Atomic Tunneling Systems in Solids

real crystals

glasses

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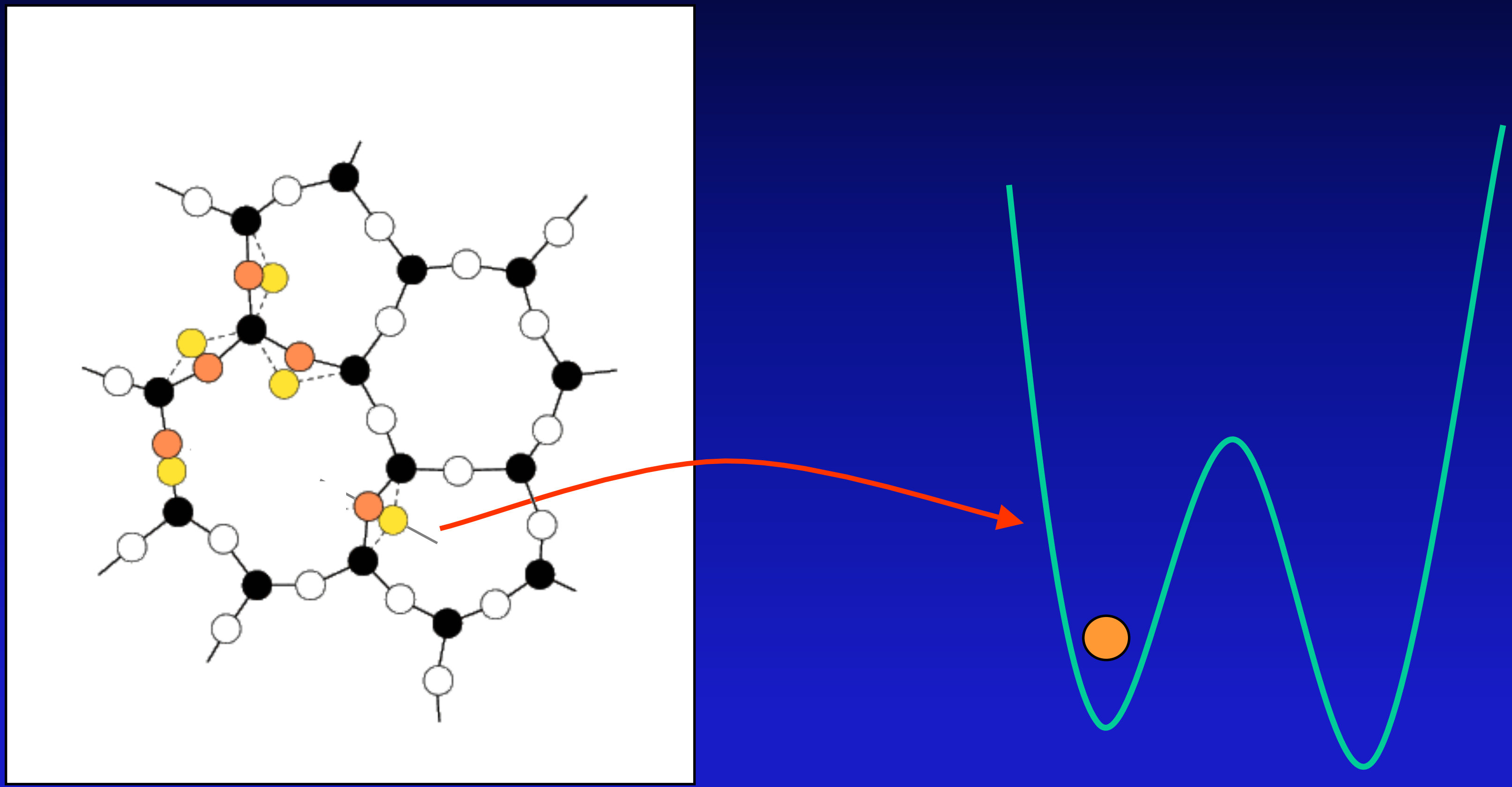
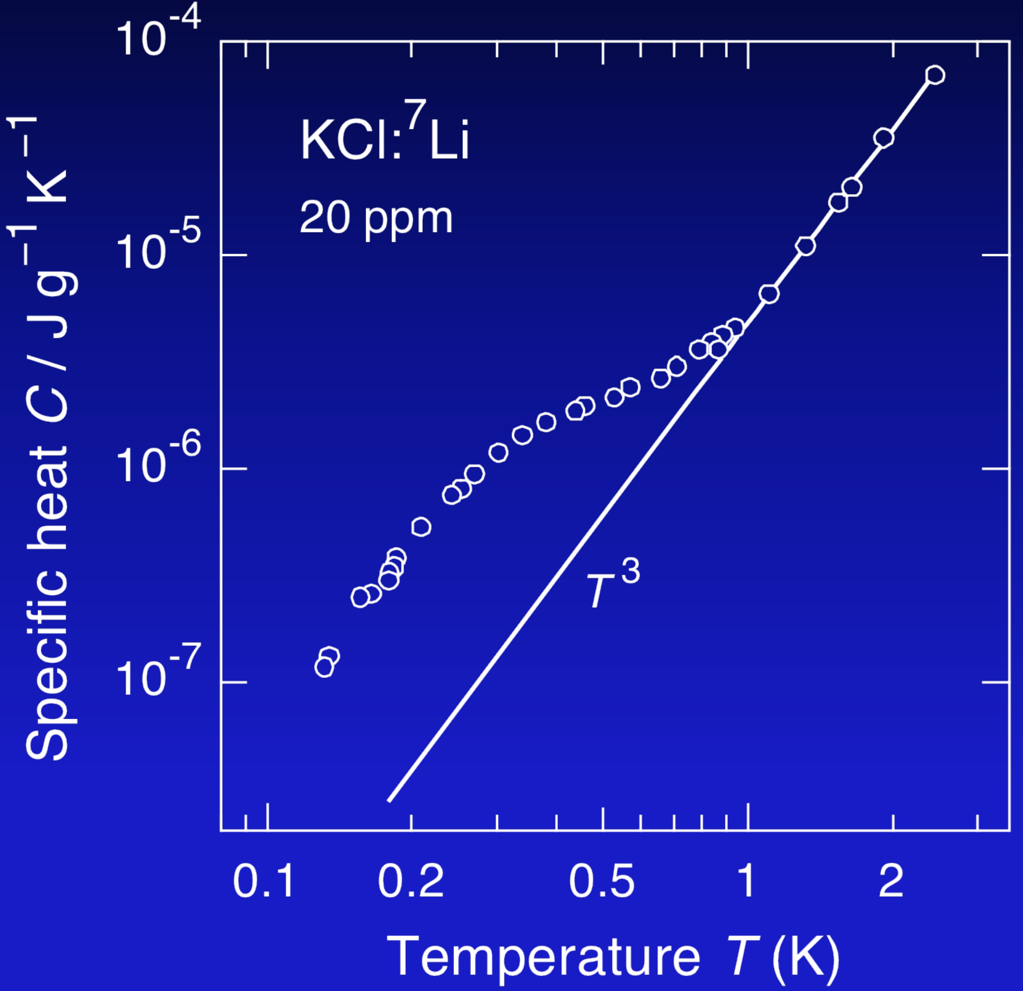
Tunneling of Atoms in Solids



KCl:Li   Specific Heat

specific heat roughly a

factor of 10 higher at 0.5 K



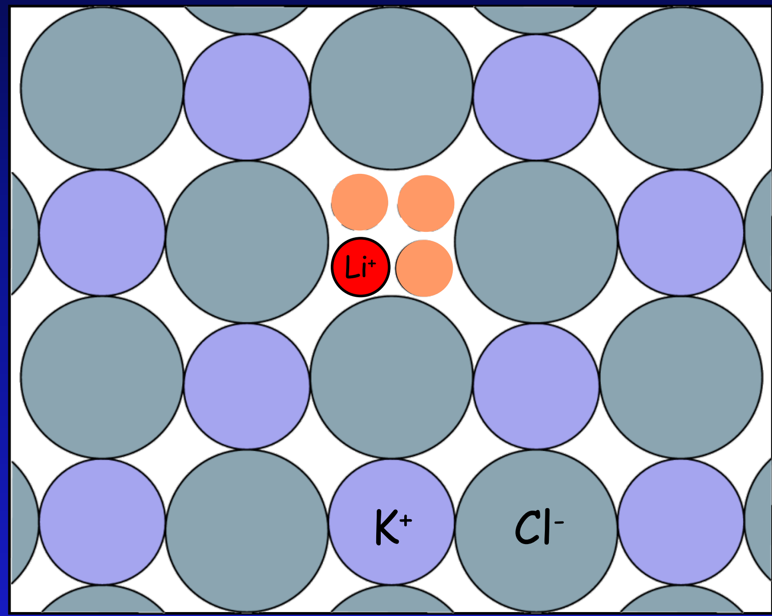
J.P. Harrison, Phys. Rev. 171, 1037 (1968)

Li-Tunneling Systems in KCl-Crystals

Li^+ substitutes K^+

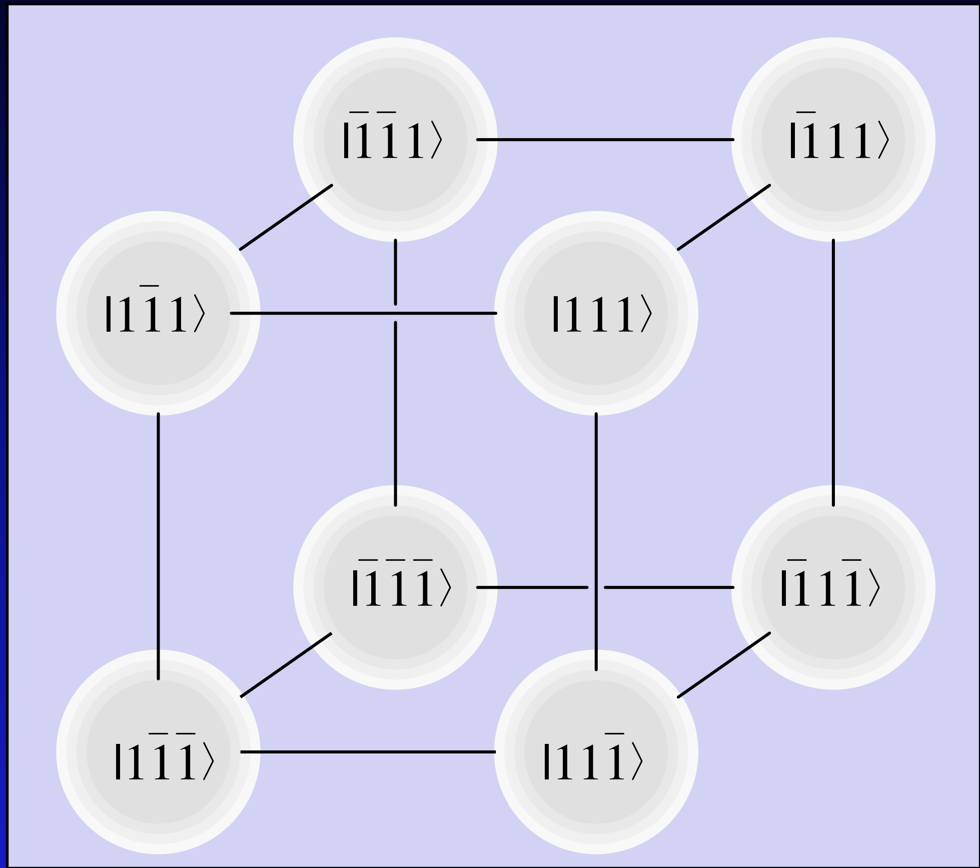
ionic radius: $r_{\mathrm{Li}^+} < r_{\mathrm{K}^+}$

➡ 8 off-center positions in <111> direction



(100)-plane

Quantum States of <111> Systems

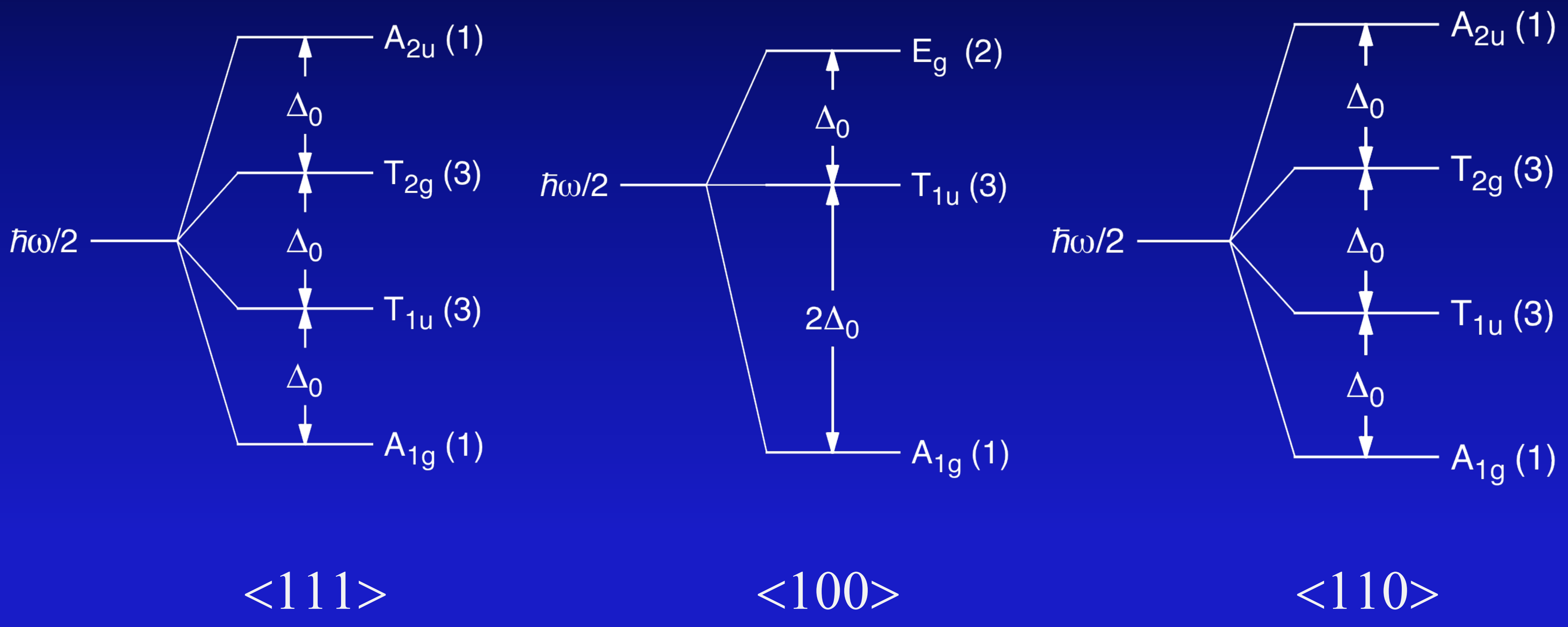


$\eta' = \langle 1\,1\,1|\boldsymbol{H}|1\,1\,\bar{1}\rangle = \langle \bar{1}\,1\,1|\boldsymbol{H}|\bar{1}\,\bar{1}\,1\rangle = \ldots$ edge tunneling

$\mu' = \langle 1\,1\,1|\boldsymbol{H}|1\,\bar{1}\,\bar{1}\rangle = \langle 1\,\bar{1}\,1|\boldsymbol{H}|1\,1\,\bar{1}\rangle = \ldots$ face diagonal tunneling

$\nu' = \langle 1\,1\,1|\boldsymbol{H}|\bar{1}\,\bar{1}\,\bar{1}\rangle = \langle 1\,\bar{1}\,1|\boldsymbol{H}|\bar{1}\,1\,\bar{1}\rangle = \ldots$ room diagonal tunneling

Tunneling Systems with Cubic Symmetry



KCl:Li, KBr:CN

KCl:OH, LiF:OH

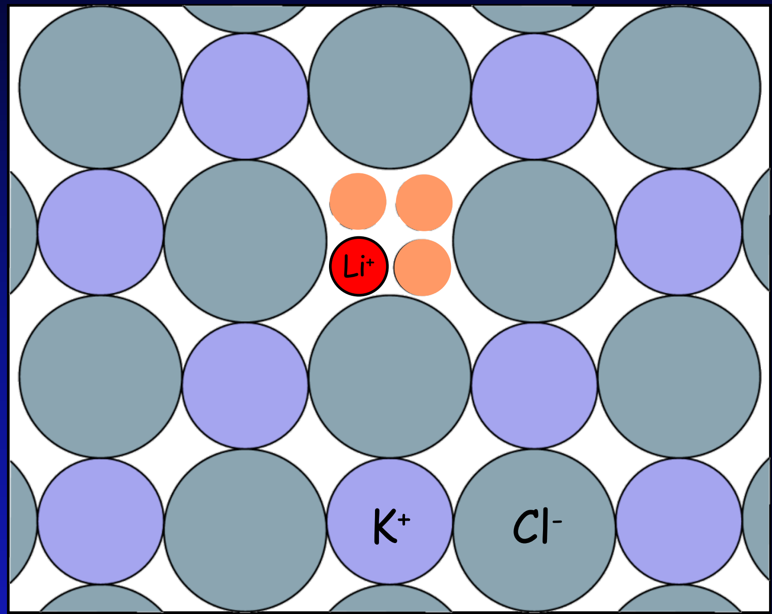
NaBr:F

Li-Tunneling Systems in KCl-Crystals

Li^+ substitutes K^+

ionic radius: $r_{\mathrm{Li}^+} < r_{\mathrm{K}^+}$

➡ 8 off-center positions in <111> direction



(100)-plane

tunnel splitting:

$$\Delta_0 = \hbar \Omega \, \mathrm{e}^{-\lambda} \quad \text{with} \quad \lambda = \frac{d}{2\hbar}\sqrt{2mV}$$

$$^7\mathrm{Li}: \quad {}^7\Delta_0/k_\mathrm{B} = 1{,}1\ \mathrm{K}$$

$$^6\mathrm{Li}: \quad {}^6\Delta_0/k_\mathrm{B} = 1{,}6\ \mathrm{K}$$

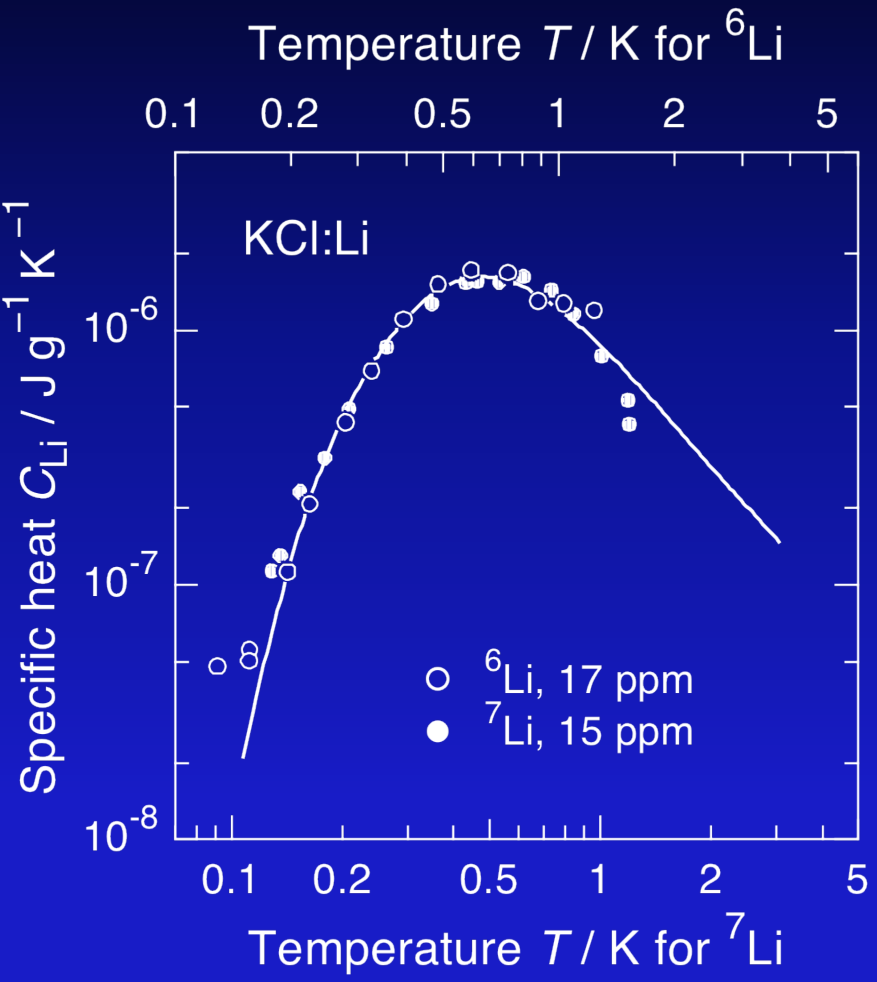
X. Wang, F. Bridges, Phys. Rev. **B46**, 5122 (1992)

