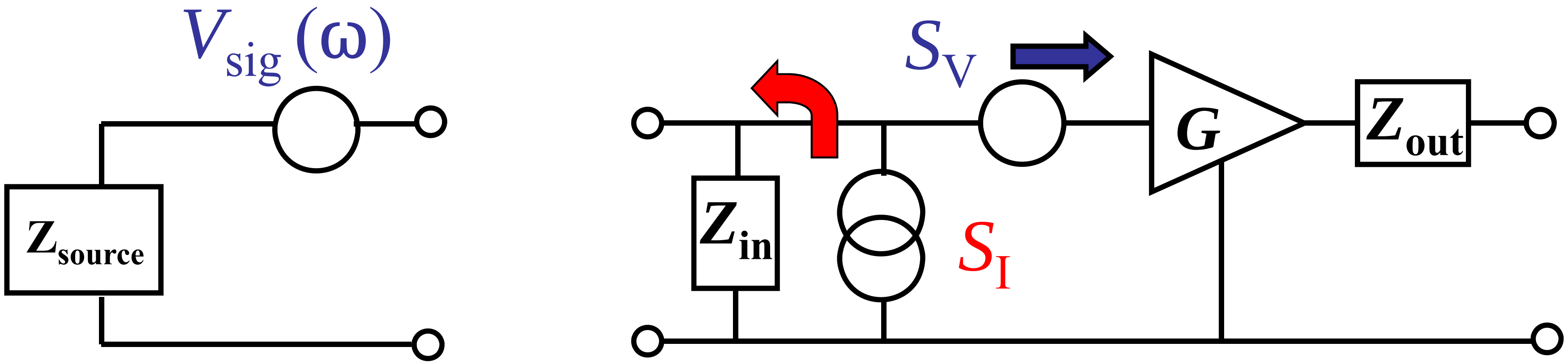
$S_V(\omega)$ = output noise referred to input

$S_I(\omega)$ = a real ”back action” noise (A^2/Hz)

may be strongly correlated with S_V

Noise Temperature of an Amplifier

- Beware: definition varies



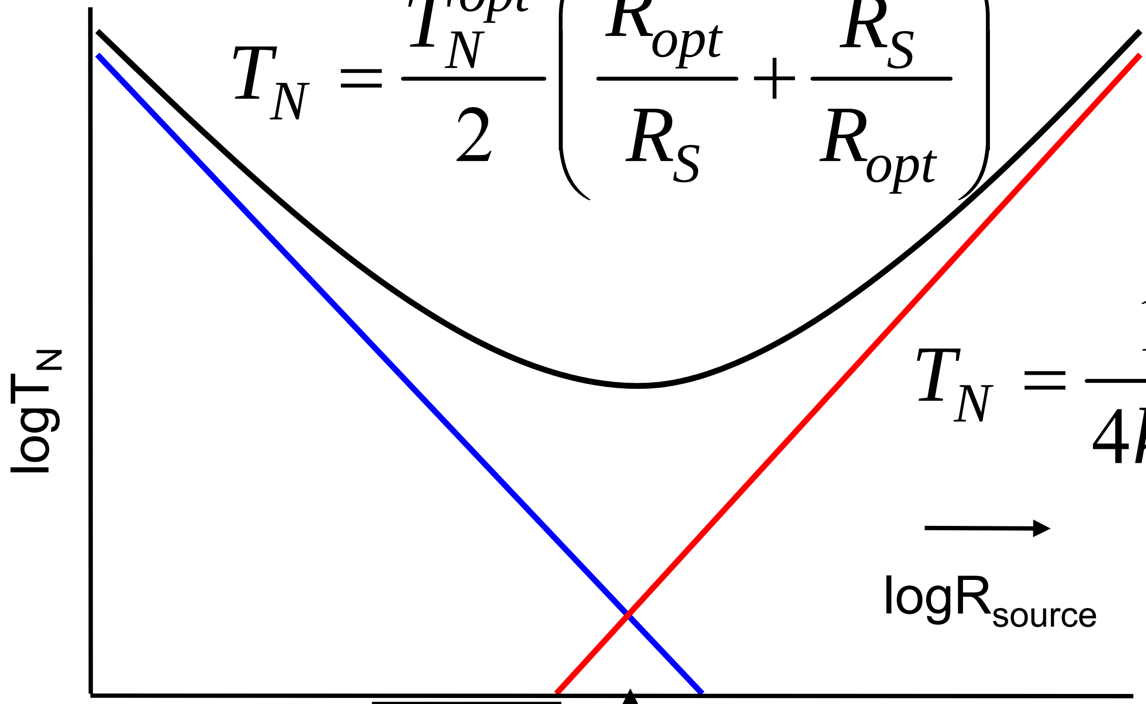
Total noise at the input : $S_V^{tot} = S_V + S_I \left| \frac{Z_{in} Z_S}{Z_{in} + Z_S} \right|^2$