Isotope effect

Schottky-Anomaly

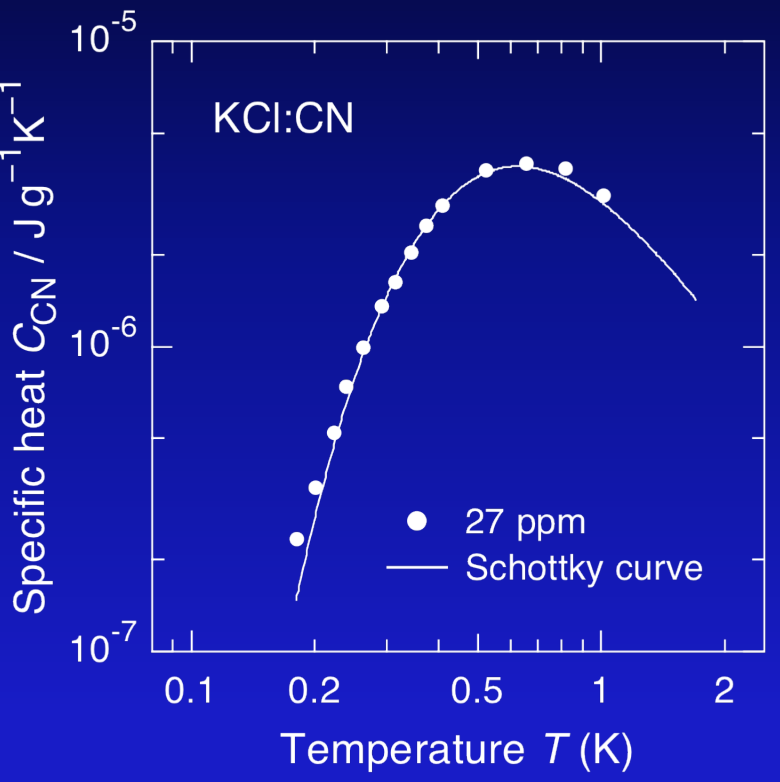
$$C_{\mathrm{TS}} = \frac{3nk_{\mathrm{B}}}{\varrho} \left(\frac{\Delta_0}{2k_{\mathrm{B}}T}\right)^2 \operatorname{sech}^2\left(\frac{\Delta_0}{2k_{\mathrm{B}}T}\right)$$

number density $n = N/V$

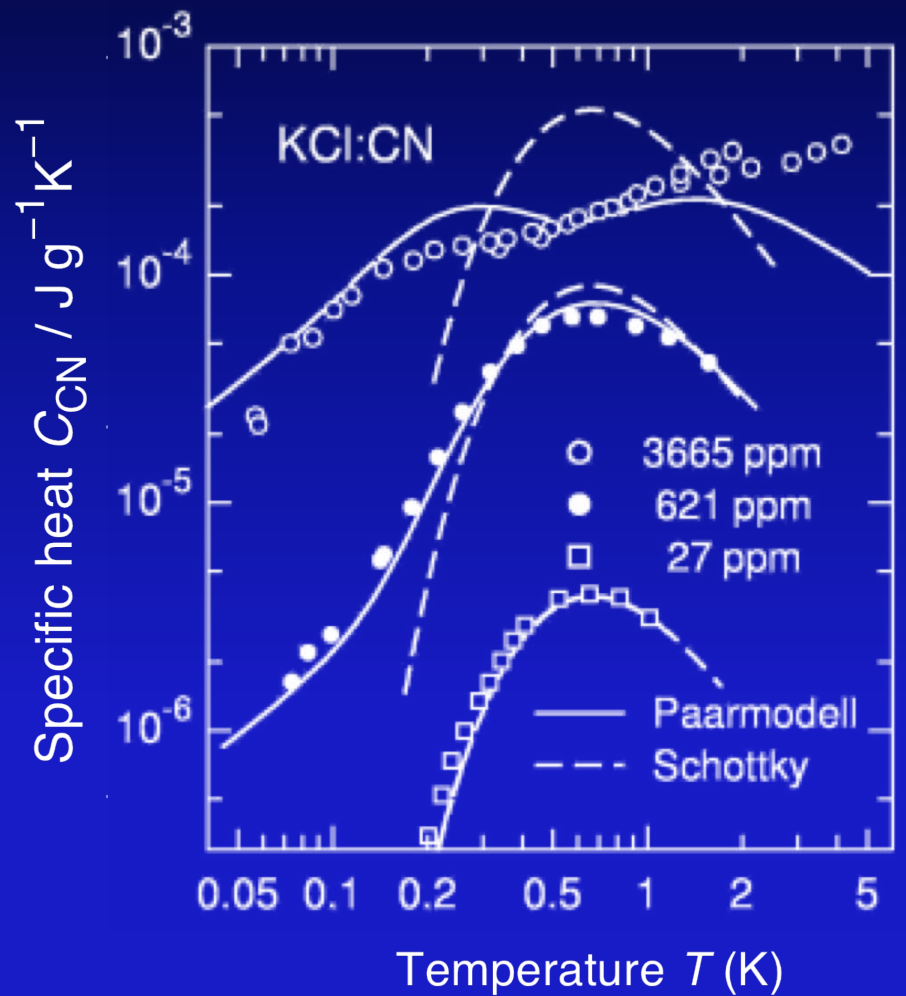


J.P. Harrison, Phys. Rev. 171, 1037 (1968)

KCl:CN



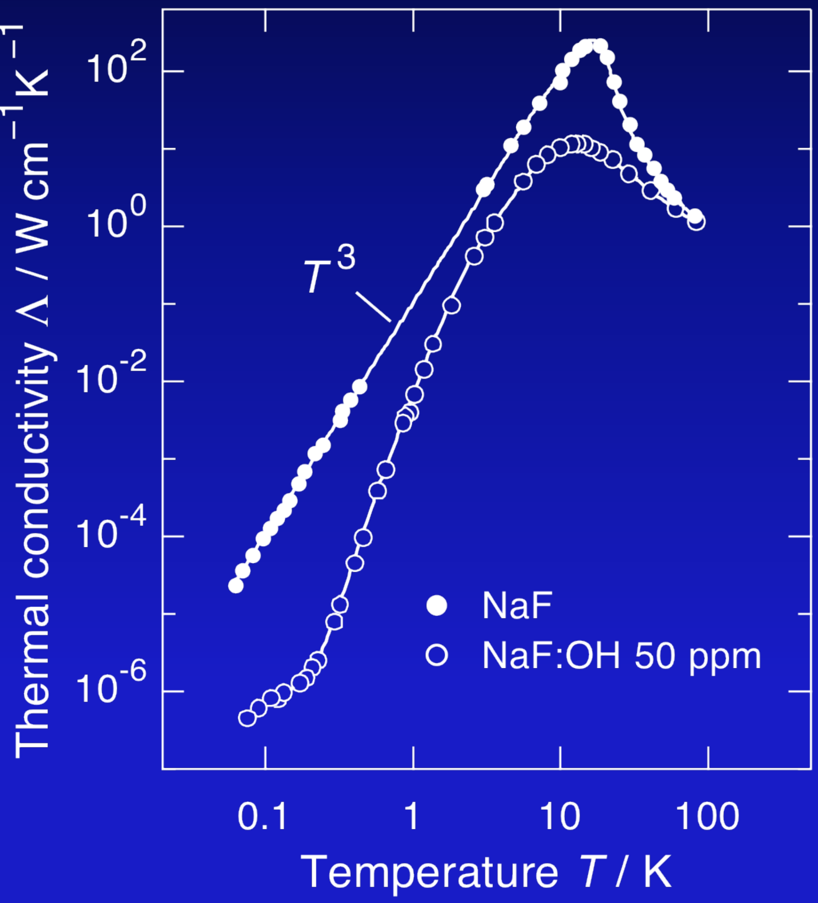
concentration dependence



P.P. Peressini, J.P. Harrison, R.O. Pohl,
Phys. Rev. **182**, 939 (1969)

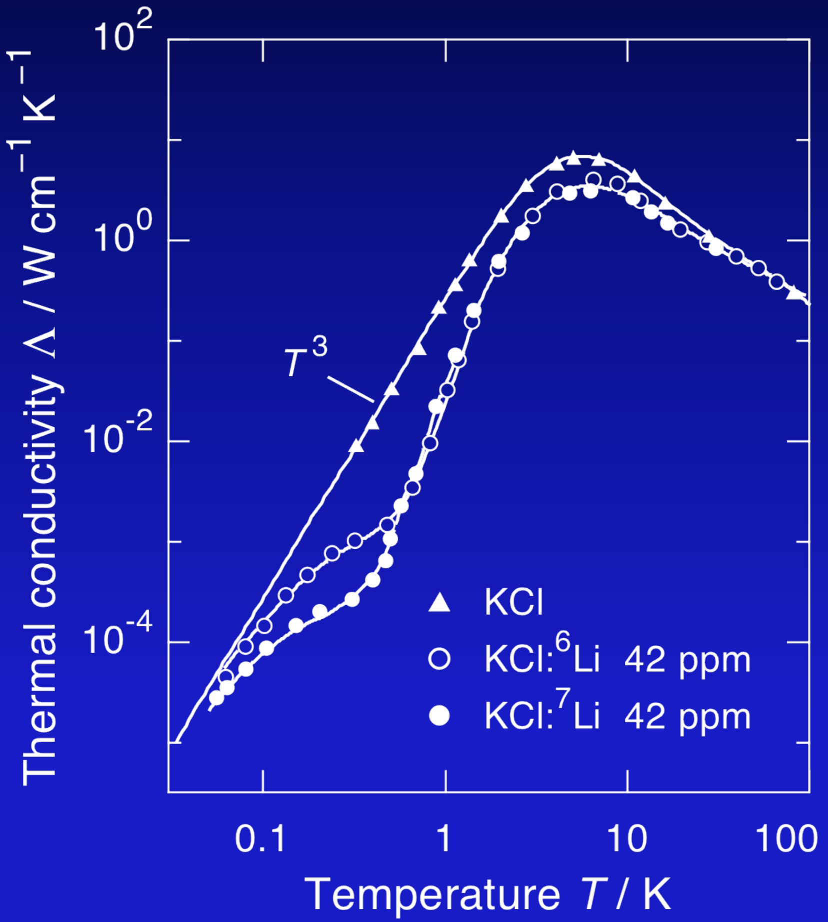
Thermal Conductivity

NaF: OH^-



T.F. McNelly, Ph.D. thesis
(Cornell University 1974)

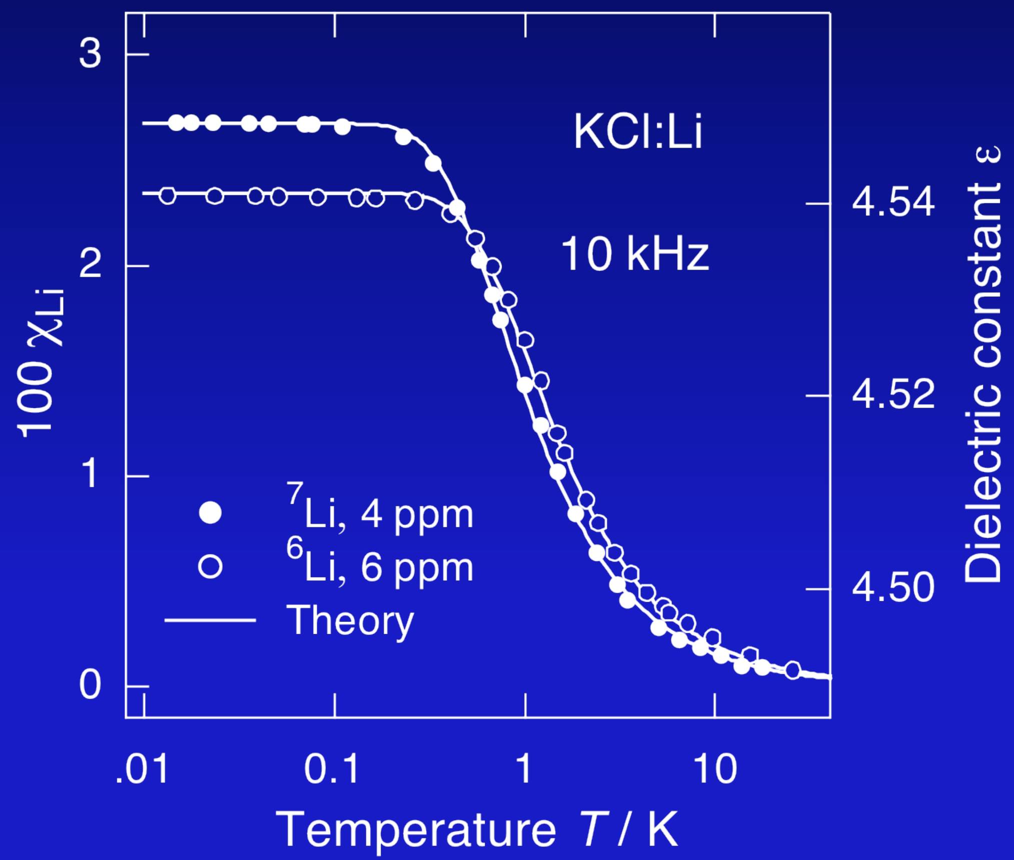
KCl:Li



P.P. Peressini, J.P. Harrison, R.O. Pohl,
Phys. Rev. 180, 926 (1969)

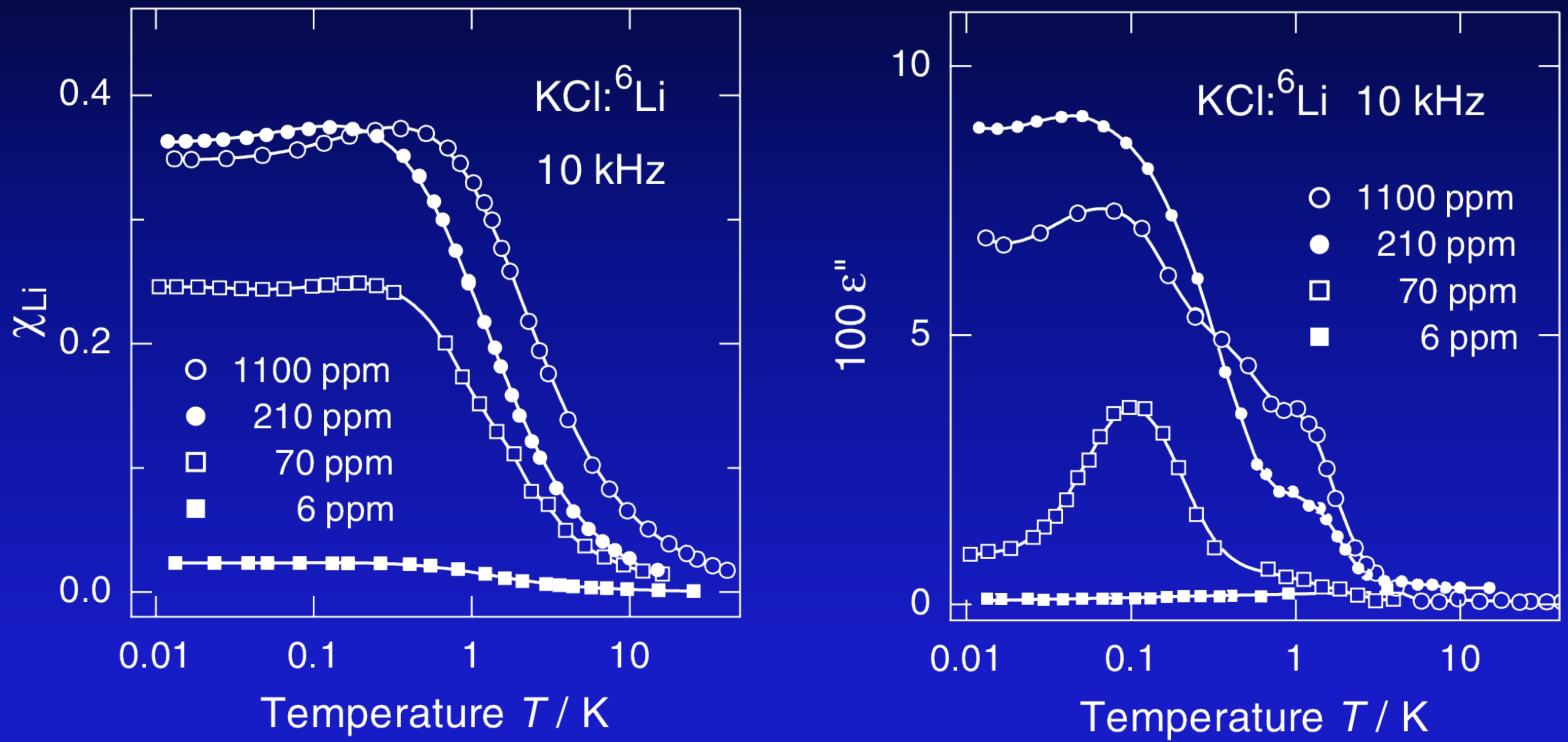
Dielectric Susceptibility

$$\chi_{\text{iso}} = \frac{2}{3}\,\frac{np^2}{\varepsilon_0 \Delta_0}\tanh\left(\frac{\Delta_0}{2k_{\text{B}}T}\right)$$



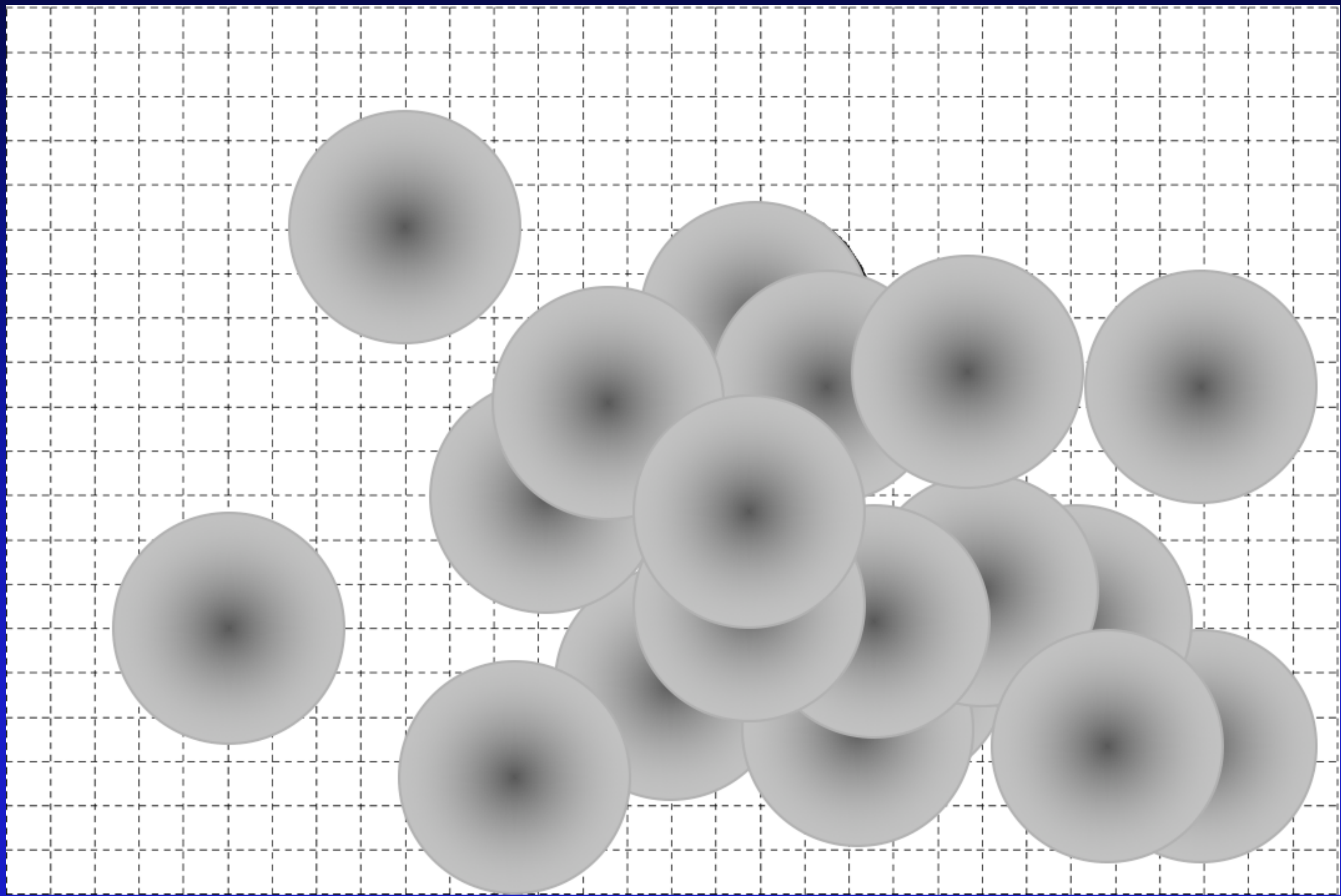
C. Enss, M. Gaukler, S. Hunklinger, M. Tornow, R. Weis, A. W'urger, Phys. Rev. B 53, 12094 (1996)