Thermal noise of the source : $S_V^{tot} = 4kT_N \, \mathrm{Re}[Z_S]$

Assume: $Z_{in} = R_{in} >> R_S = Z_S$

$$T_N = \frac{1}{4k_B}\left(\frac{S_V}{R_S} + S_I R_S \right)$$

Optimum Noise Temperature



$$T_N = \frac{T_N^{opt}}{2}\left(\frac{R_{opt}}{R_S} + \frac{R_S}{R_{opt}}\right)$$

$$T_N = \frac{1}{4k_B}\left(\frac{S_V}{R_S} + S_I R_S\right)$$

$$R_{opt} = \sqrt{S_V / S_I}$$

$$T_N^{opt} = \sqrt{S_V S_I} / 2k \qquad E_N^{opt} = kT_N^{opt} = \sqrt{S_V S_I} / 2$$

E_N is the signal energy that can be detected with SNR = 1

Quantum mechanics: $E_N \geq \hbar\omega / 2$

T_n of cascaded amplifiers

C. D. Motchenbacher and J. A. Connelly, *Low noise eletronic system design*

$$T_N = T_{N_1} + T_{N_2} / G_1 + T_{N_3} / G_1 G_2 + \ldots$$

*- **T_N** of the first amplifier dominates if it has sufficient gain*

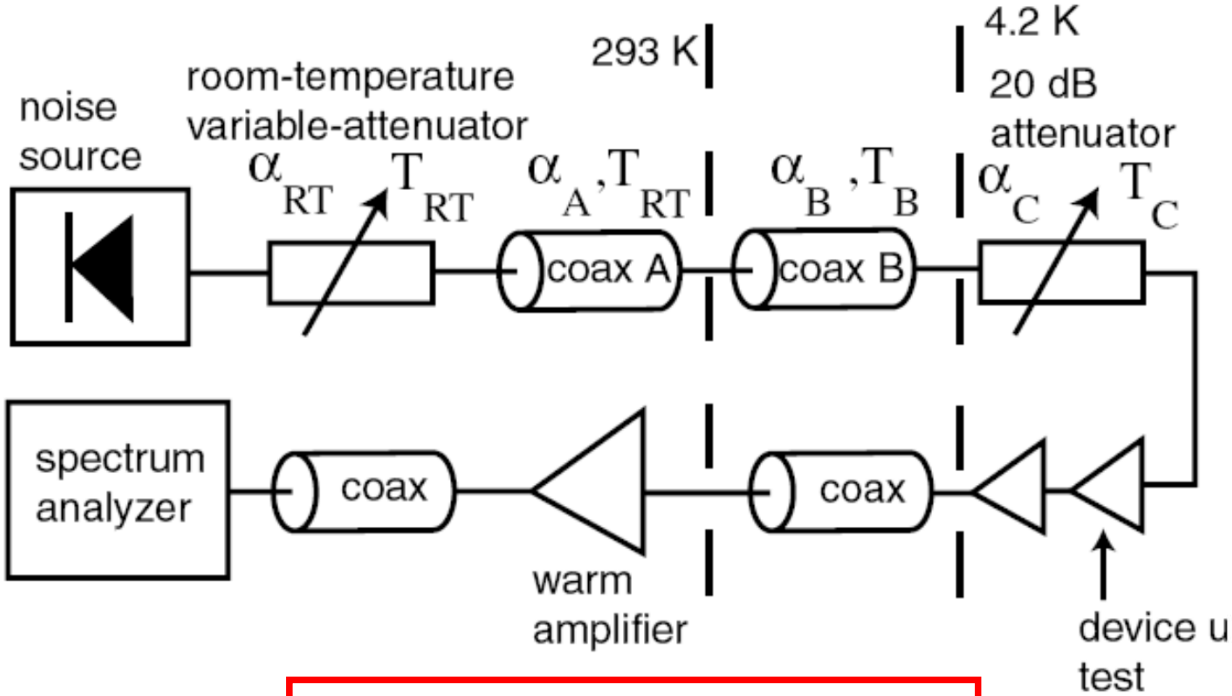
$T_{N_1} = 100\ \text{mK}$ SQUID amplifier

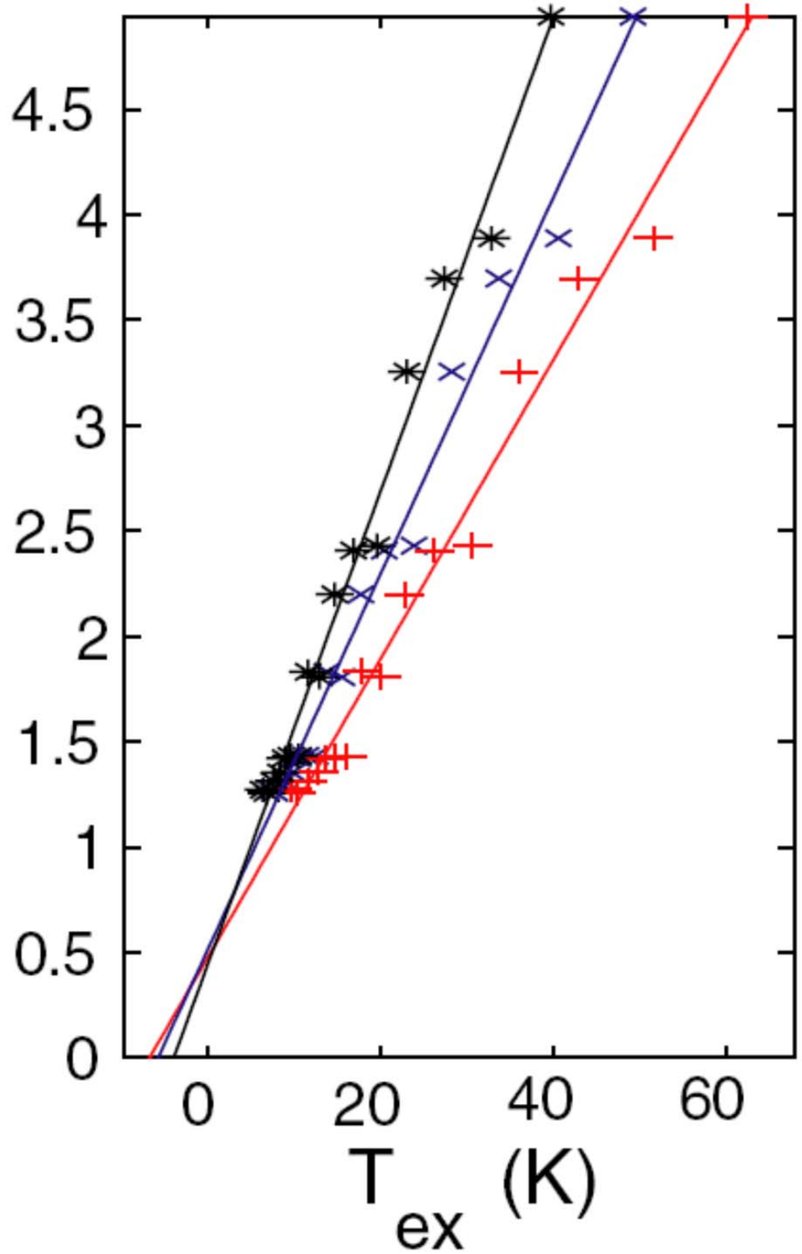
$T_{N_2} = 10\ \text{K}$ HEMT amplifier

Desirable to have the gain of
the SQUID amplifier ~30 dB

Measurement of Noise Temperature T_N

***T_N* is the value of the temperature of the source resistance that generates thermal noise equal to the amplifier noise.**





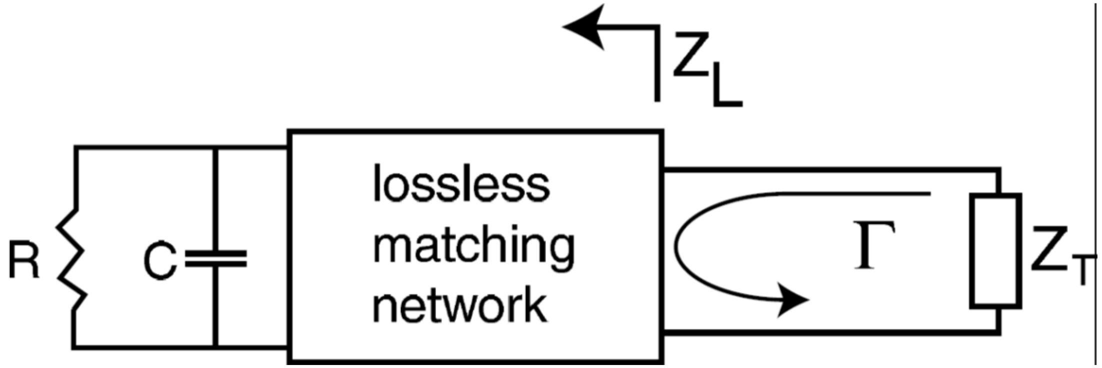
$$N = G(T_N + T_{ex})$$

Matching

- How to preserve the band width when you connect your measuring apparatus to your sensor?