Dielectric Susceptibity: High Concentrations



C. Enss, M. Gaukler, S. Hunklinger, M. Tornow, R. Weis, A. Würger, Phys. Rev. B **53**, 12094 (1996)

Interacting Tunneling Systems



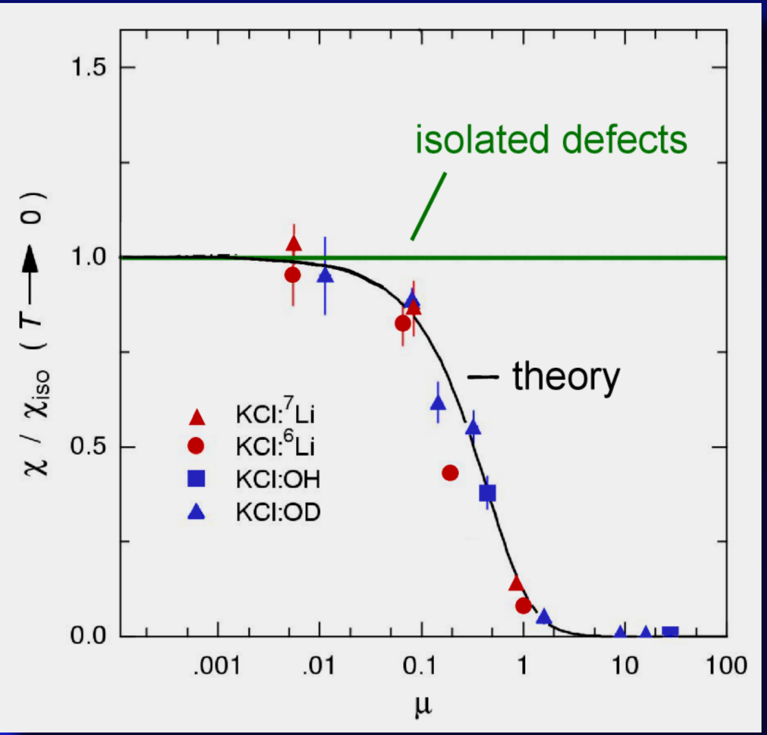
Transition to Incoherent Tunneling

- defects in crystals:

  at high concentrations → cross over to **incoherent tunneling**

  consequences:
  - reduced **resonant** contribution
  - new **phononless** relaxation channel

$$\mu = \frac{\overline{J}}{\Delta_0}$$

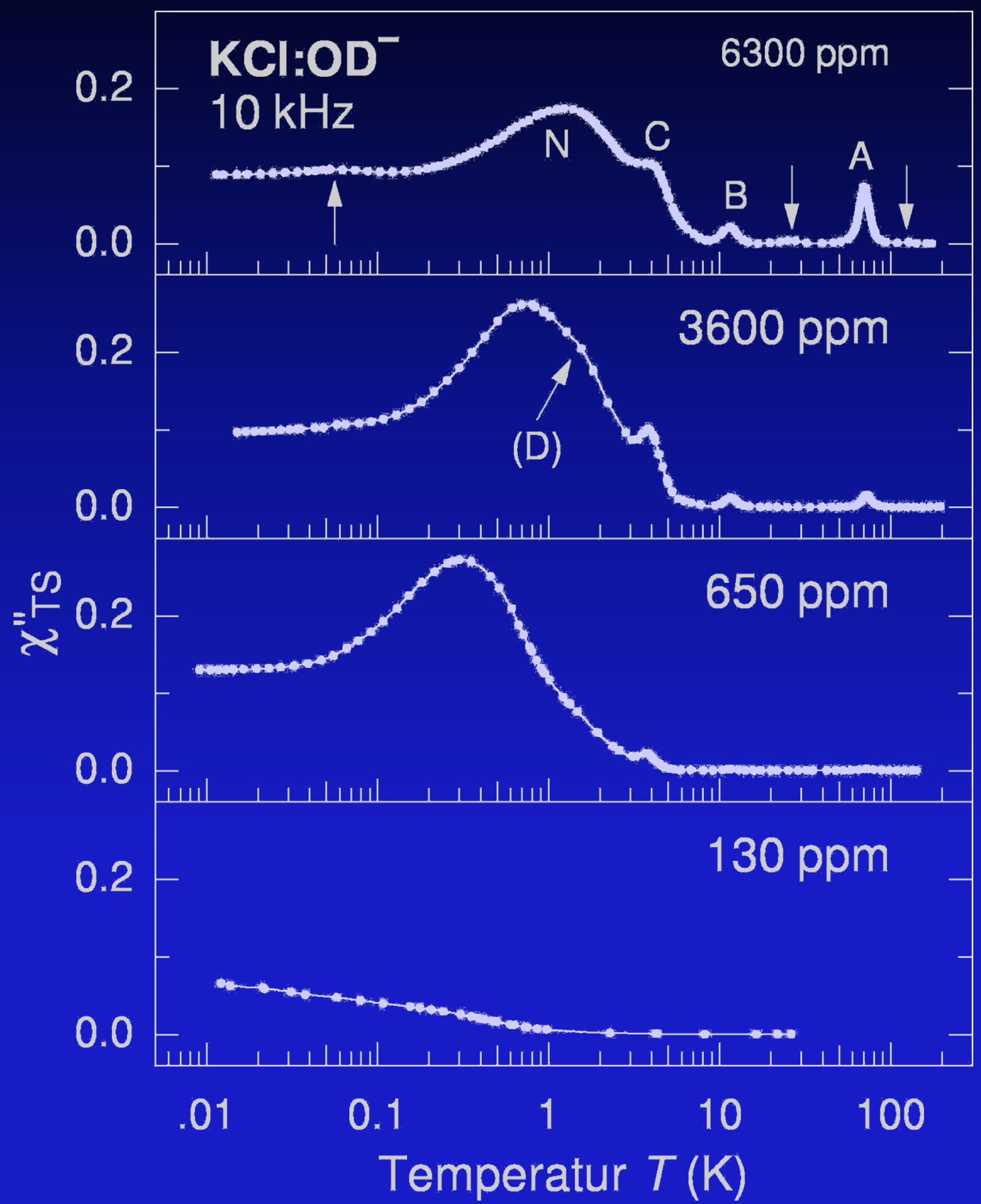


- incoherent tunneling in glasses at very low temperatures?

$$\overline{J} > \Delta_0 \approx k_B T$$

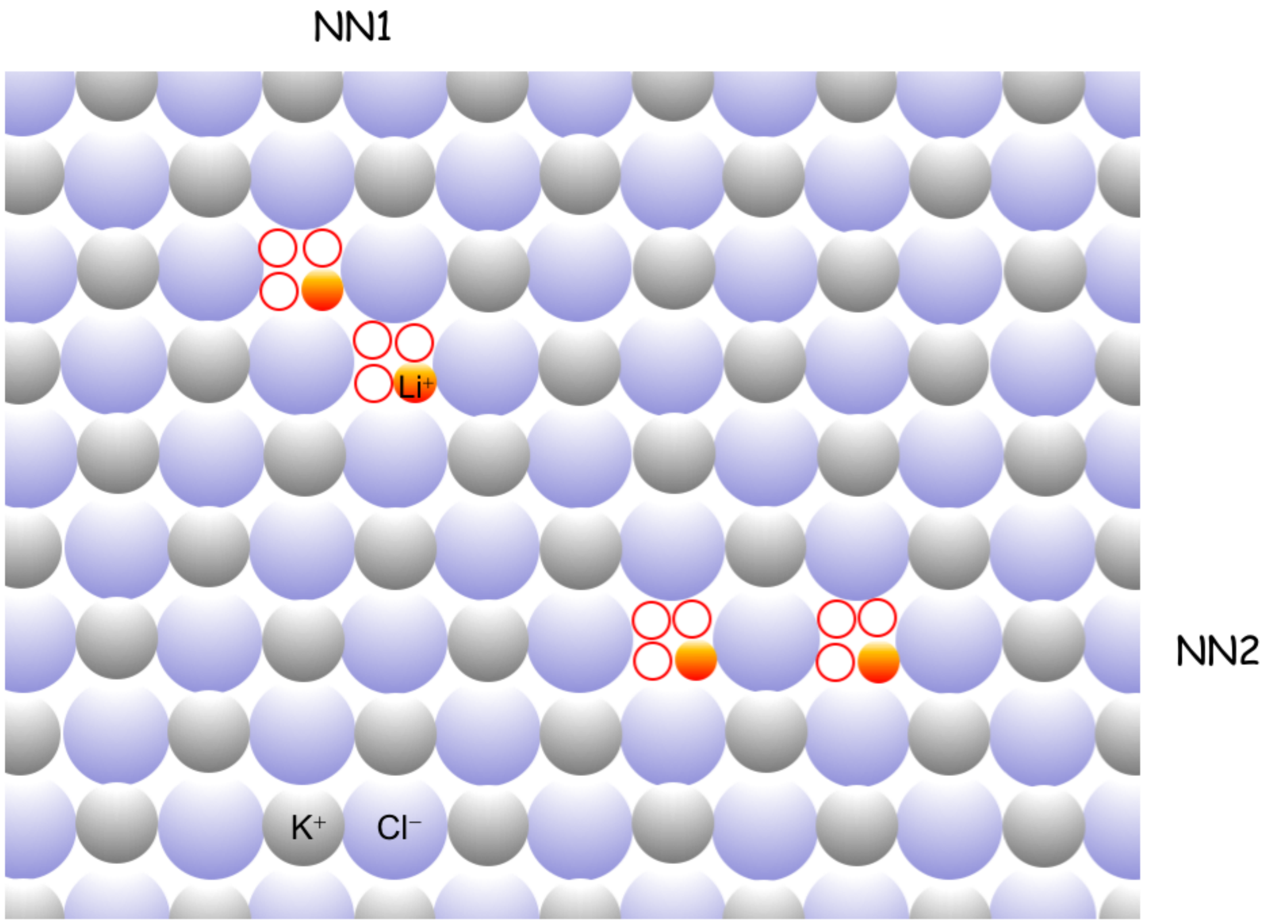
A. Würger, R. Weis, M. Gaukler, C. Enss, Europhys. Lett. **33**, 533 (1996)

Thermally Activated Pairs



S. Ludwig, C. Enss, J. Low Temp. Phys. **137**, 371 (2004)

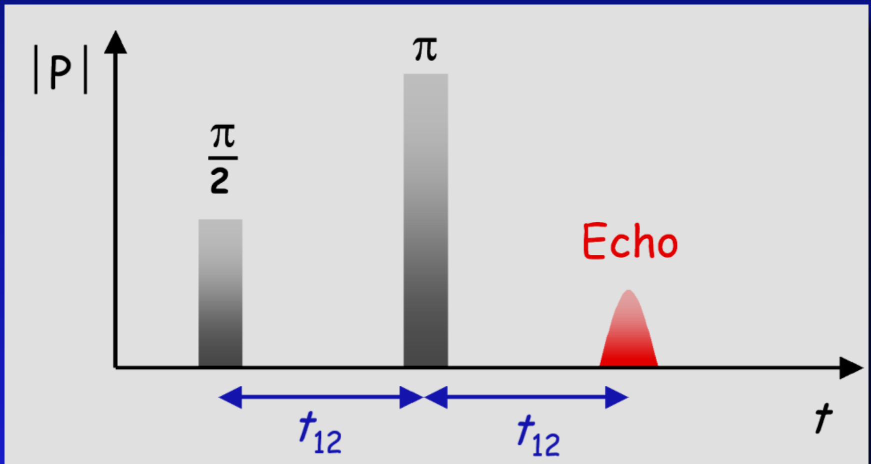
Tunneling of Li Pairs



Coherent Properties

$t \ll \tau_1, \tau_2 \;\rightarrow\; \infty$ → coherent regime

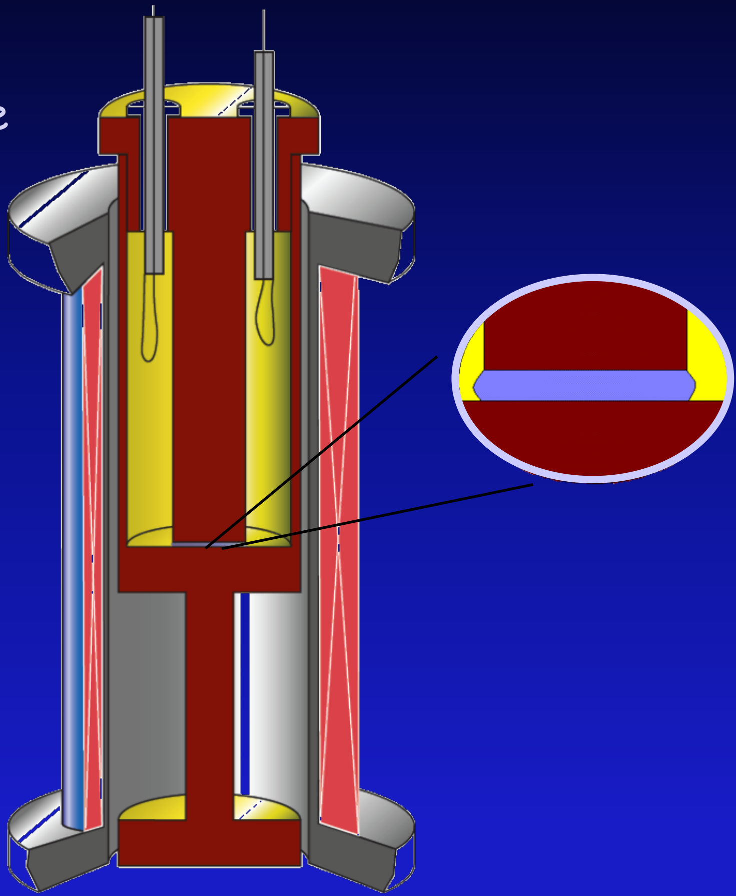
two-pulse polarization echoes:



$\Theta_\mathrm{p} = \Omega_\mathrm{R} t_\mathrm{p}$

Rabi frequency

$$\Omega_\mathrm{R} = \frac{1}{\hbar}\frac{\Delta_0}{E}\,\boldsymbol{p}\cdot\boldsymbol{F}$$



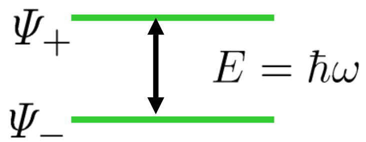
microwave cavity

1 GHz → 50 mK

Echo – Theoretical Background I

coherent regime: $t \ll \tau_1, \tau_2 \;\rightarrow\; \infty$

two level approximation:



applied field: $\boldsymbol{F} = \boldsymbol{F}_0 \left(\mathrm{e}^{\mathrm{i}\omega_\mathrm{r} t} + \mathrm{e}^{-\mathrm{i}\omega_\mathrm{r} t}\right)$

Schrödinger equation: $\mathrm{i}\hbar \dfrac{\partial \Psi}{\partial t} = \left[H_0 + H_\mathrm{S}\right] \Psi$ mit $H_\mathrm{S} = 2\dfrac{\Delta_0}{E}\,\boldsymbol{p}\cdot\boldsymbol{F}$

ansatz: $\Psi(t) = a_1(t)\,\Psi_- + a_2(t)\,\Psi_+$

$$\left.\begin{aligned} a_1(t) &= \cos\left(\Omega_\mathrm{R} t\right)\,\mathrm{e}^{-\mathrm{i}\omega_\mathrm{r} t} \\ a_2(t) &= -\mathrm{i}\sin\left(\Omega_\mathrm{R} t\right)\,\mathrm{e}^{-\mathrm{i}\omega_\mathrm{r} t} \end{aligned}\right\}$$

Rabi frequency: $\Omega_\mathrm{R} = \dfrac{1}{\hbar}\dfrac{\Delta_0}{E}\,\boldsymbol{p}\cdot\boldsymbol{F}_0$

Phase Jump Rotary Echoes

coherent regime:

$$t \ll \tau_1, \tau_2 \;\rightarrow\; \infty$$

applied field:

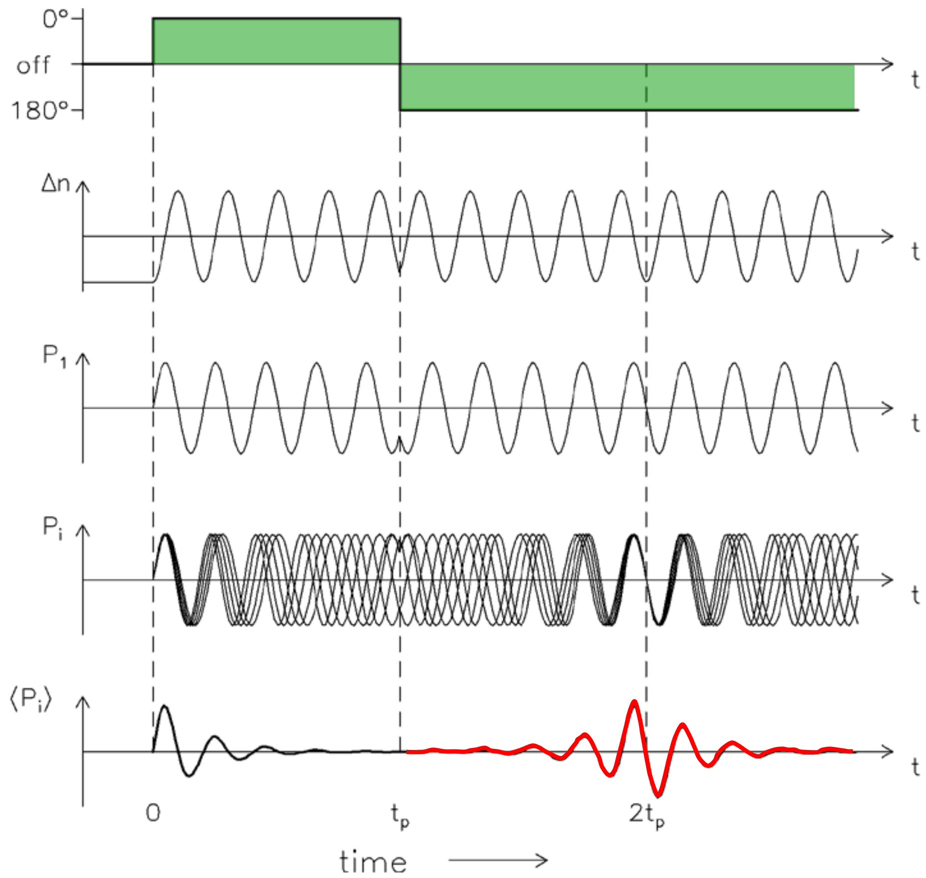
$$\boldsymbol{F} = 2\,\boldsymbol{F}_0 \cos(\omega t)$$

occupation number difference

$$\Delta n = \cos\left(\Omega_{\mathrm{R}} t\right)$$

↓

periodic change of polarisation

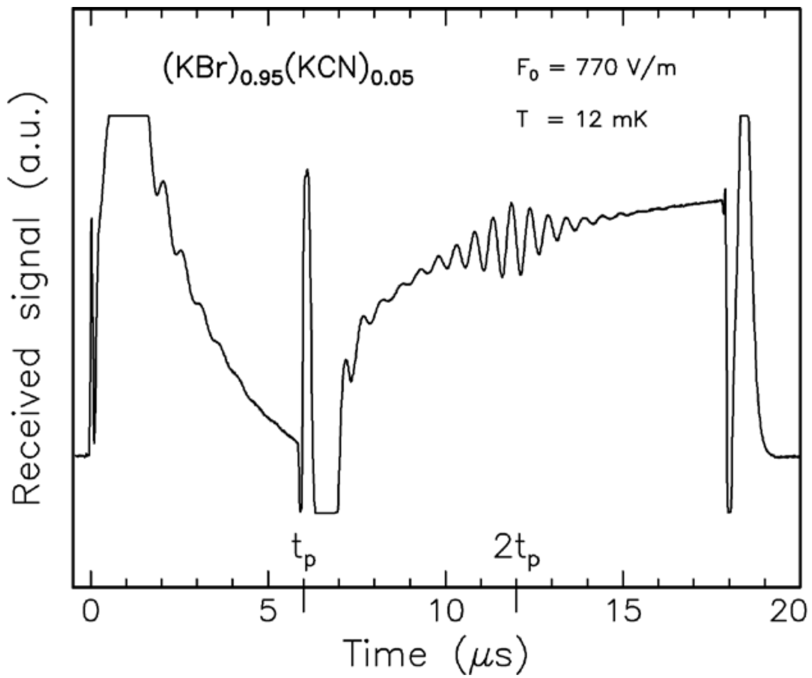


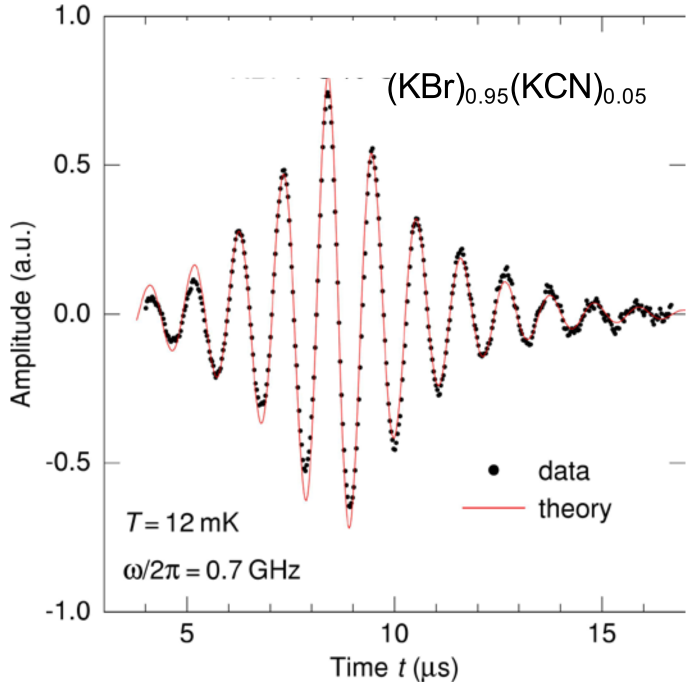
$$A_{\mathrm{r}}(t) \propto \int_0^{+\infty}\int_0^{E} P(E,\Delta_0)\,\frac{\Delta_0}{E}\,\frac{\boldsymbol{p}\cdot\boldsymbol{F}_0}{|\boldsymbol{F}_0|}\left[\frac{\Omega_{\mathrm{R}}}{\Omega}\right]^3 \mathrm{e}^{-\beta}\sin\left[\Omega\left(t-2t_{\mathrm{p}}\right)\right]\,\mathrm{d}\Delta_0\,\mathrm{d}E$$

with $\Omega = \sqrt{\Omega_{\mathrm{R}}^2 - \omega_{\mathrm{d}}^2}$ $\qquad \omega_{\mathrm{d}} = \omega_{\mathrm{r}} - \omega$ $\qquad \beta = \frac{1}{\Omega^2}\left(\frac{\omega_{\mathrm{d}}^2}{\tau_2} + \frac{\Omega_{\mathrm{R}}^2}{2\tau_1} + \frac{\Omega_{\mathrm{R}}^2}{2\tau_2}\right)$

Phase Jump Rotary Echoes

$(KBr)_{1-x}(KCN)_x$





many oscillations ⟶ well defined Ω_R

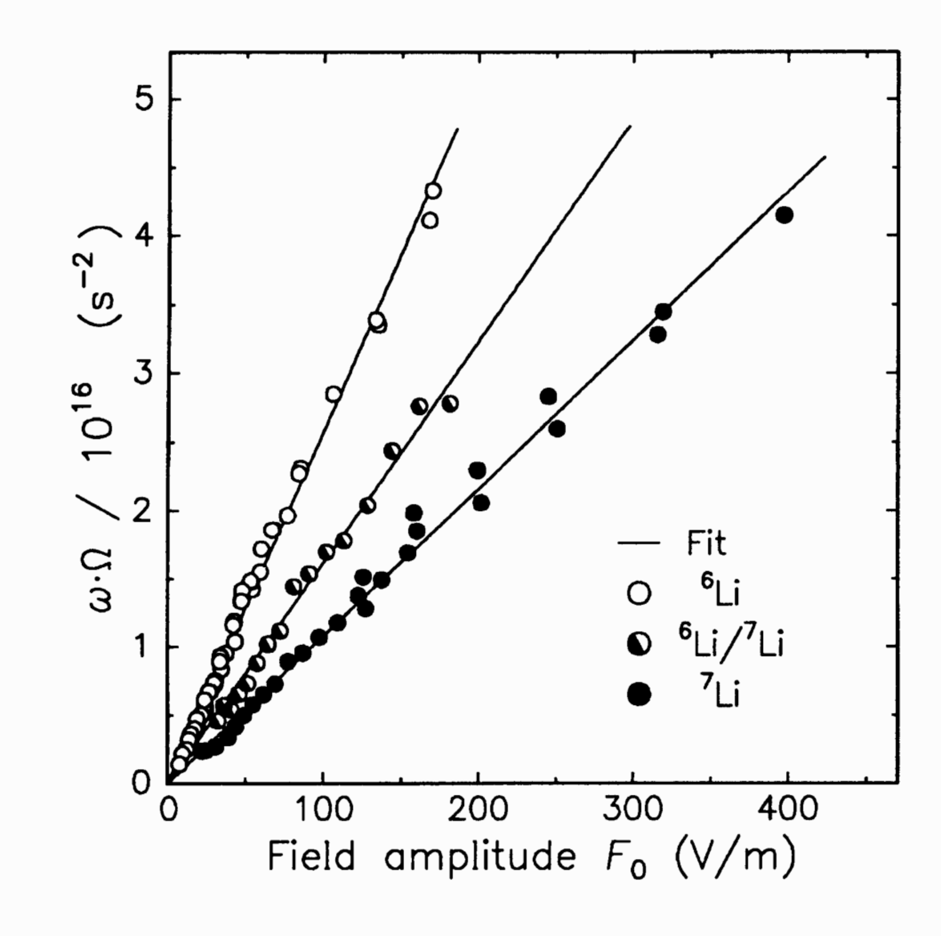
fit: fixed values of E, p

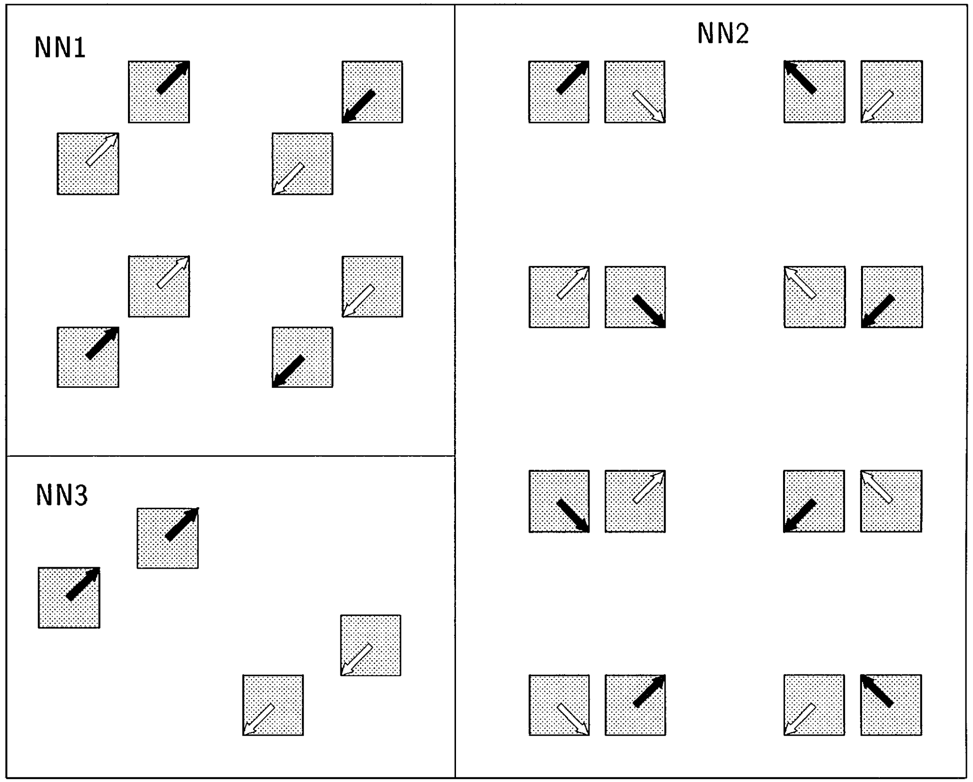
Lorentz distribution of Δ_0 width 10%

G. Baier, M.v. Schickfus, C. Enss, Europhys. Lett. **8**, 487 (1989)

Coherent Pair Tunneling

Rabi-Frequency of NN2 pairs of different isotopes





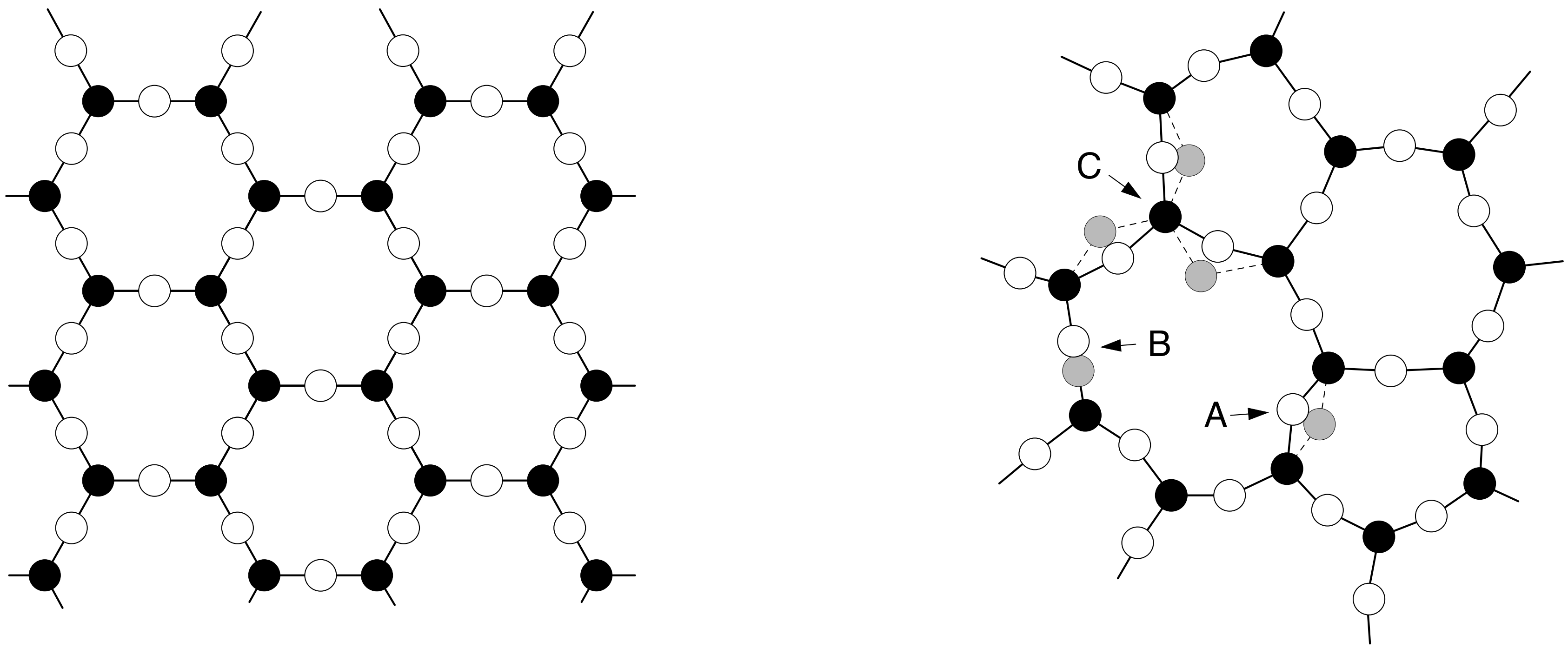
only NN2 pairs contribute

R. Weis, C. Enss, A. Würger, F. Lüty, Ann. Phys. **6**, 263 (1956)

Atomic Tunneling Systems in Amorphous Solids

glasses

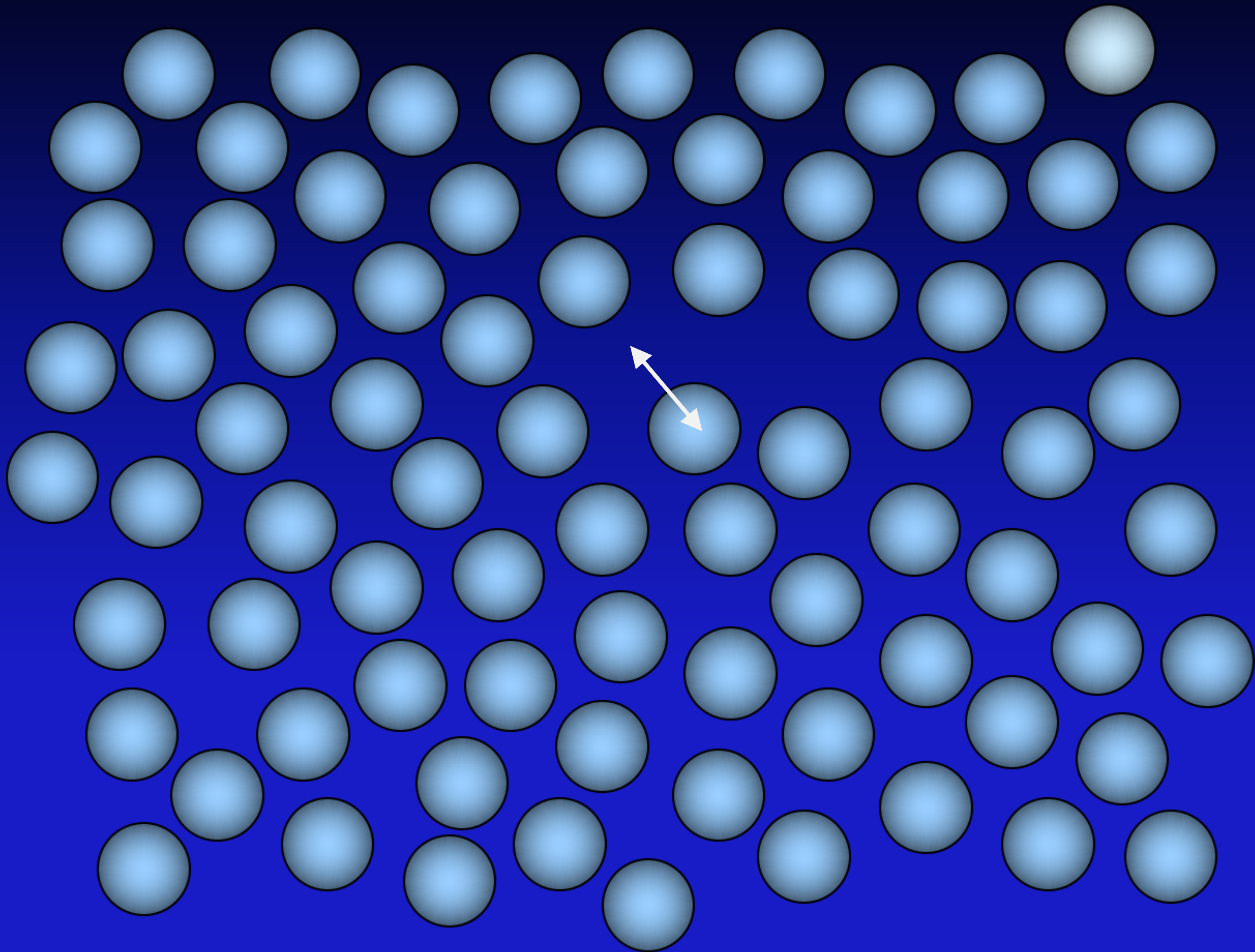
Structure of Crystalline and Amorphous Materials



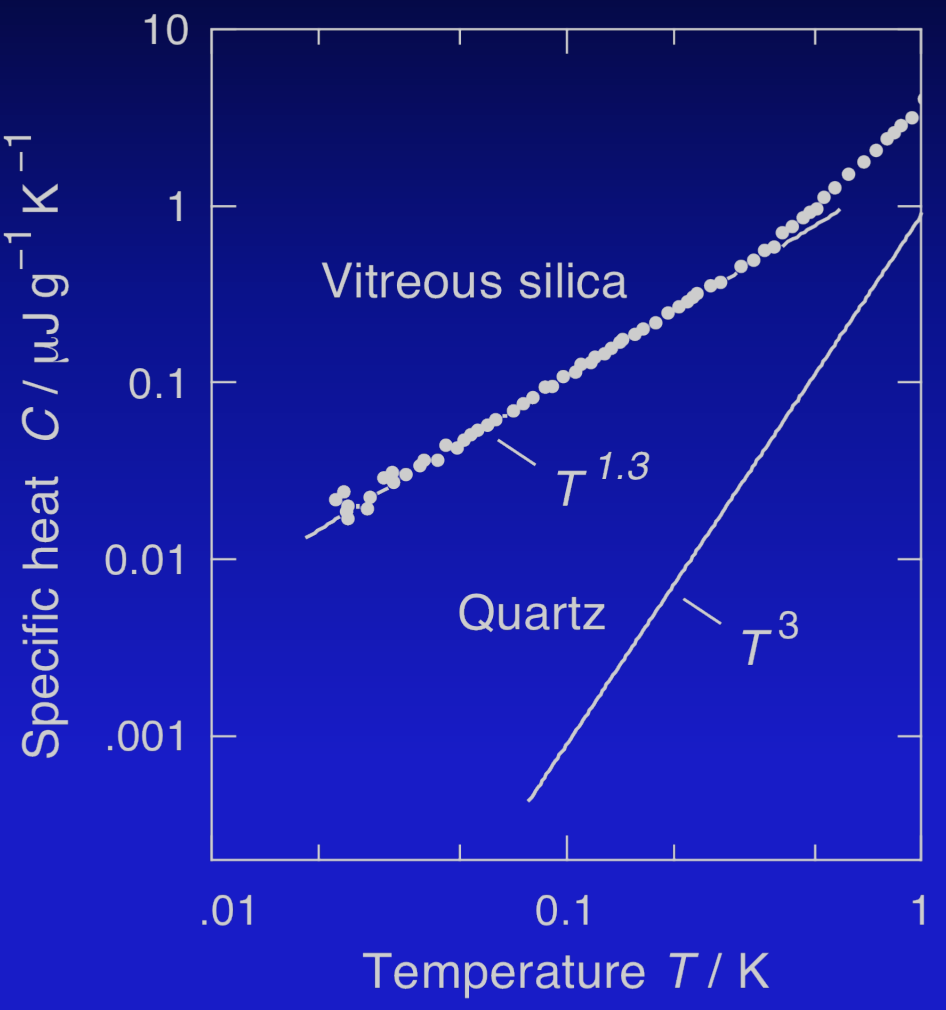
Crystal

Glass

Atomic Tunneling Systems in Glasses



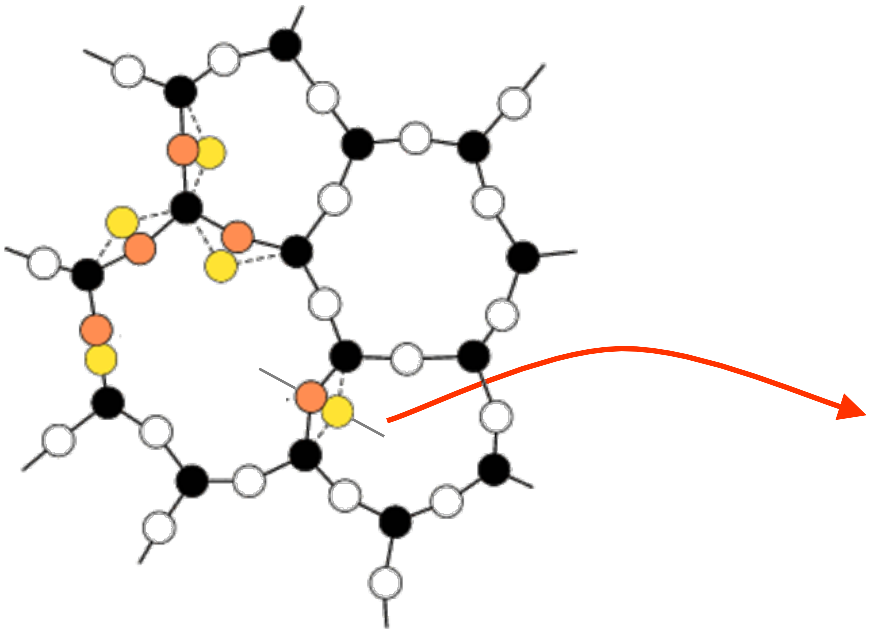
Specific Heat



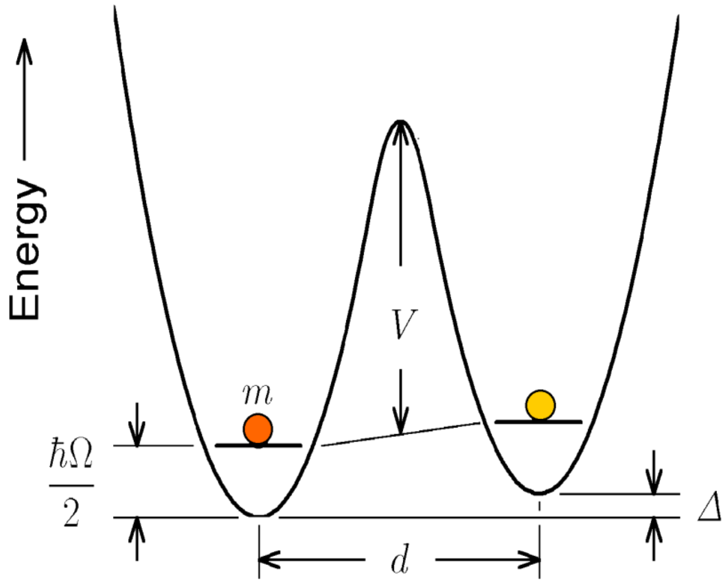
broad distribution of
low-energy excitations

J.C. Lasjaunias, et al., Solid State Commun. **17**, 1045 (1975)