The theoretical maximum bandwidth:

$$\int_0^\infty \ln \frac{1}{|\Gamma(\omega)|} d\omega \le \frac{\pi}{RC},$$



Bode--Fano criterion

$$\Gamma(\omega) = \frac{Z_L(\omega) - Z_0}{Z_L(\omega) + Z_0}$$

”You cannot exceed inverse of RC time constant”

References

D. M. Pozar, *Microwave Engineering*, 1st ed. (Addison-Wesley, New York, 1990).

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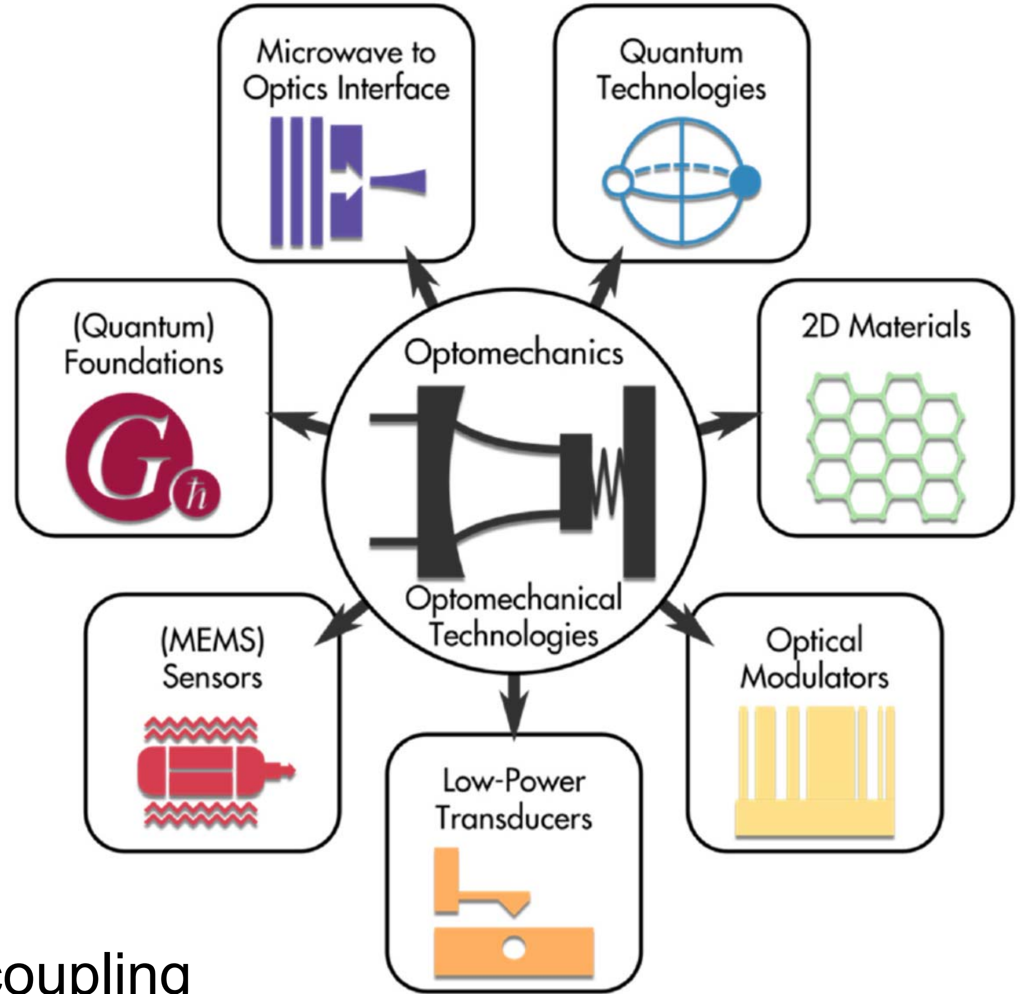

- Graduate student position available
- Optomechanics with coupling based on Josephson inductance

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