Atomic Tunneling Systems in Glasses

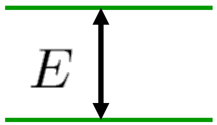


W.A. Phillips, J. Low. Temp. Phys. **7**, 351 (1972)
P.W. Anderson et al., Philos. Mag. **25**, 1 (1972)



energy splitting

$$E = \sqrt{\Delta_0^2 + \Delta^2}$$



tunnel splitting

$$\Delta_0 = \hbar\Omega\, \mathrm{e}^{-\lambda}$$

$$\lambda = \frac{d}{2\hbar}\sqrt{2mV}$$

distribution function

$$P(\lambda, \Delta)\, \mathrm{d}\lambda\, \mathrm{d}\Delta = \overline{P}\, \mathrm{d}\lambda\, \mathrm{d}\Delta$$

⇨ elastic, dielectric und thermal properities

Thermal Properties in the Tunneling Modell

Specific heat:

$$D(E) = \int_0^{\lambda_{\max}} P(E,\lambda)\,\mathrm{d}\lambda = \overline{P}\lambda_{\max}\ln\frac{2E}{\hbar\Omega} \approx D_0 = \text{const.}$$

$$P(E,\lambda)\,\mathrm{d}E\,\mathrm{d}\lambda = P(\Delta,\lambda)\,\frac{\partial\Delta}{\partial E}\,\mathrm{d}E\,\mathrm{d}\lambda$$

$$C_V = \frac{1}{6}D_0\pi^2k_\mathrm{B}^2\,T \propto T$$

Thermal Conductivity:

$$\Lambda = \frac{1}{3}C_V\,v\,\ell \qquad C_V \propto T^3$$

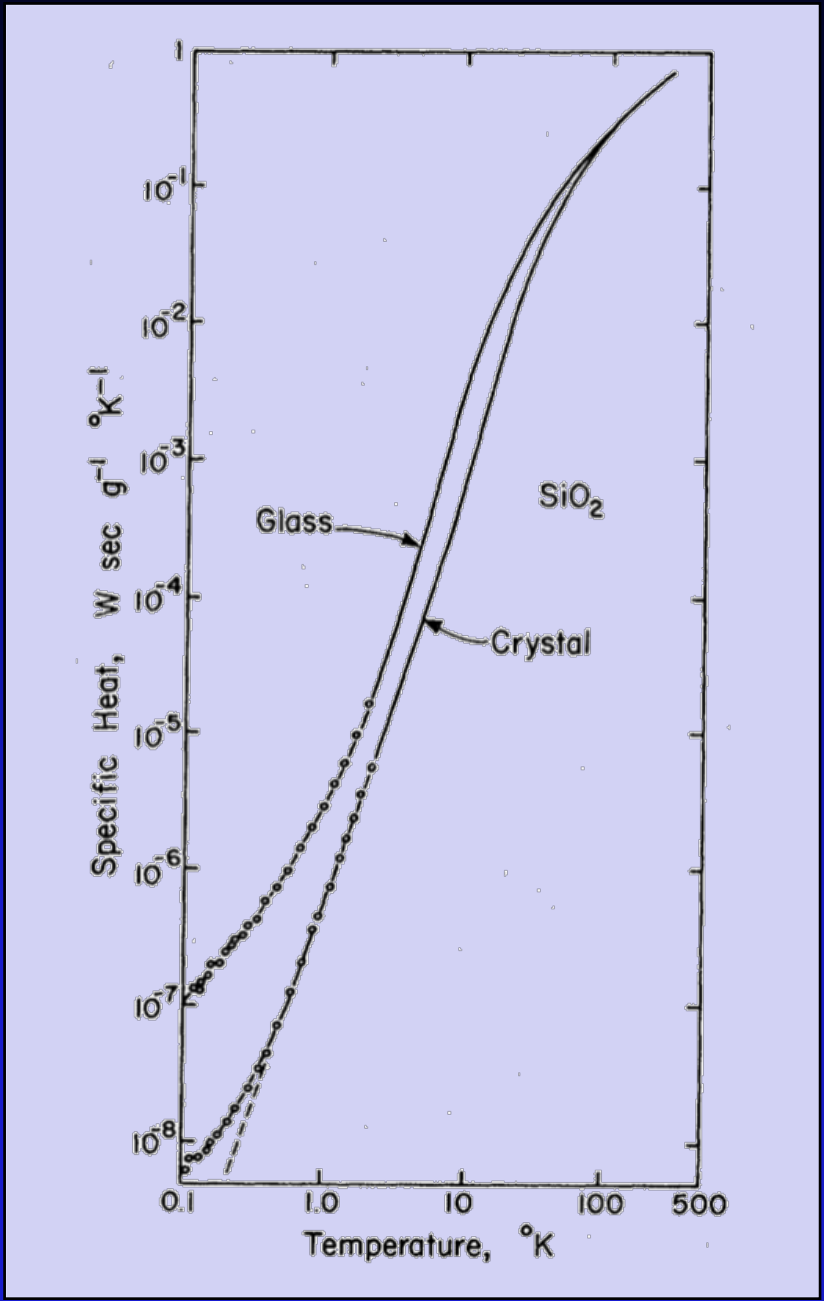
$$\ell^{-1} \propto E\,\tanh\left(\frac{E}{2k_\mathrm{B}T}\right) \propto \overline{\omega} \propto T$$

$$E = \hbar\overline{\omega} = k_\mathrm{B}T$$

$$\Lambda \propto T^2$$

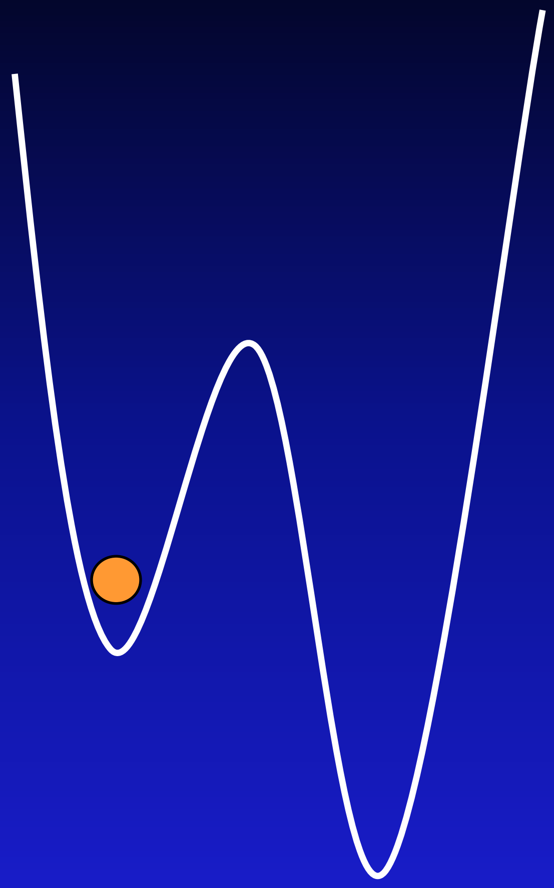
Specific Heat

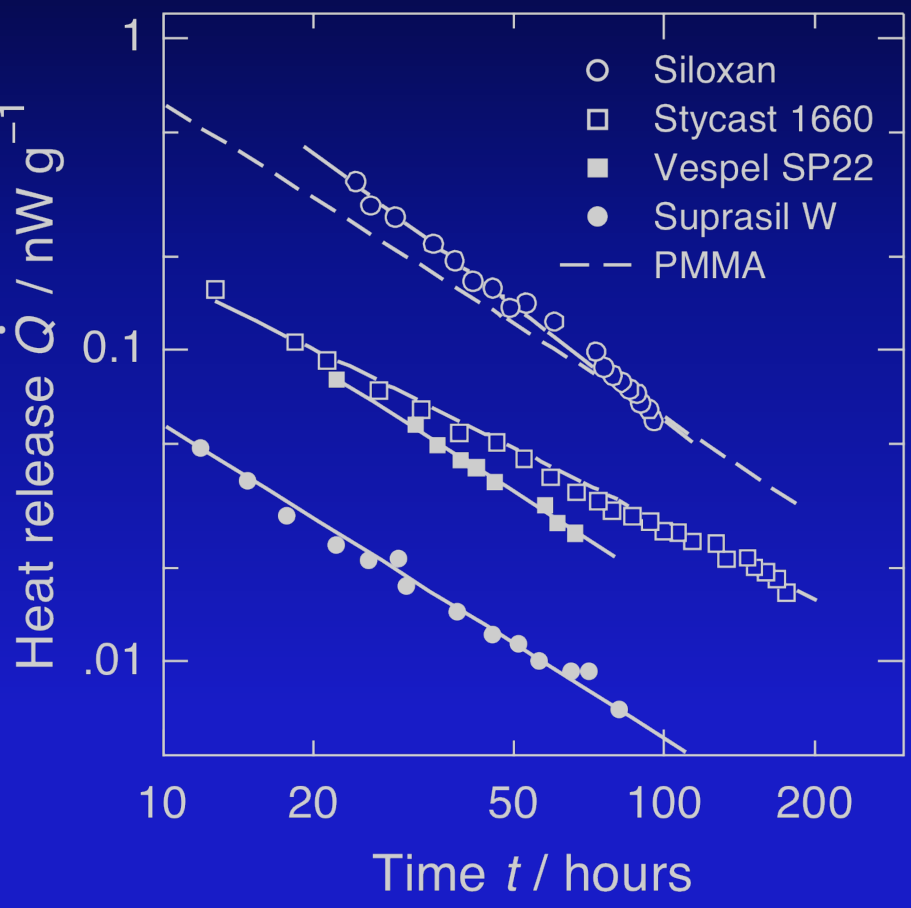
$$C = aT + bT^3 + C_{\text{Debye}}$$



R. C. Zeller and R. O. Pohl. Phys. Rev. B **4**, 2029 (1971)

Heat Release of Amorphous Solids

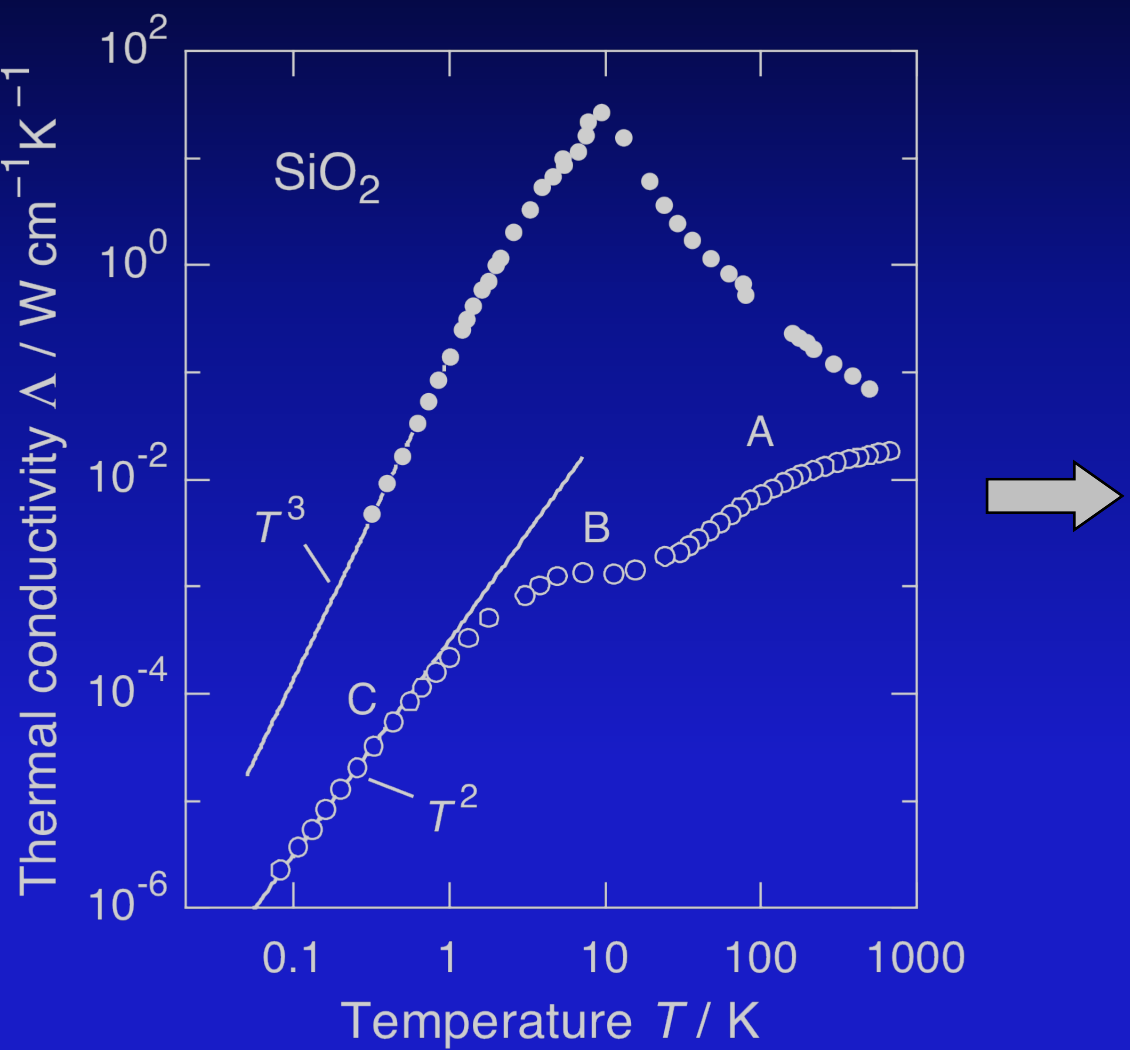




$$\dot{Q} = \frac{\pi^2 k_B^2}{24} P_0 (T_1^2 - T_0^2) \frac{1}{t}$$

M. Schwark, et al., J. Low Temp. Phys. **58**, 171 (1985)

Thermal Conductivity

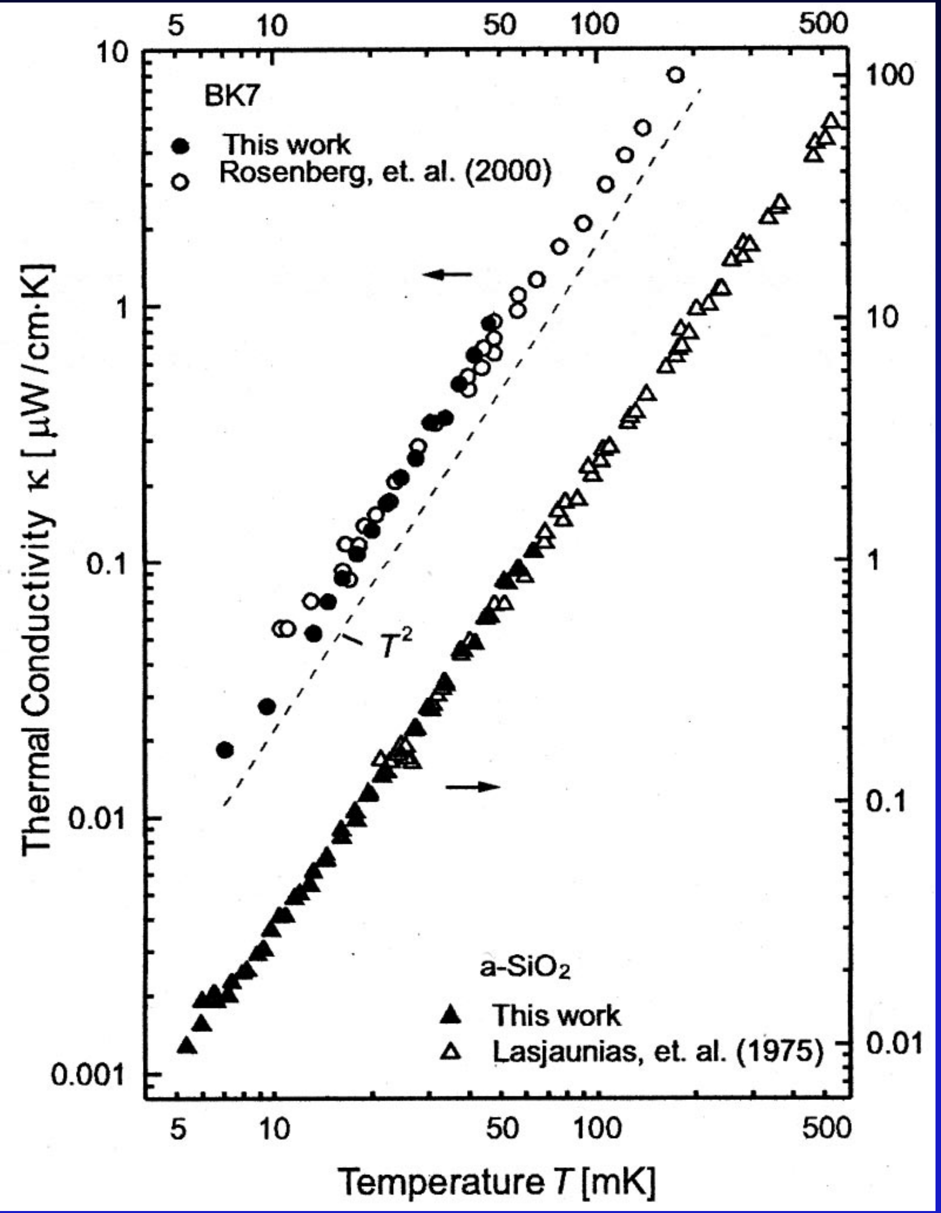


strong coupling to phonons

systems are localized

R. C. Zeller and R. O. Pohl. Phys. Rev. B **4**, 2029 (1971)

Thermal Conductivity

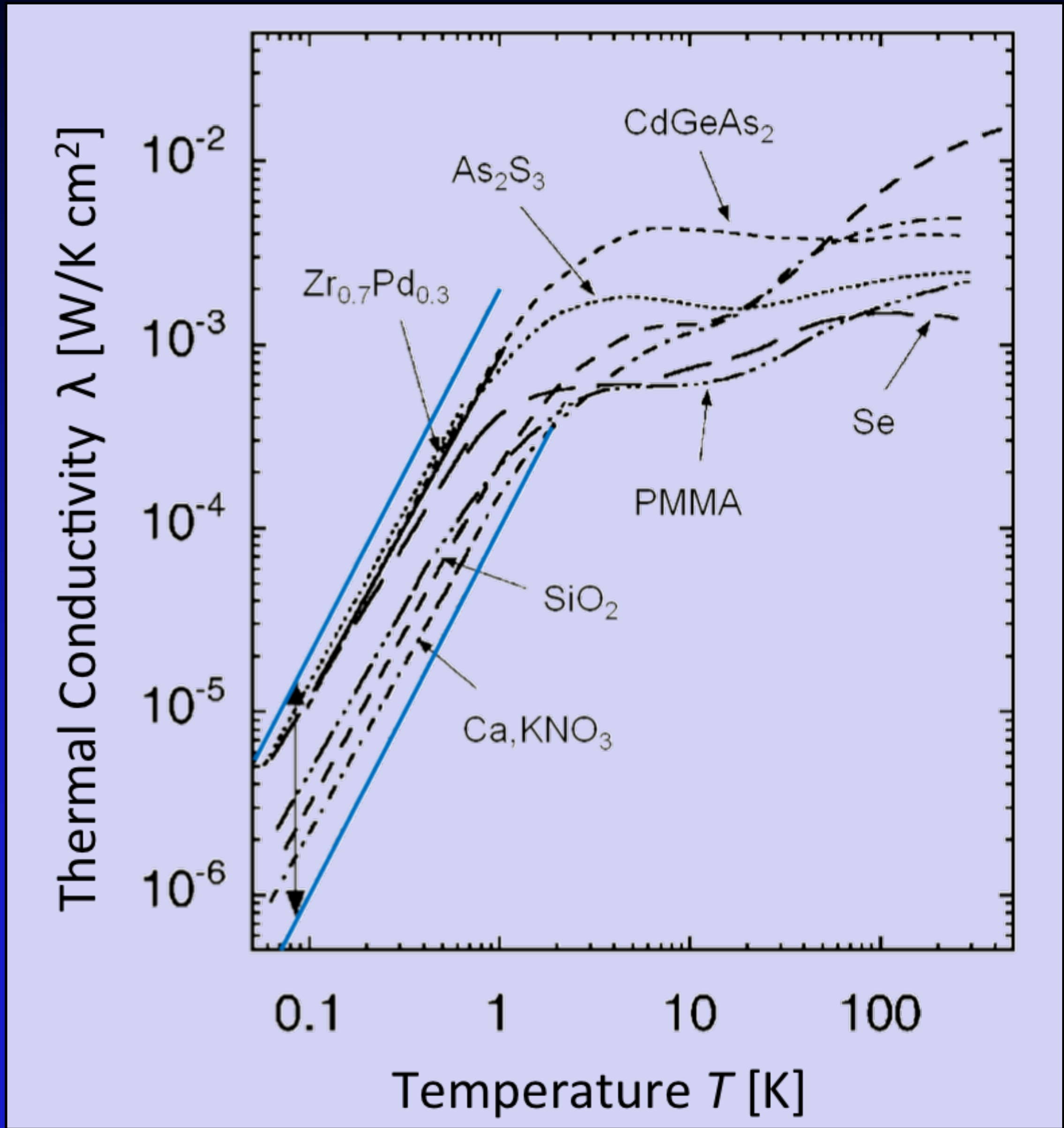


strong coupling to phonons

systems are localized

H.-Y. Hao, *et al.*, Rev. Sci. Instrum. **75**, 2718 (2004)

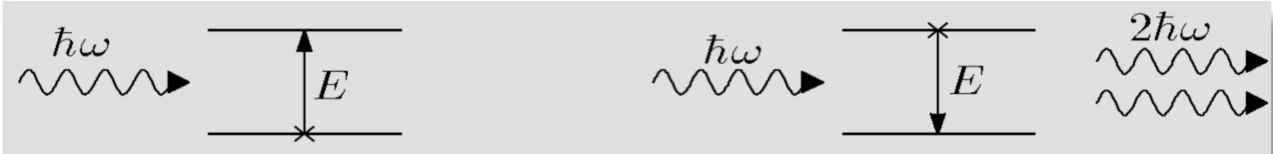
Universality



R. B. Stephens, Phys. Rev. B **8**, 2896 (1973)

Elastic and Dielectric Properties

resonant processes

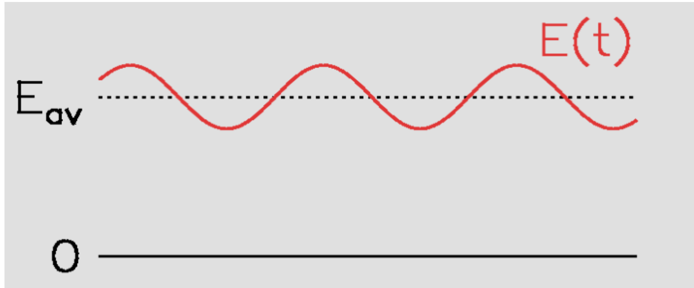


relaxational processes

⟶ modulation of Δ

$$\delta\Delta = 2\gamma\, e$$

$$\delta\Delta = 2\,\boldsymbol{p}\cdot\boldsymbol{F}$$



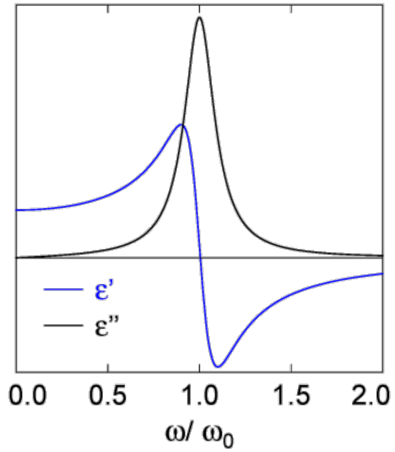
$T < 1$ K one-phonon relaxation → wide distribution even for fixed E

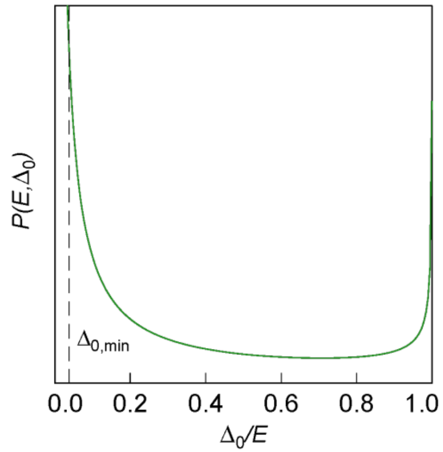
$$\tau_1 = \mathcal{A}\left(\frac{E}{\Delta_0}\right)^2 \frac{1}{E^3}\tanh\left(\frac{E}{2k_\mathrm{B}T}\right)$$

Dielectric Susceptibility in the STM

Resonant Part

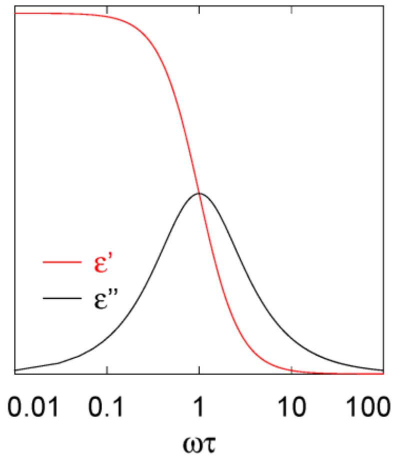
$$\frac{\delta\varepsilon'_{res}}{\varepsilon'} = \frac{2}{3\varepsilon_0\varepsilon'}\frac{p_0^2\overline{P}}{\hbar}\int_{\Delta_{0,\min}}^{E_{\max}} \mathrm{d}E \int_{\Delta_{0,\min}}^{E} \mathrm{d}\Delta_0 \left(\frac{\Delta_0^2}{E^2}\right) \tanh\left(\frac{E}{2k_\mathrm{B}T}\right)\left(\frac{(\omega+\omega_0)\tau_2^2}{1+(\omega+\omega_0)^2\tau_2^2} - \frac{(\omega-\omega_0)\tau_2^2}{1+(\omega-\omega_0)^2\tau_2^2}\right) \frac{E}{\Delta_0\sqrt{E^2-\Delta_0^2}}$$





Relaxational Part

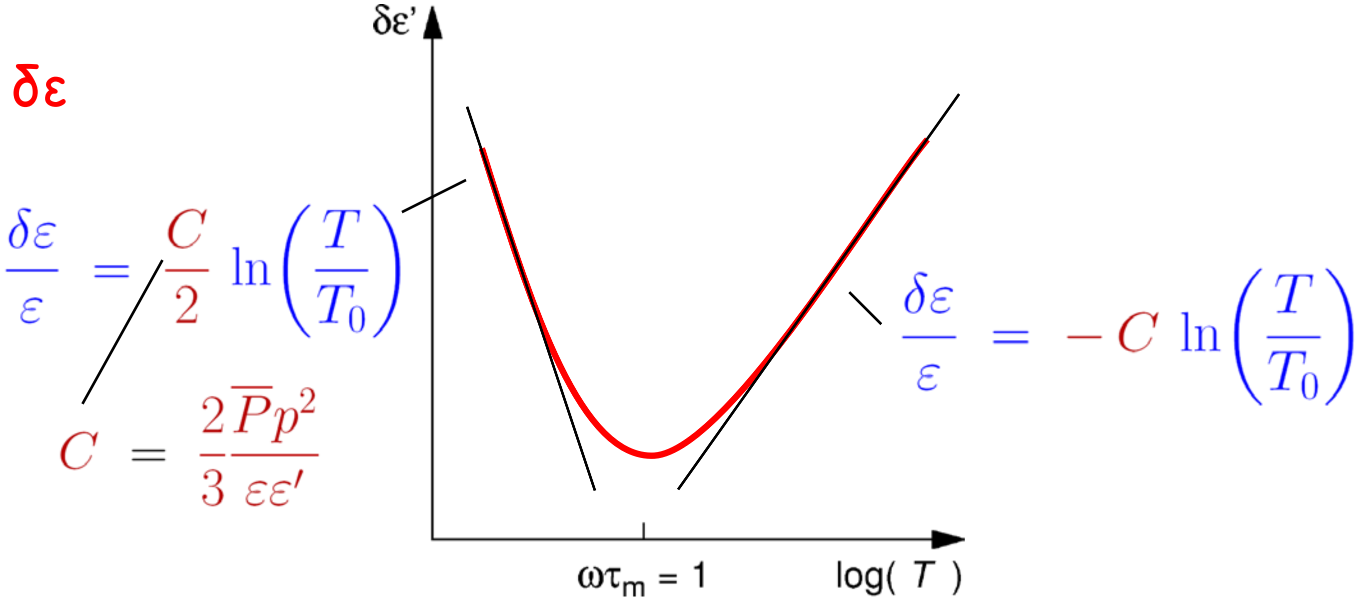
$$\frac{\delta\varepsilon'_{rel}}{\varepsilon'} = \frac{2}{3\varepsilon_0\varepsilon'}\frac{p_0^2\overline{P}}{k_\mathrm{B}T}\int_{\Delta_{0,\min}}^{E_{\max}} \mathrm{d}E \int_{\Delta_{0,\min}}^{E} \mathrm{d}\Delta_0 \left(1-\frac{\Delta_0^2}{E^2}\right) \mathrm{sech}^2\left(\frac{E}{2k_\mathrm{B}T}\right)\frac{1}{1+\omega^2\tau_1^2}\frac{E}{\Delta_0\sqrt{E^2-\Delta_0^2}}$$



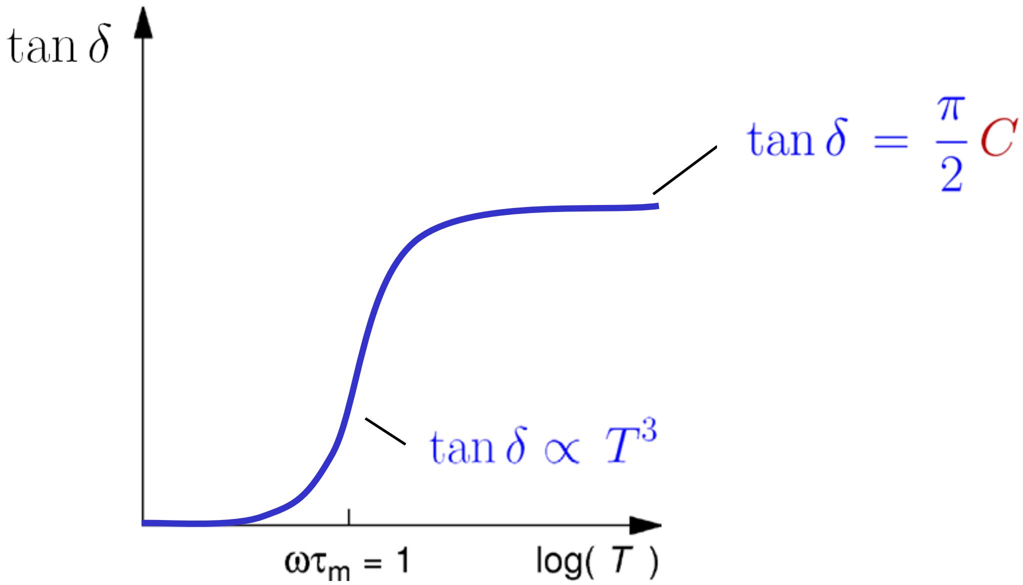
$$\tau_\mathrm{tot}^{-1} = \tau_\mathrm{1ph}^{-1} + \tau_\mathrm{2ph}^{-1} + \tau_?^{-1} + \ldots$$

Dielectric Constant and Dielectric Loss

Dielectric constant: δε



Dielectric loss: tan δ

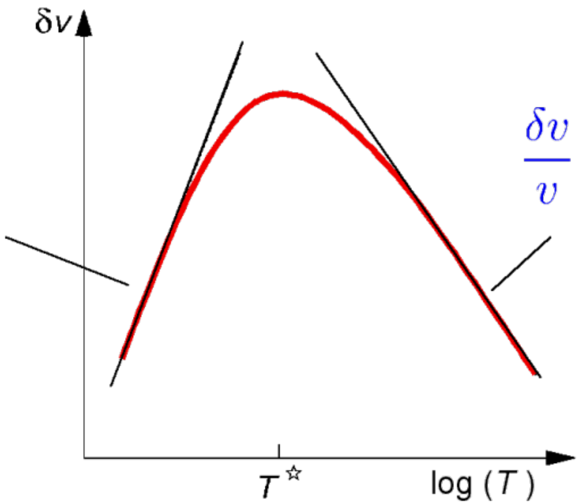


Sound Velocity and Internal Friction

sound velocity $\delta v / v$

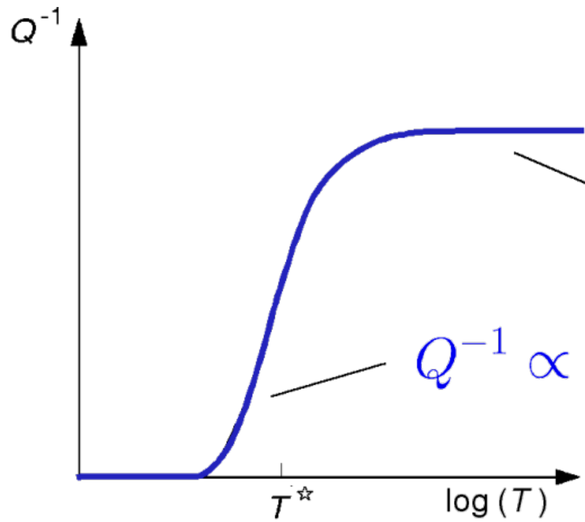
$$\frac{\delta v}{v} = C \ln\left(\frac{T}{T_0}\right)$$

$$C = \frac{\overline{P}\gamma^2}{\rho v^2}$$



$$\frac{\delta v}{v} = -\frac{C}{2} \ln\left(\frac{T}{T_0}\right)$$

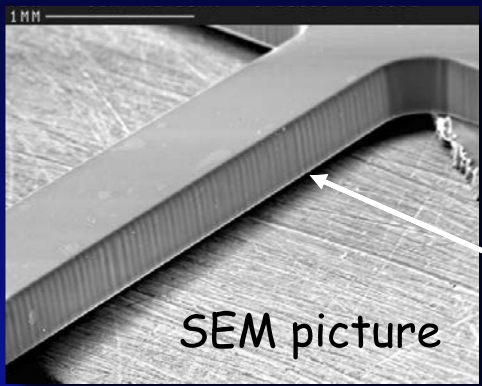
internal friction Q^{-1}



$$Q^{-1} = \frac{\pi}{2} C$$

$$Q^{-1} \propto T^3$$

Elastic Measurements with Mechanical Oscillators

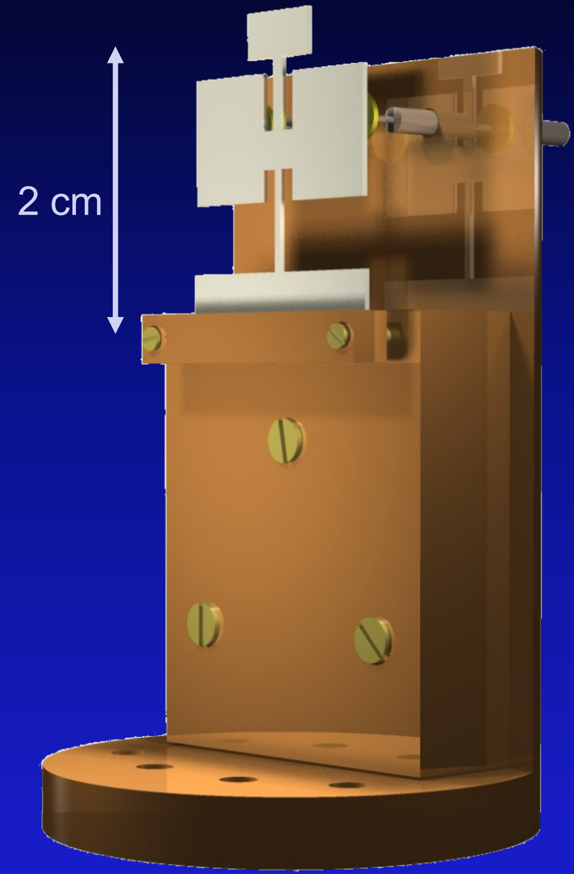


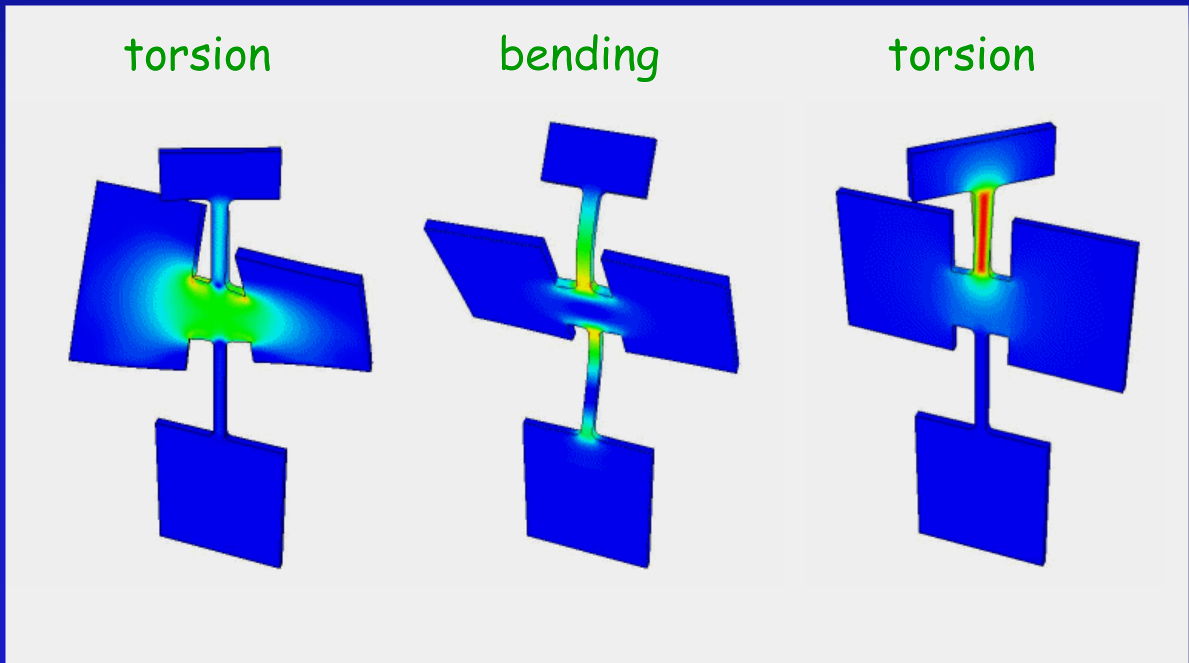
1 μm silver film

→ good thermalization

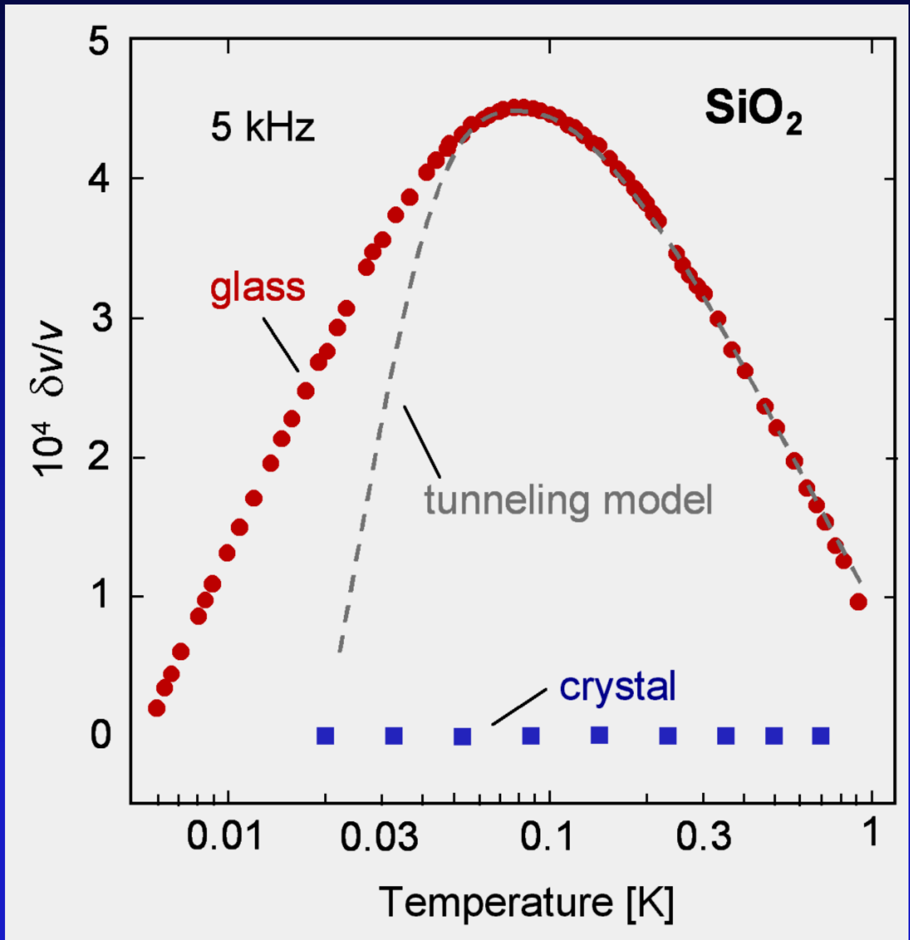
laser-cut glass

neck





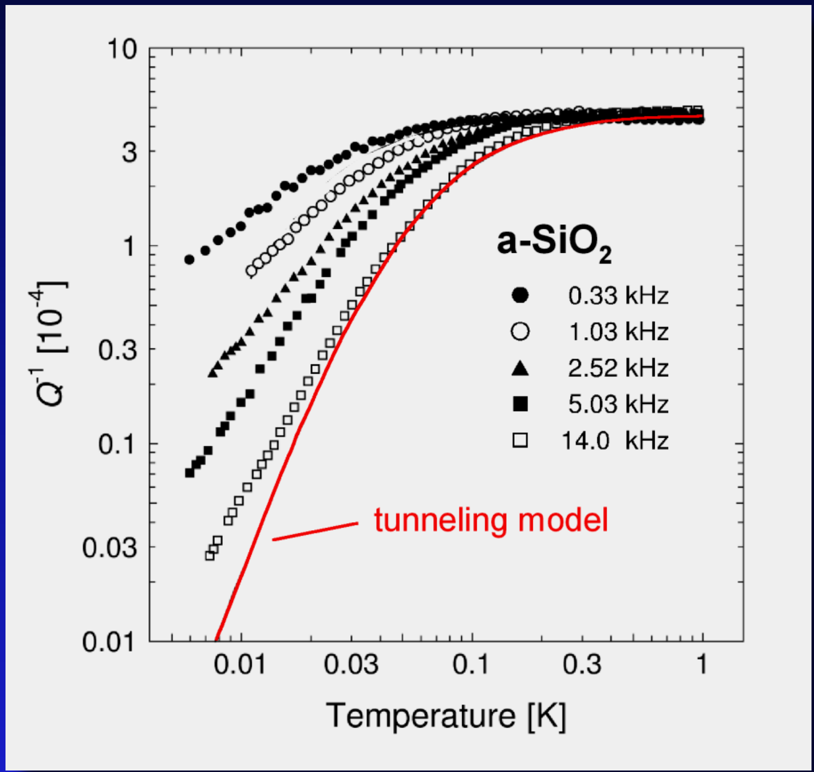
Sound Velocity

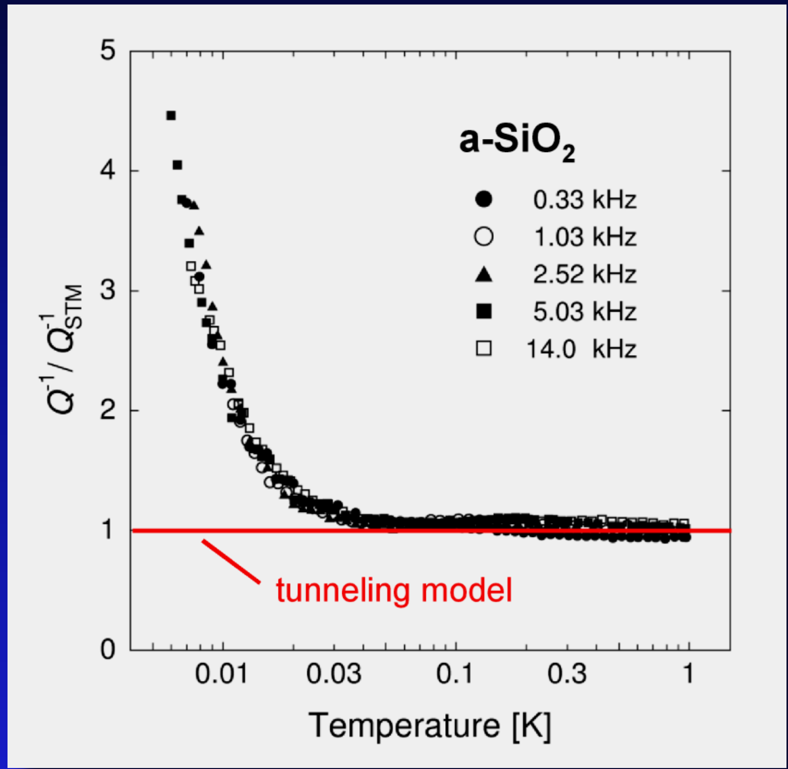


discrepancy at low temperatures

J. Classen, T. Burkert, C. Enss, S. Hunklinger
Phys. Rev. Lett. **84**, 2176 (2000)

Internal Friction





$T < 30$ mK

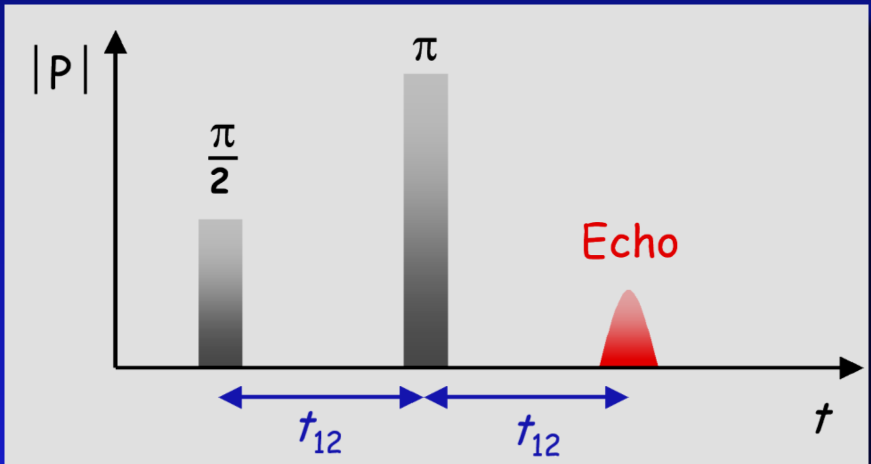
J. Classen, T. Burkert, C. Enss, S. Hunklinger
Phys. Rev. Lett. **84**, 2176 (2000)

→ additional relaxation channel

Coherent Properties

$$t \ll \tau_1, \tau_2 \;\rightarrow\; \infty$$ → coherent regime

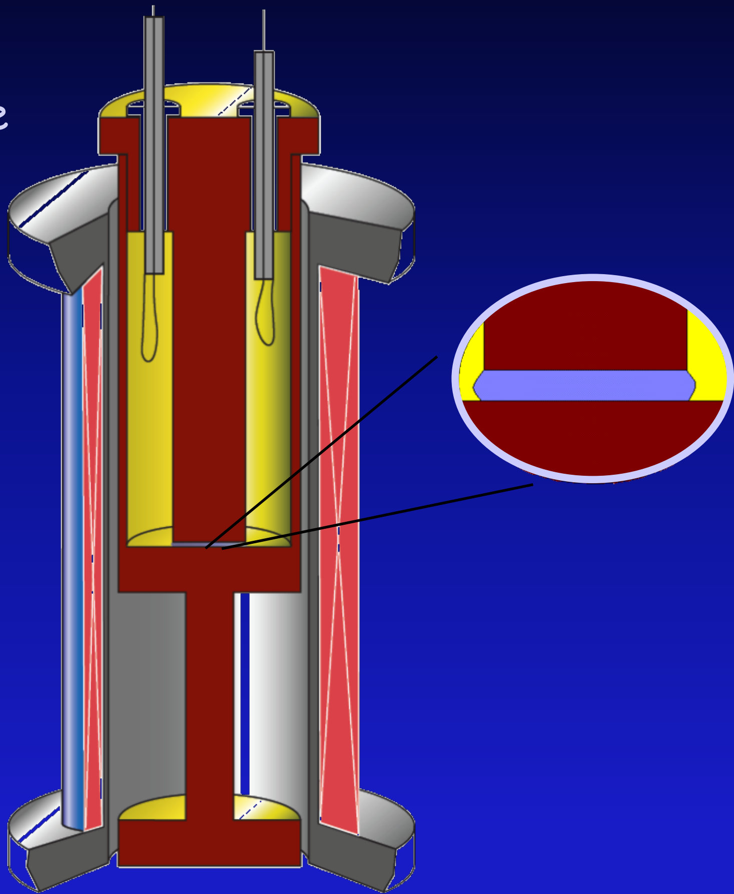
two-pulse polarization echoes:



$$\Theta_\mathrm{p} = \Omega_\mathrm{R} t_\mathrm{p}$$

Rabi frequency

$$\Omega_\mathrm{R} = \frac{1}{\hbar}\frac{\Delta_0}{E}\,\boldsymbol{p}\cdot\boldsymbol{F}$$



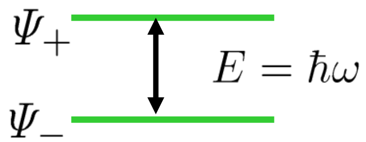
microwave cavity

1 GHz → 50 mK

Echo – Theoretical Background I

coherent regime: $t \ll \tau_1, \tau_2 \;\rightarrow\; \infty$

two level approximation:



applied field: $\boldsymbol{F} = \boldsymbol{F}_0 \left(\mathrm{e}^{\mathrm{i}\omega_\mathrm{r} t} + \mathrm{e}^{-\mathrm{i}\omega_\mathrm{r} t} \right)$

Schrödinger equation: $\mathrm{i}\hbar \frac{\partial \Psi}{\partial t} = \left[H_0 + H_\mathrm{S} \right] \Psi$ mit $H_\mathrm{S} = 2 \frac{\Delta_0}{E} \boldsymbol{p} \cdot \boldsymbol{F}$

ansatz: $\Psi(t) = a_1(t)\, \Psi_- + a_2(t)\, \Psi_+$

$$\left.\begin{aligned} a_1(t) &= \cos\left(\Omega_\mathrm{R} t\right)\, \mathrm{e}^{-\mathrm{i}\omega_\mathrm{r} t} \\ a_2(t) &= -\mathrm{i} \sin\left(\Omega_\mathrm{R} t\right)\, \mathrm{e}^{-\mathrm{i}\omega_\mathrm{r} t} \end{aligned}\right.$$

Rabi frequency: $\Omega_\mathrm{R} = \frac{1}{\hbar} \frac{\Delta_0}{E}\, \boldsymbol{p} \cdot \boldsymbol{F}_0$

Echo —Theoretical Background II

polarisation vector:

$$\boldsymbol{P} = \begin{pmatrix} ab^* + ba^* \\ \mathrm{i}(ab^* - ba^*) \\ aa^* - bb^* \end{pmatrix} = \begin{pmatrix} -\sin(\Omega_\mathrm{R} t)\sin(\omega t) \\ \sin(\Omega_\mathrm{R} t)\cos(\omega t) \\ \cos(\Omega_\mathrm{R} t) \end{pmatrix} = \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix}$$

Bloch equations:

$$\frac{\mathrm{d}\langle S_x\rangle}{\mathrm{d}t} = -\frac{2}{\hbar}\left(\frac{E}{2} + \frac{\Delta}{E}\boldsymbol{p}\cdot\boldsymbol{F}\right)\langle S_y\rangle - \frac{\langle S_x\rangle}{\tau_2}$$

$$\frac{\mathrm{d}\langle S_y\rangle}{\mathrm{d}t} = \frac{2}{\hbar}\left(\frac{E}{2} + \frac{\Delta}{E}\boldsymbol{p}\cdot\boldsymbol{F}\right)\langle S_x\rangle - \frac{2}{\hbar}\left(\frac{\Delta_0}{E}\boldsymbol{p}\cdot\boldsymbol{F}\right)\langle S_z\rangle - \frac{\langle S_x\rangle}{\tau_2}$$

$$\frac{\mathrm{d}\langle S_z\rangle}{\mathrm{d}t} = \frac{2}{\hbar}\left(\frac{\Delta_0}{E}\boldsymbol{p}\cdot\boldsymbol{F}\right)\langle S_y\rangle - \frac{\left[\langle S_z\rangle - S_z^0(\boldsymbol{F})\right]}{\tau_1}$$

τ_1 energy relaxation — $T < 1\,\mathrm{K}$ ⟶ one phonon prozess

..... ?

τ_2 phase coherence time —

τ_1 processes

spectral diffusion ⟵

spin diffusion

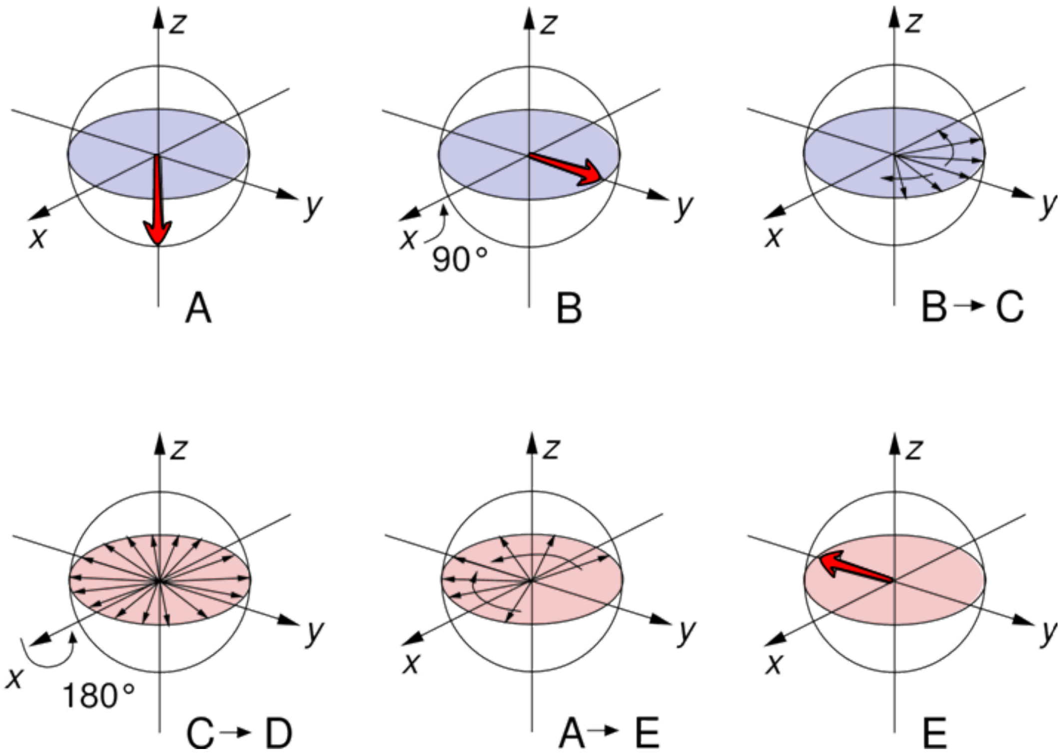
..... ?

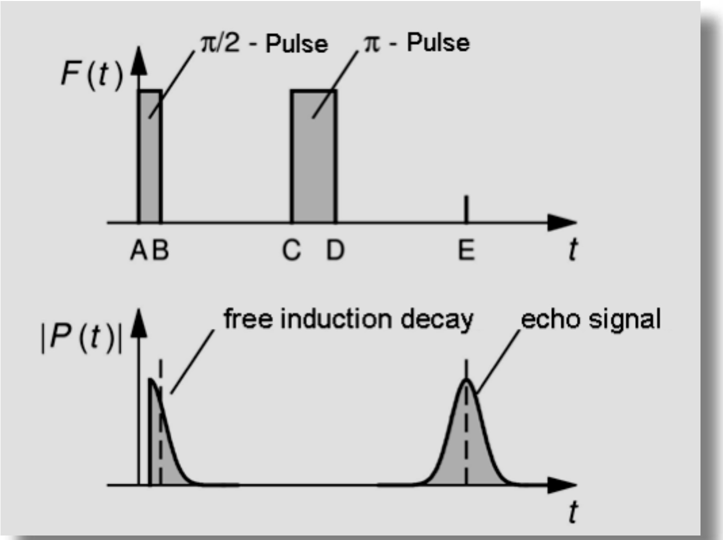
Two Pulse Echo

polarization vector

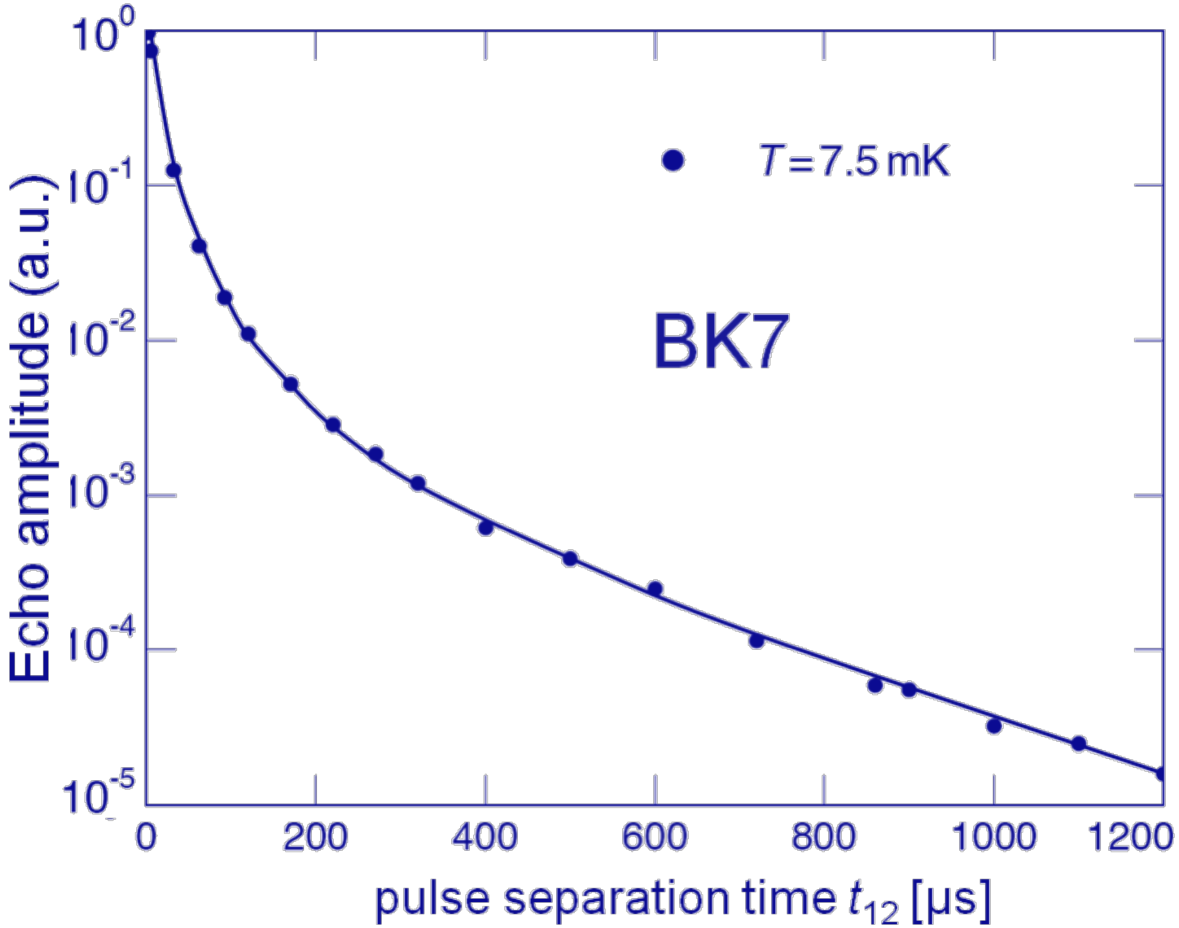
$$\boldsymbol{P} = (S_x, S_y, S_z)$$

rotating frame



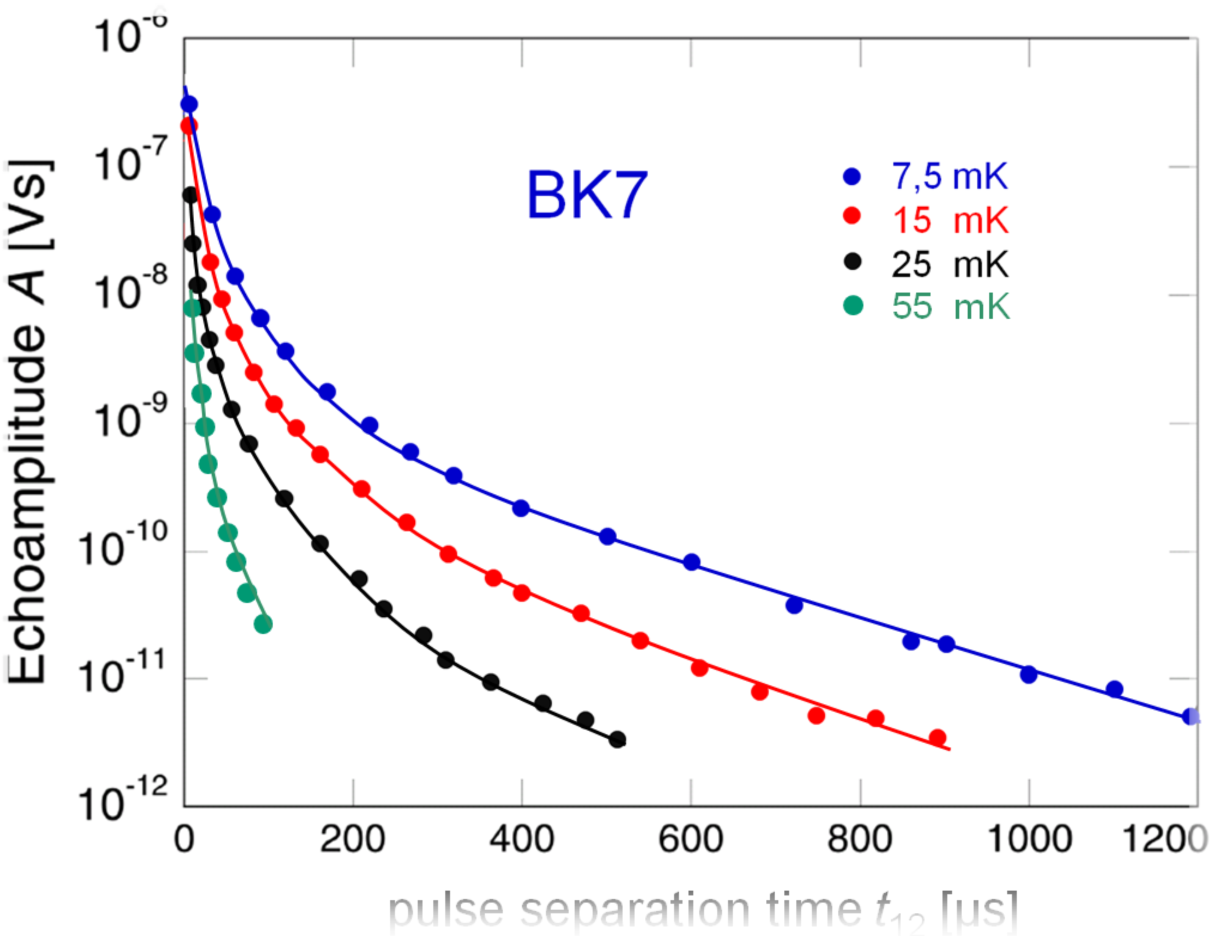


Echo Decay

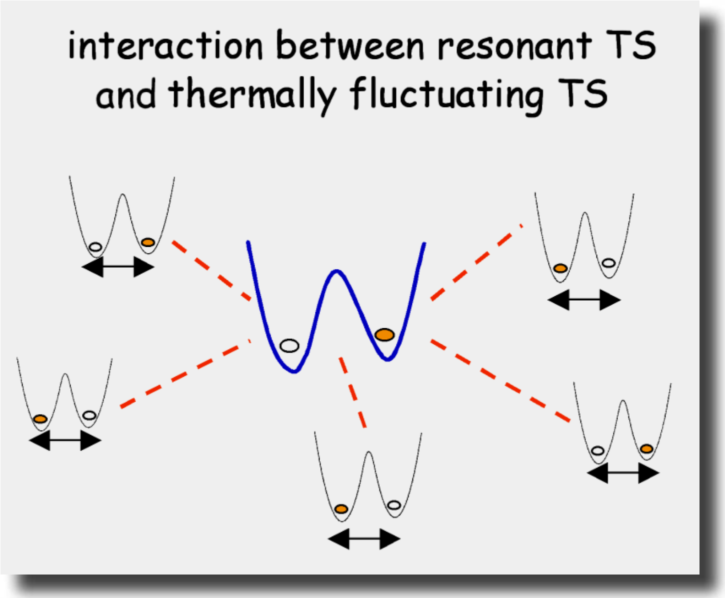


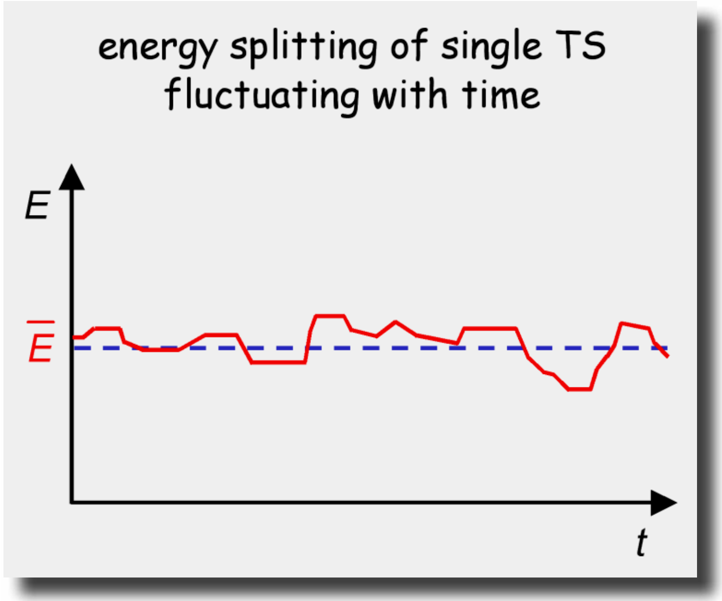
- time window 1200 µs
- sensitivity: five orders of magnitude

Temperature Dependence



Spectral Diffusion





- short time limit (no flip limit): $t_{12} \ll \tau_{\min}$

  $\longrightarrow$ $$A(2t_{12}) = A(0)\,\mathrm{e}^{-(2t_{12}/\tau_2)^2}$$ Gaussian decay

- long time limit (multiple flip limit): $t_{12} \gg \tau_{\min}$

  $\longrightarrow$ $$A(2t_{12}) = A(0)\,\mathrm{e}^{-2t_{12}/\tau_2}$$ exponential decay

Temperature Dependence

short time limit (no flip limit): $t_{12} \ll \tau_{\min}$

$$A(2t_{12},T) \;\propto\; \tanh\left(\frac{E}{2k_{\rm B}T}\right) \mathrm{e}^{-m_0 T^4 (\Delta/E) t_{12}^2}, = \; A_0(T)\, \mathrm{e}^{-m(T)(\Delta/E) t_{12}^2}$$

$$\tau_2 \;\propto 1/\sqrt{m(T)} \;\propto\; T^{-2}$$

long time limit (multiple flip limit): $t_{12} \gg \tau_{\min}$

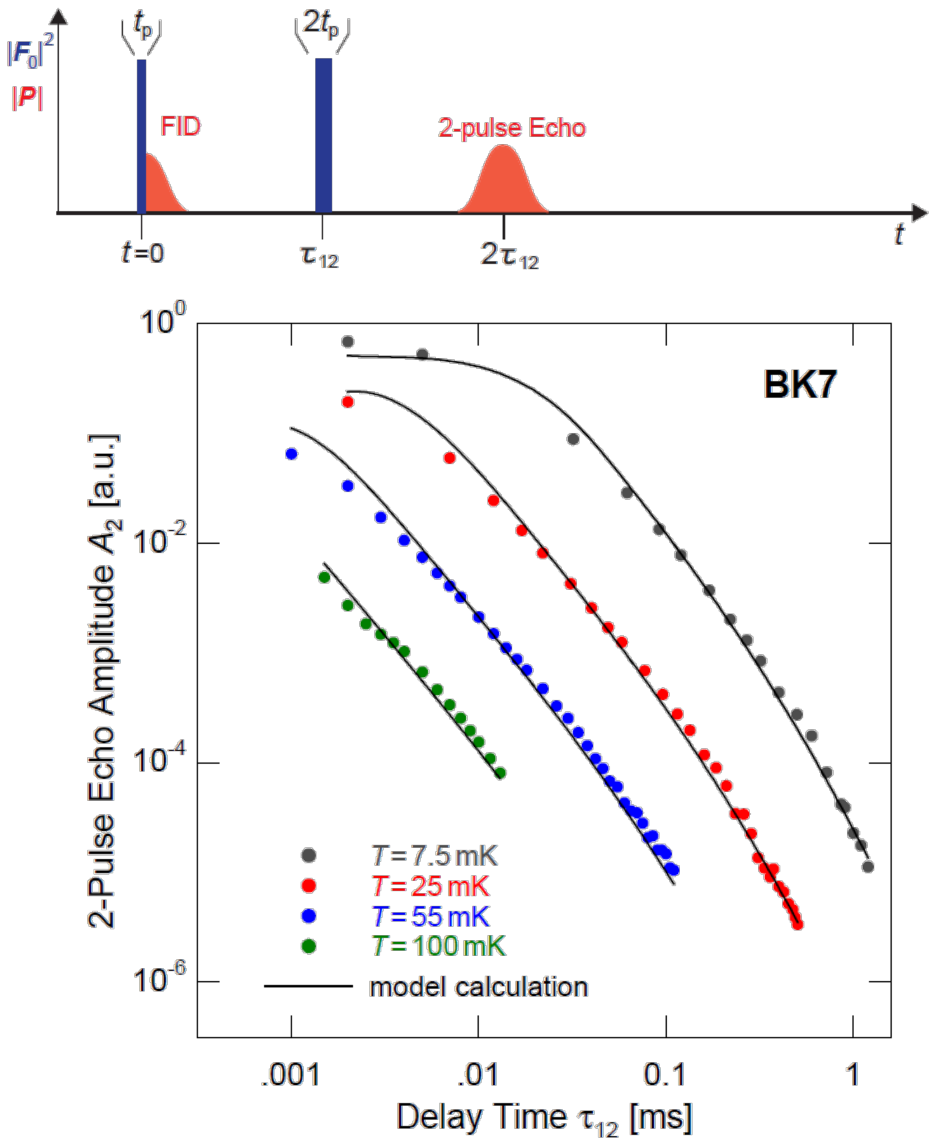
$$A(2t_{12},T) \;\propto A_0(T)\, \mathrm{e}^{-2t_{12}^2/\tau_2}$$

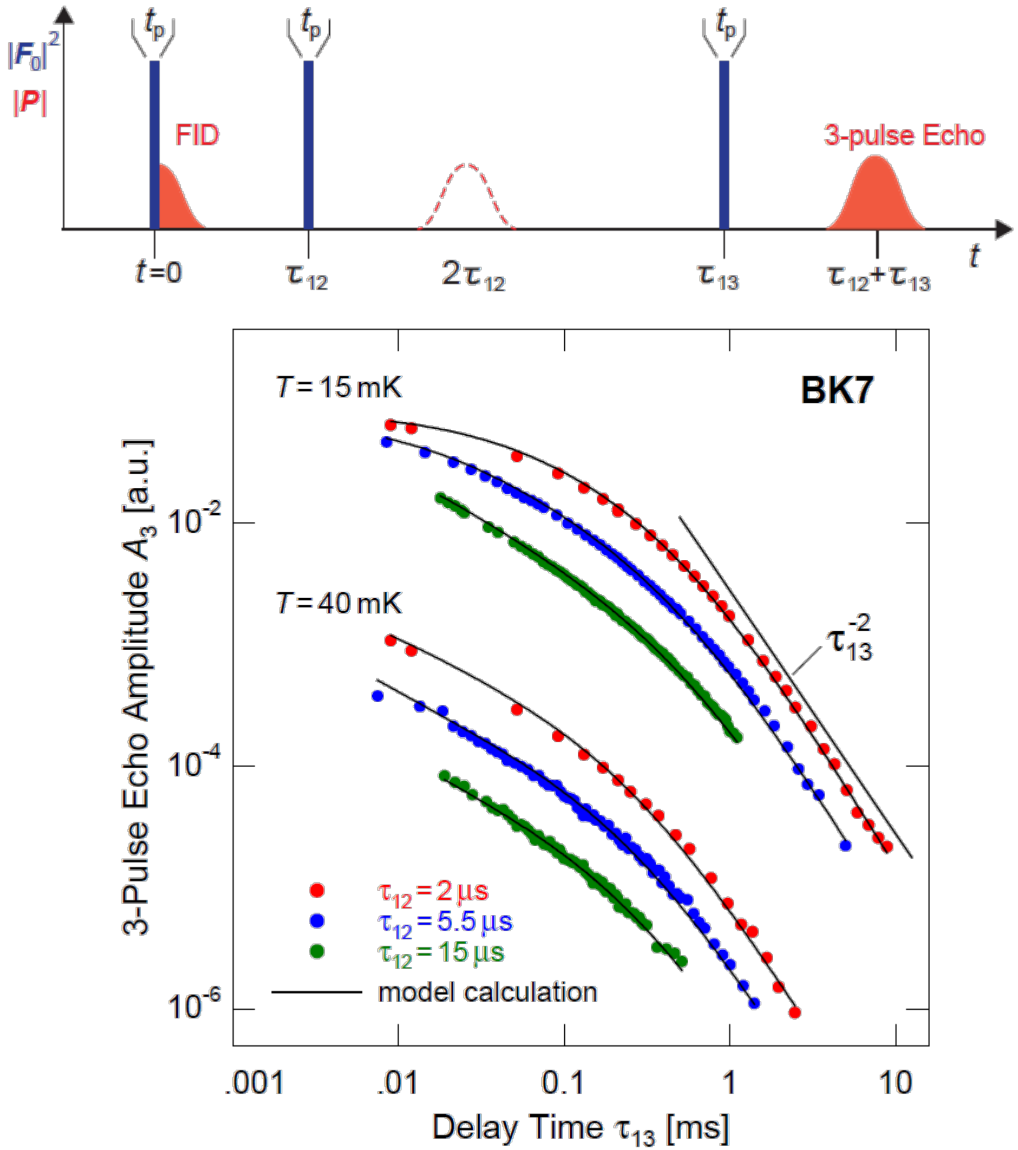
$$\tau_2 \;\propto\; T^{-1}$$

Theoretical papers:

P. Hu, S.R. Hartmann, PRB **9**, 1 (1974)
J.L. Black, B.I. Halperin, PRB **16**, 2879 (1976)
P. Hu, L.R. Walker, PRB **18**, 1300 (1978)
R. Maynard, R. Rammal, R. Suchail, J. Phys. Paris Lett. **41**, L-291 (1980)
B.D. Laikhtman, PRB **31**, 400 (1985)
Yu.M. Galperin, V.L. Gurevich, D.A. Parshin, PRB **37**, 10339 (1988)

Temperature Dependence





A. L. Burin, J. M. Leveritt III, G. Ruyters, C. Schötz, M. Bazrafshan, P. Faßl, M. von Schickfus, A. Fleischmann, C. Enss, Europhys. Lett. **104**, 57006 (2013)

Phase Jump Rotary Echoes

coherent regime:

$$t \ll \tau_1, \tau_2 \;\rightarrow\; \infty$$

applied field:

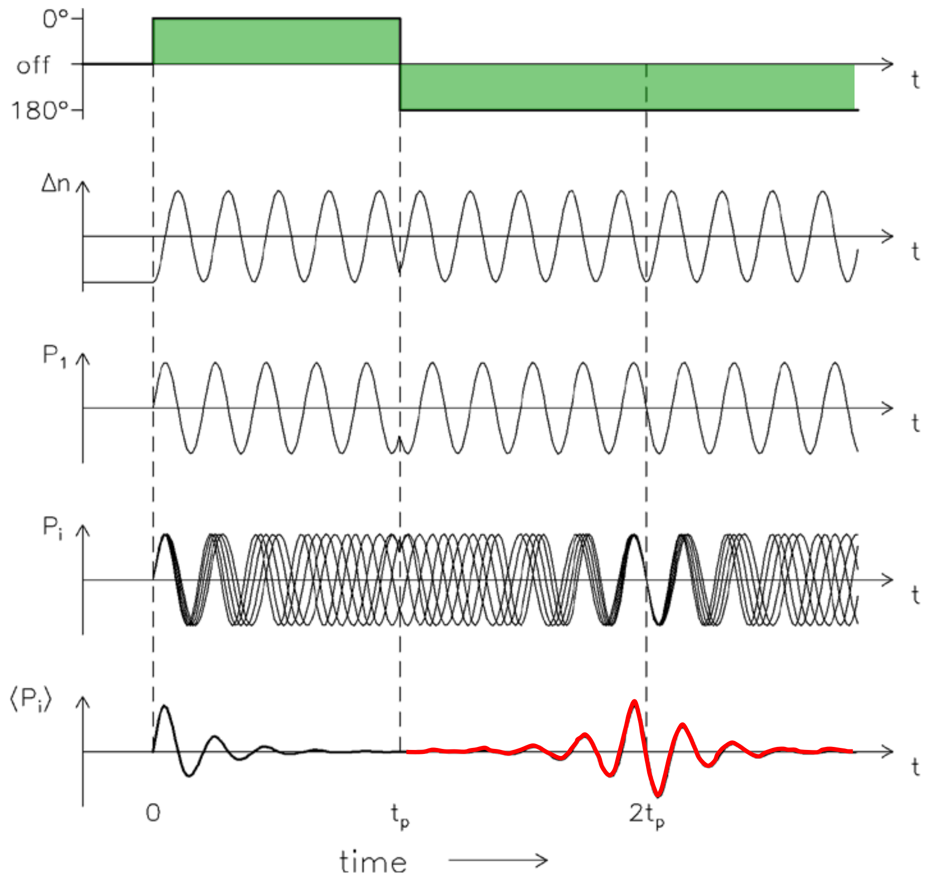
$$\boldsymbol{F} = 2\,\boldsymbol{F}_0 \cos(\omega t)$$

occupation number difference

$$\Delta n = \cos\left(\Omega_{\rm R} t\right)$$

↓

periodic change of polarisation

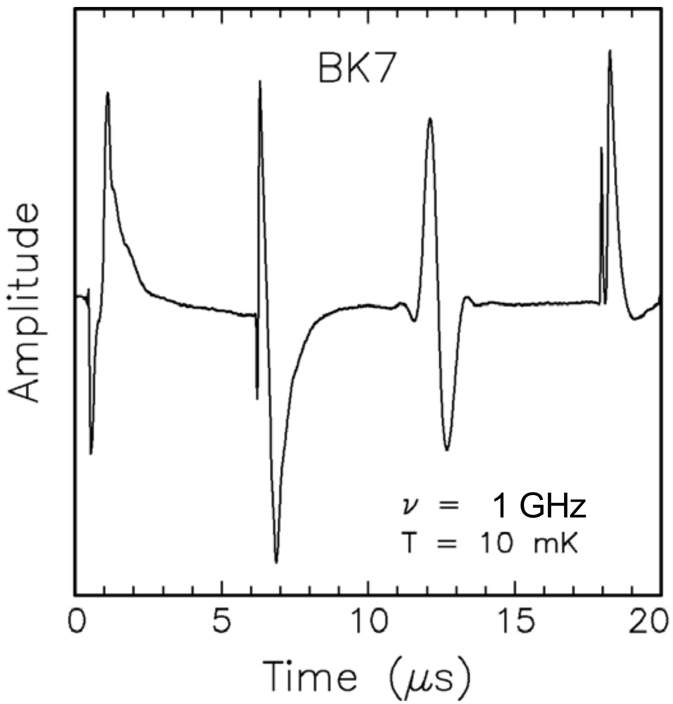


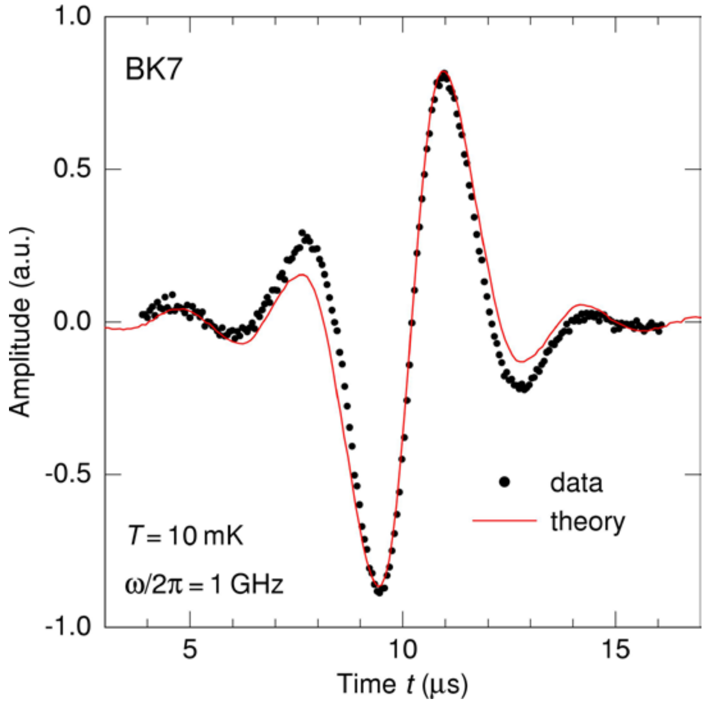
$$A_{\rm r}(t) \propto \int_0^{+\infty}\!\!\int_0^{E} P(E,\Delta_0)\,\frac{\Delta_0}{E}\,\frac{\boldsymbol{p}\cdot\boldsymbol{F}_0}{|\boldsymbol{F}_0|}\left[\frac{\Omega_{\rm R}}{\Omega}\right]^3 {\rm e}^{-\beta}\sin\left[\Omega\,(t-2t_{\rm p})\right]\,{\rm d}\Delta_0\,{\rm d}E$$

with $\Omega = \sqrt{\Omega_{\rm R}^2 - \omega_{\rm d}^2}$ $\qquad \omega_{\rm d} = \omega_{\rm r} - \omega$ $\qquad \beta = \frac{1}{\Omega^2}\left(\frac{\omega_{\rm d}^2}{\tau_2} + \frac{\Omega_{\rm R}^2}{2\tau_1} + \frac{\Omega_{\rm R}^2}{2\tau_2}\right)$

Phase Jump Rotary Echoes

BK7: (SiO_2, B_2O_3, NaO, CaO, ...)





few oscillations ⟶ broad distribution of Ω_R

Fit: tunneling model distribution
uniform distribution
fixed dipole moment

C. Enss, R. Weis, S. Ludwig, S. Hunklinger, Czech. J. Phys. **46**, 3287 (1996)